Stochastic Multi-Period OLG Model

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Econ821

January 28, 2016

Contents

Introduction

We develop a realistically calibrated OLG model with heterogeneous agents.
Based on Huggett (1996)
We study the implications for

- consumption/saving puzzles
- wealth distribution.

Model Features

Households:

- Live for many (a_D) periods
- Earnings are random
- Age of retirement is fixed (a_R).

Government:

- Pays transfers to retired households (annuitized income in the data)

Simplifying assumptions:

- Steady state
- No random mortality
- No intergenerational links
- No labor-leisure choice.

Model Primitives

Demographics

Households live for exactly a_D periods.
Total mass of households is $N = 1$.
In each period, $1/a_D$ households are born.
Mass of households aged a: $N_a = 1/a_D$.
Mass of retired households: $N_R = (a_D - a_R)/a_D$.

Preferences

$$\mathbb{E} \sum_{a=1}^{a_D} \beta^a u(c_a) \tag{1}$$

Technologies

$$F(K, L) = (1 - \delta) K + C + G + K' \tag{2}$$

Endowments

Working agents are endowed with labor efficiency $\eta_a e_a$
η_a: age-efficiency profile
e_a: labor efficiency (wage) shock

- governed by a Markov chain: $\Pr(e' = \varepsilon_k | e = \varepsilon_j) = P_e(k, j)$.
- new agents draw labor endowments from a fixed distribution.
- number of states: N_e.

Markets

Labor: wage w
Capital rental: r
Goods: numeraire.

Government

Balances the budget in each period: $G + X = T$

Tax revenues: $T = \tau_w \, w \, L$.

Government consumption is thrown into the ocean (G).

Transfers are paid equally to all households who are retired: $\varpi\left(a\right) = \varpi$ if $a > a_R$.

Aggregate transfers: $X = N_R \, \varpi$.

Household Problem

Exogenous state variables are age a and labor endowment e: $s = (a, e)$.
Endogenous state variable: wealth k.
Borrowing constraint: $k \geq 0$.

Sequence problem

$$\max E \sum_{a=1}^{a_D} \beta^a u(c_a)$$

subject to

$$k_{a+1} = y_a - c_a \geq 0$$

$$y_a = R\,k_a + w(1 - \tau_w)\eta_a e_a + \varpi(a) \tag{3}$$

Household Dynamic Program

$$V\left(k,s\right)=\max u\left(y\left(k,s\right)-k'\right)+\beta\mathbb{E}V\left(k',s'\right) \tag{4}$$

with

$$y\left(k,s\right)=Rk+w\left(1-\tau_{w}\right)\eta_{a}e+\varpi\left(s\right) \tag{5}$$

subject to $k'\geq 0$.

Euler equation:

$$u'(c)\geq\beta R\mathbb{E}u'(c') \tag{6}$$

with equality if $k'>0$.

Household Solution

Solution is a consumption function $c\,(k,a,e)$ which satisfies

$$u'(c\,[k,a,e]) \geq \beta R \sum_{e'} P_e(e,e') u'\left(c\,[y - c\,(k,a,e)\,, a+1, e']\right)$$

In the last period, consume all income:

$$c\,(k,a_D,e) = y(k,a_D,e) \tag{7}$$

Stationary Equilibrium

Objects:

Distribution of households over exogenous types:

- $\Lambda(s)$ denotes fraction of households of type s.

Distribution over all types:

- $\Gamma(k, s)$ denotes the density.

Household policy function $c(k, s)$ and value function $V(k, s)$.
Aggregate quantities: K, L, X.
Price functions: $r(K, L), w(K, L)$.

Equilibrium conditions

Household policy and value functions are optimal.

Prices equal marginal products:

- $r = F_K(K, L), w = F_L(K, L)$.

Goods market clears: $Y = C + I + G$.

Labor market clears: $L = \sum_s e(s)\eta\,(s)\,\Lambda(s)$.

Capital market clears: $K = \sum_s \int_k \Gamma(k, s)\,k\,dk$.

Distribution of households is stationary.

Identities and definitions:

Set of states where households work: $S_w = \{s : a \leq a_R\}$.

Set of states where households are retired: $S_R = \{s : a > a_R\}$.

Aggregate investment: $I = K' - (1 - \delta)\,K$.

$K' = \sum_s \int_k \Gamma(k, s)\,k'(k, s)\,dk$.

Household rate of return: $R = 1 + r - \delta$.

Remarks

The distribution of household types Γ is complicated (an infinite dimensional object).
It must be approximated on a grid for k.

Why not restrict k to lie on a grid?
This might greatly simply computations.

Parameter Choices

Calibrated parameters: β, δ, A.
Calibration targets: K/Y, $w = 1$, R.

Period length: λ years per model period.

Preferences

$u(c) = c^{1-\sigma}/(1-\sigma)$
$\sigma = 2$.
Choose β to match $K/Y = 2.9/\lambda$.

Demographics

Households live from age 20 to 79.
Work from 20 to 64 (45 years).
Retire for 15 years.
$a_R = round\,(45/\lambda)$
$a_D = round(60/\lambda)$

Production Function

$F(K, L) = A\,K^{\alpha}\,L^{1-\alpha}$.
$\alpha = 0.36$.
Choose δ and A to match

- $w = 1$
- $R = 1.04$

Government

$\tau_w = 0.4$ (Trostel 1993).
Set transfers to 40% of average earnings.

- This can be done before computing equilibrium.

Labor Endowments

Can be set before equilibrium is computed.

Empirical studies estimate AR(1) processes for [log earnings] minus [mean log earnings, η_a] by age.

New agents draw endowments from exogenous distribution:

$\ln(e_1) \backsim N(0, \sigma_1^2)$.

Over time, endowments are drawn from an AR(1):

$\ln(e_a) = \eta_a + \gamma \ln(e_{a-1}) + \varepsilon_a$.

$\ln(\varepsilon_a) \backsim N(0, \sigma_\varepsilon^2)$.

We follow Huggett (1996):

- $\sigma_1^2 = 0.38$, $\sigma_\varepsilon^2 = 0.045$, $\gamma = 0.96$.
- Approximate the AR(1) on a grid of 18 states equally spaced over $\pm 4\,\sigma_1$.
- Add an additional state at $+6\,\sigma_1$ to capture skewness of earnings distribution.
- Use Tauchen (1986) (we have code for that)

Age-efficiency profile

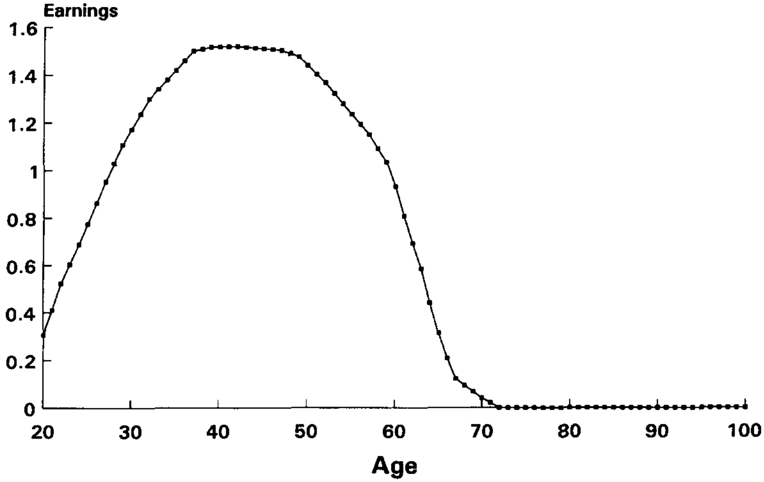

Fig. 1. Earnings profile (ratio to overall mean).

From PSID data (Huggett, 1996)

References

HUGGETT, M. (1996): "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 38, 469–494.

TAUCHEN, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics letters*, 20, 177–181.