Wealth Distribution: Extensions

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Introduction

We have seen that the stochastic life-cycle model goes a long way towards accounting for U.S. wealth inequality.

But the model fails to account for the concentration of wealth within the top 5% or 1% of the population.

Two candidate solutions:

1. Change the labor earnings process
   (a) there are not enough earnings rich households in panel data
   (b) specify a process that undoes this sample selection problem (Castaneda, Diaz-Gimenez, and Rios-Rull, 2003)
2. Preference heterogeneity
   (a) that might also address the problem that wealth and earnings are too highly correlated

Castenda et al. (2003)

Main innovations relative to Huggett (1996):

- Households are *altruistic* (additional source of wealth and motive for saving).
- *Earnings process* is chosen to match SCF data on earnings and wealth inequality.
- *Social Security* system modeled in more detail (to give high retirement incomes to low earnings households; helps account for low wealth observations).
- *Progressive* income tax system (found important for wealth distribution).
- *Stochastic aging*.

Main finding: The model accounts for distribution of earnings and wealth.

Model

There is a continuum of families.

Each family consists of non-overlapping individuals.

In each period, a person:

- draws a stochastic labor endowment e,
- chooses consumption and saving,
- retires with some probability,
- dies with some probability.

New individuals inherit assets and labor endowments from their parents.

Household Problem

State variables:

- "age": working or retired (there is no symbol for age).
- labor endowment e.
- wealth a.

The exogenous states are collected in $s = (age, e)$.
s_t evolves according to a transition matrix Γ.

Households solve

$$\max\ E\left\{\sum_{t=0}^{\infty}\beta^t\, u\left(c_t, \ell - l_t\right)|s_0\right\}$$

subject to the budget constraint

$$c + z = y - \tau(y) + a \tag{1}$$

$$y = a\, r + w\, e_s\, l(s, a) + \omega(s) \tag{2}$$

$$a'(z) = \left\{ \begin{array}{c} z \text{ if survive} \\ (1 - \tau_E(z))\ z \text{ if death} \end{array} \right. \tag{3}$$

Remarks

Households are modeled as infinitely lived.

- This is a reduced form for a sequence of non-overlapping individuals linked by altruistic bequests.
- There is no separate age state variable.

Labor endowments are drawn from $S = \epsilon \cup \Re$.

- $e \in \epsilon$ means "working".
- $e \in \Re$ means retired.

Agents age stochastically

Individuals are born as working ($e \in \epsilon, \omega = 0$).

In each period, they draw a new e.

If $e \in \Re$, the household retired.

If retired and household draws $e \in \epsilon$, he dies and is replaced by a child.

Benefits:

- Small state vector: (s, a).
- Value function must be computed for only 2 "ages"

Drawbacks:

- Some households have very long or short working lives.
- Hard to match life-cycle features (age-earnings profile, mortality rates)

Household Dynamic Program

$$v(s,a) = \max \ u\,(c, \ell - l) + \beta \sum_{s' \in S} \Gamma_{ss'} \ v\,(s', a'\,(z)) \tag{4}$$

$$c + z = y - \tau(y) + a \tag{5}$$

$$y = a\ r + e\,(s)\,lw + \omega\,(s) \tag{6}$$

$$a'(z) = \{ \begin{array}{c} (1 - \tau_E(z))\ z \text{ if } s \in \Re \text{ and } s' \in \epsilon \\ z \text{ otherwise} \end{array} \tag{7}$$

Other Model Agents

Firms

Firms maximize period profits.

Production technology is $F(K, L)$.

Government

Taxes bequests at rate $\tau_E(z)$, where z is the bequest amount.

Taxes income at rate $\tau(y)$.

Provides retirement transfers to households.

Balances the budget in each period: $G_t + Tr_t = T_t$.

Steady State Equilibrium

Objects:

- Policy functions: $c(s,a), z(s,a), l(s,a)$.
- Government policies: $\tau(y), \tau_E(z), \omega(s), G$.
- A stationary probability distribution over household types: x.
- Aggregate quantities: K, L, T, Tr.

These satisfy:

- Policy functions are optimal decision rules.
- Factor market clearing: $K = \int a \; dx, \quad L = \int e(s) \; l(s,a) \; dx$.
- Goods market clearing: $F(K,L) + (1-\delta) \; K = G + \int [c(s,a) + z(s,a)] \; dx$.
- Firm's first-order conditions.
- Government budget constraints.
- Measure of households is stationary.

Calibration

Preferences ($\beta, \sigma_1, \sigma_2, \ell$):

$$u(c, l) = \frac{c^{1-\sigma_1}}{1-\sigma_1} + \chi \frac{(\ell - l)^{1-\sigma_2}}{1-\sigma_2}$$

Technology (θ, δ):

$$F(K, L) = K^{\theta} L^{1-\theta}$$

Government Policy (8 parameters): Income and estate tax schedule mimick U.S. progressive tax system.

Labor endowments (24 parameters): Endowments are realizations of a Markov chain. Nearly unrestricted transition matrix

$$\Gamma_{SS} = \begin{bmatrix} \Gamma_{\varepsilon\varepsilon} & p_{\varepsilon,\rho} I \\ \Gamma_{\Re\varepsilon} & (1 - p_{\rho,\rho}) I \end{bmatrix}$$

ε states represent work; $\Re$ states represent retirement.
Transition matrix matches:

- points on the Lorenz curves for earnings and wealth ($\Gamma_{\varepsilon\varepsilon}, e(s)$).
- intergenerational persistence of labor endowments ($\Gamma_{\Re\varepsilon}$).
- length of working lives ($p_{\varepsilon,\rho}$).
- life expectancy ($p_{\rho,\rho}$).

Total number of parameters: **39** (unusually large [for macro])

Calibration targets

Various features of U.S. tax schedules.

Aggregate ratios: $K/Y, I/Y, G/Y, Tr/Y, l/\ell$

Normalization: ℓ

$\sigma_1 = 1.5$

Ratio of standard deviations for c and l.

Average length of work life: 45 years.

Average length of retirement: 18 years.

Average earnings middle age / young: 1.3

Intergenerational correlation of log lifetime earnings: 0.4

Gini coefficients of earnings and wealth.

13 points on the Lorenz curves of earnings and wealth (SCF).

Findings

Model economy matches calibration targets, except

- earnings growth over the life-cycle
- intergenerational earnings correlation

Earnings and Wealth Distributions

Model economy matches cross-sectional earnings distribution very well.

Wealth distribution match is good, not perfect.

TABLE 7

DISTRIBUTIONS OF EARNINGS AND OF WEALTH IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES (%)

		Quintile					Top Groups (Percentile)		
Economy	Gini	First	Second	Third	Fourth	Fifth	90th–95th	95th–99th	99th–100th
				A. Distributions of Earnings					
United States	.63	−.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Benchmark	.63	.00	3.74	14.59	15.99	65.68	15.15	17.65	14.93
				B. Distributions of Wealth					
United States	.78	−.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Benchmark	.79	.21	1.21	1.93	14.68	81.97	16.97	18.21	29.85

Mobility

Model economies overstate wealth persistence over time.

TABLE 9
EARNINGS AND WEALTH PERSISTENCE IN THE UNITED STATES AND IN THE BENCHMARK MODEL ECONOMIES: FRACTIONS OF HOUSEHOLDS THAT REMAIN IN THE SAME QUINTILE AFTER FIVE YEARS

	QUINTILE				
ECONOMY	First	Second	Third	Fourth	Fifth
	A. Earnings Persistence				
United States	.86	.41	.47	.46	.66
Benchmark	.76	.55	.65	.80	.80
	B. Wealth Persistence				
United States	.67	.47	.45	.50	.71
Benchmark	.81	.80	.80	.75	.89

Success or Failure?

The model successfully replicates the cross-sectional distribution of wealth.

No departure from standard theory is needed.

Key features for the model's success:

- Intended bequests permit households to accumulate wealth over longer time periods.
- Earnings process consistent with cross-sectional SCF data.

Reservations:

The earnings process.

- Calibration does not use information on persistence of earnings.
- The earnings process is "cooked" to match the wealth distribution.

The paper shoes that it is **possible** to write down a standard life-cycle model that matches wealth concentration based on an earnings process with the right amount of cross-sectional inequality.

It does not show that a life-cycle model generates the right wealth distribution when a "**realistic**" earnings process is imposed.

Could one fix this?

- why not combine info on the process for the bottom 99% from the PSID with info for the cross-sectional distribution for everyone from SCF?

Features of the Earnings Process

The lower 3 earnings states "look like" something estimated from the PSID (though persistence is very high).

The top earnings state is totally transitory.

TABLE 5
RELATIVE ENDOWMENTS OF EFFICIENCY LABOR UNITS, $e(s)$, AND THE STATIONARY DISTRIBUTION OF WORKING-AGE HOUSEHOLDS, γ^*_ε

	$s = 1$	$s = 2$	$s = 3$	$s = 4$
$e(s)$	1.00	3.15	9.78	1,061.00
γ^*_ε (%)	61.11	22.35	16.50	.0389

The top earnings level is very large

TABLE 4
TRANSITION PROBABILITIES OF THE PROCESS ON THE ENDOWMENT OF EFFICIENCY LABOR UNITS FOR WORKING-AGE HOUSEHOLDS THAT REMAIN AT WORKING AGE ONE PERIOD LATER, $\Gamma_{\varepsilon\varepsilon}$ (%)

	To s'			
FROM s	$s' = 1$	$s' = 2$	$s' = 3$	$s' = 4$
$s = 1$	96.24	1.14	.39	.006
$s = 2$	3.07	94.33	.37	.000
$s = 3$	1.50	.43	95.82	.020
$s = 4$	10.66	.49	6.11	80.51

Intuition:

- households win the lottery once every 25 years
- lottery winners save everything because the top state is so transitory

Preference Heterogeneity

Motivation

Life-cycle models have trouble accounting for observed wealth inequality. In the data:

- The richest 1% of households hold 35% of wealth.
- "Similar" households hold very different amounts of wealth.

Several authors interpret this as evidence of preference heterogeneity (Venti and Wise, 2000)

Survey data suggest a fair bit of preference heterogeneity (Kimball, Sahm, and Shapiro, 2009)

How important is preference heterogeneity for these observations?

A first step towards an answer: Hendricks (2007)

How to discipline the analysis?

Unrestricted preference heterogeneity can account for any behavior.

Key assumption:

- Each household is endowed with time-invariant preferences.

To measure the importance of preference heterogeneity: exploit that preference heterogeneity affects how consumption/wealth inequality changes with age.

How else could the analysis be disciplined?

- Use empirical estimates of preference parameters (Cozzi, 2014)

Outline

Write down a life-cycle model of the type commonly used to study wealth inequality.

Show that it has the two problems mentioned before:

1. The richest households do not hold enough wealth.
2. Households with similar earnings hold similar wealth.

Add heterogeneity in time preference or risk aversion.

Choose the distribution of preference parameters to match how wealth or consumption inequality change with age.

Examine to what extent preference heterogeneity helps resolve the two problems.

Model

Based on Huggett (1996)

Households

Households live for up to a_D periods.
$P_s(a)$ is probability of living to $a+1$ given a.
There is no population growth.
Exogenous state variables are $s = (a, e, q)$.
Endogenous state variable: wealth k.
Labor endowments are governed by a Markov chain: $P_e(e, e')$.

- New agents draw labor endowments from a fixed distribution, P_{e_1}.
- Permanent labor endowments are drawn from the distribution $P_q(q)$.

During *retirement* $(a > a_R)$:

- Household does not work $(e = 0)$.
- Household receives transfers ϖ.

Household Problem

$$\max\ E \sum_{a=1}^{a_D} \beta^a\, u\,(c_a)$$

subject to

$$k_{a+1} = y_a - c_a \tag{8}$$

$$y_a = R\, k_a + w\ l_a + \varpi_a \tag{9}$$

$$l_a = h_a\ q\ e_a \tag{10}$$

$$k_{a+1} \geq 0 \tag{11}$$

Other Agents

Firms

A representative firm maximizes period profits:

$$\max F(K, L) - q_K K - q_L L$$

Government

Balances the budget in each period: $G + X = T$.

Tax revenues: $T = \tau_w q_L L$.

Government consumption is thrown into the ocean (G).

Transfers are paid equally to all households who are retired: $\varpi(a) = \varpi$ if $a > a_R$.

Stationary Equilibrium

Objects:

Distribution of households over exogenous types: $\Lambda(s)$ denotes fraction of households of type s.

Distribution over all types: $\Gamma(k, s)$ denotes the density.

Household policy function $c(k, s)$ and value function $V(k, s)$.

Price functions: $q_K(K, L), q_L(K, L)$.

Aggregate quantities: K, L, X.

Equilibrium conditions:

Household policy and value functions are optimal.

Prices equal marginal products: $q_K = F_K(K, L), q_L = F_L(K, L)$.

Household prices: $R = 1 + q_K - \delta, w = (1 - \tau_w)\, q_L$.

Goods market clears: $Y = C + I + G$.

Labor market clears: $L = \sum_{s \in S_W} l(s)\Lambda(s)$.

Capital market clears: $K = \sum_s \int_k \Gamma(k, s)\, k\, dk$.

Distribution of households is stationary.

Identities and definitions:

Aggregate investment: $I = K' - (1 - \delta)\, K$.

$K' = \sum_s \int_k \Gamma(k, s)\, k'(k, s)\, dk$.

Parameters

Preferences: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. $\sigma = 1.5$ (Huggett 1996). β set such that $K/Y = 2.9$.

Demographics: Mortality rates from Social Security Life Tables.

Technology: $F(K, L) = A\ K^{\alpha}\ L^{1-\alpha}$. Choose α, δ to match $w = 1$ and interest rate of 4%.

Labor endowments: Baseline process taken from Huggett (1996); has no permanent labor endowment (q).

STY process taken from Storesletten, Telmer, and Yaron (2004)

Government: $\tau_w = 0.4$. Transfers equal 40% of average household earnings.

Experiments

Fix preference parameters on a grid:

- $\beta_b \in \{\bar{\beta}_1, ..., \bar{\beta}_B\}$ and $\sigma = 1.5$, or
- $\sigma_s \in \{\bar{\sigma}_1, ..., \bar{\sigma}_S\}$ and $\beta = 0.97$.

Search over fractions of households in each preference cell to match calibration targets. Targets are $K/Y = 2.9$ and one of the following:

1. Consumption inequality by age.
2. Wealth inequality by age.
3. Cross-sectional wealth distribution.

Baseline: No preference heterogeneity

Problem 1: Model fails to account for top of wealth distribution.

Same as Huggett (1996)

Problem 2: Model implies too tight relationship between lifetime earnings and retirement wealth:

	C_{WE}	**Gini**	$R_{90/20}$
Data	0.50	0.50	2.00
Baseline	0.95	0.27	4.31
STY	0.96	0.29	11.37

C_{WE}: Correlation between lifetime earnings and retirement wealth. (How tight is the relationship?)

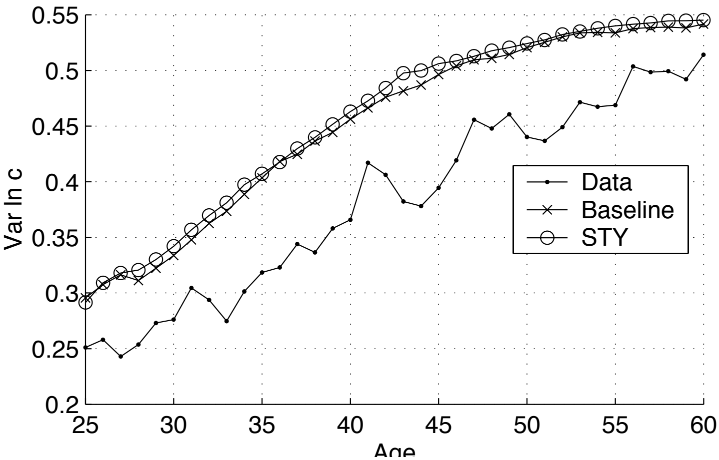
$Gini$: Mean Gini of retirement wealth within lifetime earnings deciles. (How much inequality after controlling for earnings and age?)

$R_{90/20} = x_9/x_2$ where $x_j = \frac{Median\ retirement\ wealth_j}{Mean\ lifetime\ earnings_j}$ for lifetime earnings decile j. (How large wealth gaps between earnings rich and earnings poor households?)

These are the problems that motivate interest in preference heterogeneity.

Consumption inequality by age

Too high in the model, but covaries with age in roughly the right way.
Suggests little scope for preference heterogeneity.

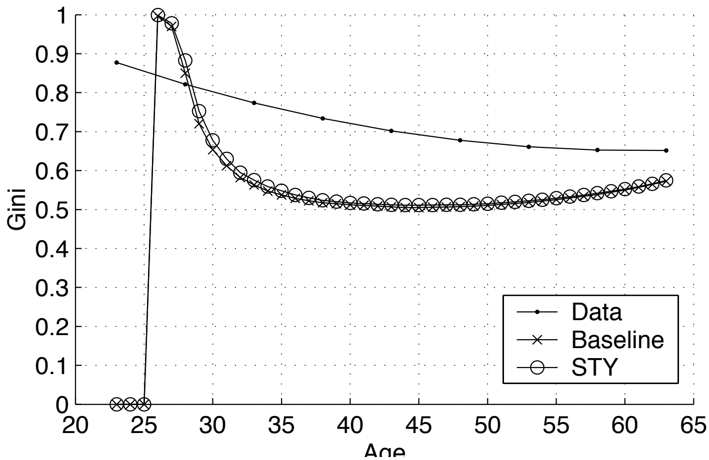


Data: Storesletten, Telmer, and Yaron (2004)

Wealth inequality by age

Too low in the model.

Suggests scope for preference heterogeneity.



A small amount of β heterogeneity

Experiment: Restrict β to lie on the 2 grid points surrounding the β value without preference heterogeneity.

Find weights that match K/Y target.

Wealth distribution:

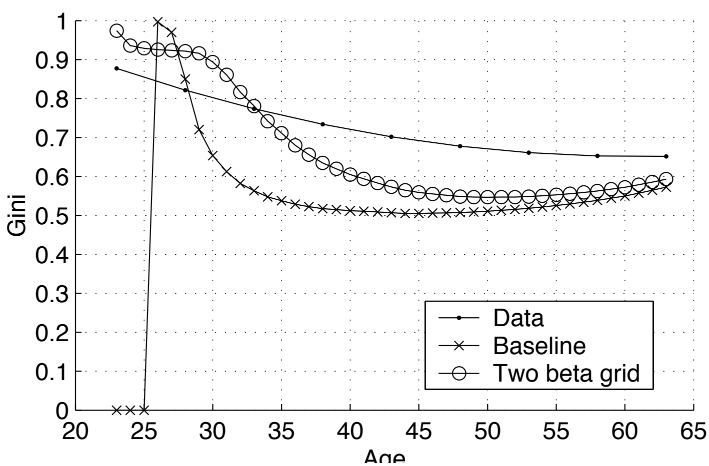
	20.0	40.0	60.0	80.0	90.0	95.0	99.0	100.0	Gini
Data	-0.7	1.1	7.7	24.2	40.5	54.4	75.6	100.0	0.75
Baseline	0.0	1.7	8.6	27.9	48.5	65.3	89.1	100.0	0.70
Two beta grid	0.0	1.1	7.1	25.5	46.1	63.5	88.1	100.0	0.72

Relationship between lifetime earnings and retirement wealth:

	C_{WE}	Gini	$R_{90/20}$
Data	0.50	0.50	2.00
Baseline	0.95	0.27	4.31
Two beta grid	0.88	0.39	6.46

Small effect on $var\left(\ln\left(c\right)\right)$ by age:

Larger effect on wealth inequality by age:

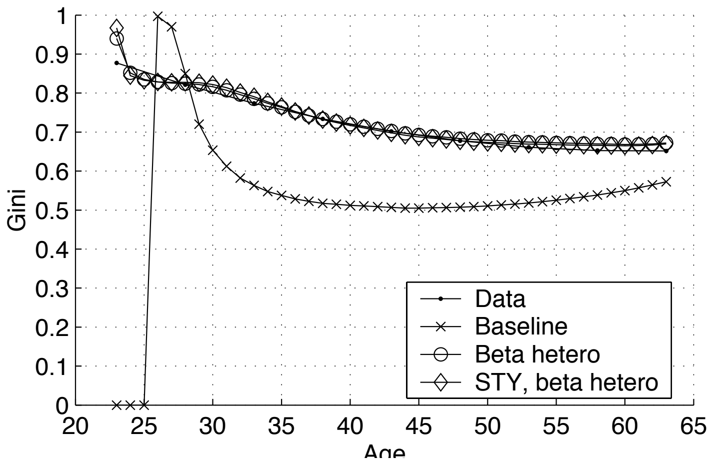


Implications

Consumption inequality by age does not offer good identification of preference parameters.
Wealth inequality by age is more promising.
Preference heterogeneity seems to

- improve relationship lifetime earnings / wealth.
- help account for high wealth holdings.
- push too many households towards zero wealth.

Experiment: Match wealth inequality by age



Wealth distribution:

	20.0	40.0	60.0	80.0	90.0	95.0	99.0	100.0	Gini
Data	-0.7	1.1	7.7	24.2	40.5	54.4	75.6	100.0	0.75
Baseline	0.0	1.7	8.6	27.9	48.5	65.3	89.1	100.0	0.70
Beta hetero	0.0	0.0	3.0	19.1	40.2	59.0	85.9	100.0	0.78
STY, beta hetero	0.0	0.1	3.1	19.4	40.5	59.0	85.3	100.0	0.78

Relationship lifetime earnings / wealth:

	C_{WE}	Gini	$R_{90/20}$
Data	0.50	0.50	2.00
Baseline	0.95	0.27	4.31
Beta hetero	0.69	0.56	6.94
STY, beta hetero	0.71	0.55	19.74

Implications

Wealth inequality by age leaves room for vast preference heterogeneity.
β heterogeneity then:

- helps account for relationship between lifetime earnings and retirement wealth;
- helps account for upper tail of wealth distribution (though not for top 1%);
- creates problems with lower tail of wealth distribution.

Experiment: Match cross-sectional wealth distribution

Upper bound for contribution of preference heterogeneity.

Wealth distribution:

	20.0	**40.0**	**60.0**	**80.0**	**90.0**	**95.0**	**99.0**	**100.0**	**Gini**
Data	-0.7	1.1	7.7	24.2	40.5	54.4	75.6	100.0	0.75
Baseline	0.0	1.7	8.6	27.9	48.5	65.3	89.1	100.0	0.70
Beta hetero	0.0	0.6	5.2	20.9	39.8	57.1	83.6	100.0	0.76
STY, beta hetero	0.0	0.6	5.3	21.2	40.2	57.4	83.0	100.0	0.76

Relationship lifetime earnings / wealth:

	C_{WE}	**Gini**	$R_{90/20}$
Data	0.50	0.50	2.00
Baseline	0.95	0.27	4.31
Beta hetero	0.71	0.49	5.89
STY, beta hetero	0.73	0.49	17.97

Interpretation

β heterogeneity helps the model replicate the observed

- cross-sectional wealth distribution,
- relationship between lifetime earnings and retirement wealth.

The contribution towards accounting for highest wealth observations is modest.

Experiment: Heterogeneity in IES/Risk aversion

Arbitrarily assign half of the households $\sigma = 1$ and $\sigma = 5$.
To understand how σ heterogeneity affects model properties.

Implications

σ Heterogeneity is not important for wealth inequality or for understanding the relationship between lifetime earnings and retirement wealth.

Intuition: High σ has two countervailing effects on consumption:

- Flatter desired consumption profile $\rightarrow$ young c rises.
- Higher buffer stock / more precautionary wealth $\rightarrow$ young c falls.

Summary

Preference heterogeneity helps account for:

- wealth inequality among households with similar lifetime earnings;
- wealth inequality among households of similar age;
- large share of wealth held by richest 5% of households.

It does not help account for:

- small wealth gaps between earnings rich and earnings poor households.

It creates problems with:

- too many households holding no wealth (regardless of lifetime earnings).

Open Issues

Is this approach promising / convincing?

How else could the importance of preference heterogeneity be measured?

Does it help account for intergenerational persistence (how to discipline this?)?

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