# Huggett (1996) Model

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Econ890

May 4, 2021

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## Model Features

We compute a simplified version of Huggett (1996)
Households:

- Live for many ($a_D$) periods
- Earnings are random
- Age of retirement is fixed ($a_R$).

Government:

- Pays transfers to retired households (annuitized income in the data)

Simplifying assumptions:

- Partial equilibrium
- No random mortality
- No intergenerational links
- No labor-leisure choice.

# Model Primitives

### Demographics

Households live for exactly $a_D$ periods.
Total mass of households is $N = 1$.

### Preferences

$$\mathbb{E} \sum_{a=1}^{a_D} \beta^a u\left(c_a\right) \tag{1}$$

### Endowments

Working agents are endowed with labor efficiency $\eta_a e_a$
$\eta_a$: age-efficiency profile
$e_a$: labor efficiency (wage) shock

- governed by a Markov chain: $\Pr(e' = \varepsilon_k | \, e = \varepsilon_j) = P_e(k, j)$.
- new agents draw labor endowments from a fixed distribution.
- number of states: $N_e$.

### Markets

Labor: wage $w$
Capital rental: $r$
Goods: numeraire.

## Government

Taxes labor income: $T = \tau_w \, w \, L$.
Pays retirement transfers: $\varpi\left(a\right) = \varpi$ if $a > a_R$.

# Household Problem

Exogenous state variables are age $a$ and labor endowment $e$: $s = (a, e)$.
Endogenous state variable: wealth $k$.
Borrowing constraint: $k \geq 0$.

**Sequence problem**

$$\max\ E \sum_{a=1}^{a_D} \beta^a\, u\left(c_a\right)$$

subject to

$$k_{a+1} = y_a - c_a \geq 0$$

$$y_a = R\, k_a + w(1-\tau_w)\eta_a e_a + \varpi\left(a\right) \tag{2}$$

## Household Dynamic Program

$$V\left(k,s\right) = \max u\left(y\left(k,s\right) - k'\right) + \beta \mathbb{E} V\left(k',s'\right) \tag{3}$$

with

$$y\left(k,s\right) = Rk + w\left(1-\tau_w\right)\eta_a e + \varpi\left(s\right) \tag{4}$$

subject to $k' \geq 0$.

Euler equation:

$$u'(c) \geq \beta R \mathbb{E} u'(c') \tag{5}$$

with equality if $k' > 0$.

## Household Solution

*Solution* is a consumption function $c\left(k,a,e\right)$ which satisfies

$$u'(c\left[k,a,e\right]) \geq \beta R \sum_{e'} P_e(e,e') u'\left(c\left[y - c\left(k,a,e\right), a+1, e'\right]\right)$$

In the last period, consume all income:

$$c\left(k,a_D,e\right) = y(k,a_D,e) \tag{6}$$

# Parameter Choices

We simply take parameters from Huggett's paper.

Exercise: implement the calibration.

Calibrated parameters: $\beta, \delta, A$.

Calibration targets: $K/Y$, $w = 1$, $R$.

### Preferences

$u(c) = c^{1-\sigma}/(1-\sigma)$
$\sigma = 2.$
Choose $\beta$ to match $K/Y = 2.9/\lambda$.

### Demographics

Households live from age 20 to 79.
Work from 20 to 64 (45 years).
Retire for 15 years.

### Prices

- $w = 1$
- $R = 1.04$

### Government

$\tau_w = 0.4$ (Trostel 1993).
Set transfers to 40% of average earnings.

- This can be done before computing equilibrium.

## Labor Endowments

Can be set before equilibrium is computed.

Empirical studies estimate AR(1) processes for [log earnings] minus [mean log earnings, $\eta_a$] by age.
New agents draw endowments from exogenous distribution:
$\ln(e_1) \backsim N(0, \sigma_1^2)$.
Over time, endowments are drawn from an AR(1):
$\ln(e_a) = \eta_a + \gamma \ln(e_{a-1}) + \varepsilon_a$.
$\ln(\varepsilon_a) \backsim N(0, \sigma_\varepsilon^2)$.
We follow Huggett (1996):

- $\sigma_1^2 = 0.38$, $\sigma_\varepsilon^2 = 0.045$, $\gamma = 0.96$.

- Approximate the AR(1) on a grid of 18 states equally spaced over $\pm 4\,\sigma_1$.
- Add an additional state at $+6\,\sigma_1$ to capture skewness of earnings distribution.
- Use Tauchen (1986) (we have code for that)

### Age-efficiency profile

From PSID data (Huggett, 1996)

## References


Huggett, M. (1996): "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 38, 469–494.

Tauchen, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 20, 177–181.