# McCall Model

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# Motivation

- We would like to study basic labor market data:
  - unemployment and its duration
  - wage heterogeneity among seemingly identical workers
  - job to job transitions
  - how do policies affect those variables?
- Frictionless models of the labor market cannot talk about these issues.
- We need models in which workers must **search for jobs**.

# Search Models

- Unemployment is a productive activity:
  workers search for a new job.
- Types of models:
  1. Decision theoretic (McCall model).
  2. Matching: A matching function creates new jobs.
  3. Search: Random encounters and bargaining.

# McCall Model

- A partial equilibrium model of a worker searching for a job.
- The worker lives forever, in discrete time.
- Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t y_t$$

- $y_t$ is income.
- When employed: $y = w$.
- When unemployed: $y = c$.

# Timing

Enter the period as unemployed worker.

- Consume $c$.

With probability $\lambda$:

- Draw a wage offer $w$ from the distribution $F(W) = \Pr(w \leq W)$.
- Support: $[0, B]$.
- Choose whether to accept or reject.
- If accept:
  - work forever at wage $w$ with lifetime income $\frac{w}{1-\beta}$.
  - starting next period
- If no offer accepted: start over next period.

# Employed

Work forever at wage $w$.

Value is present value of wages

$$v(w) = \frac{w}{1-\beta} \tag{1}$$

# Unemployed

$$V_U = c + \beta(1-\lambda)V_U + \beta\lambda\int_0^B \max\{v(w), V_U\}\, dF(w) \tag{2}$$

Reservation wage satisfies

$$v(\bar{w}) = V_U \tag{3}$$

Accept all offers that are better than being unemployed tomorrow.

# Reservation wage property

Add and subtract $\beta\lambda V_U$:

$$V_U = c + \beta(1-\lambda)V_U + \beta\lambda V_U + \beta\lambda \int_0^B \max\{v(w) - V_U, 0\}\, dF(w) \tag{4}$$

$$= c + \beta V_U + \beta\lambda \int_{\bar{w}}^B [v(w) - V_U]\, dF(w) \tag{5}$$

Replace $V_U = \bar{w}/(1-\beta)$ and $v(w) = w/(1-\beta)$

$$\bar{w} = c + \frac{\beta\lambda}{1-\beta} \int_{\bar{w}}^B (w - \bar{w})\, dF(w) \tag{6}$$

# Reservation wage

$$\bar{w} = c + \frac{\beta\lambda}{1-\beta} \int_{\bar{w}}^{B} (w - \bar{w})\, dF(w) \tag{7}$$

In words:

- the surplus from working now $(\bar{w} - c)$ equals
- the surplus from searching: the expected lifetime wage gain from perhaps finding a better job

# Implications: Unemployment Benefits

What is the effect of more generous unemployment benefits (higher $c$)?

Optimality: $c = \bar{w} - \text{expected surplus}$ or

$$c = \bar{w} - \frac{\beta\lambda}{1-\beta}\int_{\bar{w}}^{B}(w-\bar{w})\,dF(w) \tag{8}$$

Expected surplus shrinks when $\bar{w}$ rises.

RHS increases in $\bar{w}$.

Higher $c \rightarrow$ higher reservation wage $\rightarrow$ longer unemployment.

# More dispersed wage offers

- Result: A mean preserving spread in the wage offer distribution raises the reservation wage and ex ante utility.
- Intuition:
  - Making bad wage offers worse is costless - they are rejected anyway.
  - Making good wage offers better is valuable.
- Proof: Ljunqvist & Sargent.

# Extension: Job separations

Each period the worker is fired with probability $\alpha$.

A fired worker must wait 1 period before drawing a new wage.

Now we have 3 states the worker can be in:

1. unemployed, waiting for a wage offer: $V_U$
2. unemployed with a wage offer: $v(w)$
3. employed: $v_E(w)$

# Value functions

Value when unemployed without offer is unchanged:

$$V_U = c + \beta V_U + \beta \lambda \int_{\bar{w}}^{B} [v_E(w) - V_U] \, dF(w) \tag{9}$$

Value when unemployed with an offer also unchanged:

$$v(w) = \max\{v_E(w), V_U\}$$

Value when employed at wage $w$:

$$\begin{aligned} v_E(w) &= w + \beta(1-\alpha)v_E(w) + \beta \alpha v_U \\ &= \frac{w + \beta \alpha V_U}{1 - \beta(1-\alpha)} \end{aligned}$$

# Reservation wage

Recall

$$v_E(w) = \frac{w + \beta \alpha V_U}{1 - \beta(1 - \alpha)} \tag{10}$$

and

$$V_U = v_E(\bar{w}) = \frac{\bar{w} + \beta \alpha V_U}{1 - \beta(1 - \alpha)} \tag{11}$$

So we still have

$$V_U = \frac{\bar{w}}{1 - \beta} \tag{12}$$

# Reservation Wage

Sub into (9):

$$\frac{\bar{w}}{1-\beta} = c + \frac{\beta\bar{w}}{1-\beta} + \beta\lambda\int_{\bar{w}}^{B}[v_E(w) - V_U]\,dF(w) \tag{13}$$

Now compute $v_E(w) - V_U$:

$$\begin{aligned}
v_E(w) - V_U &= \frac{w + \beta\alpha V_U}{1-\beta(1-\alpha)} - V_U \\
&= \frac{w + \beta\alpha V_U - V_U[1-\beta(1-\alpha)]}{1-\beta(1-\alpha)} \\
&= \frac{w - V_U[1-\beta(1-\alpha)-\beta\alpha]}{1-\beta(1-\alpha)} \\
&= \frac{w - V_U(1-\beta)}{1-\beta(1-\alpha)} \\
&= \frac{w-\bar{w}}{1-\beta(1-\alpha)} \qquad (14)
\end{aligned}$$

# Reservation Wage

Sub back into (13):

$$\bar{w} = c + \frac{\beta\lambda}{1-\beta(1-\alpha)} \int_{\bar{w}}^{B} (w - \bar{w})\, dF(w) \tag{15}$$

The only difference relative to the model without firing: The $1-\alpha$ term in the denominator.

The reservation wage falls when $\alpha$ rises.

- If jobs do not last as long, there is no point holding out for the perfect offer.

# 4. Frictional Wage Dispersion

# Frictional Wage Dispersion

A puzzle: The McCall model implies **very small wage dispersion**.

Intuition:

- Unemployment spells tend to be short.
- Wage offers must arrive frequently.
- Waiting for a better job is cheap, but highly profitable.

Reference: Macroeconomics, ch. 20.9.

# Deriving Wage Dispersion

We will look at the ratio of mean wage to minimum (accepted) wage.

Mean accepted wage:

$$w_{mean} = \mathbb{E}\{w | w \geq \bar{w}\} = \frac{\int_{\bar{w}}^{B} w \times dF(w)}{1 - F(\bar{w})} \tag{16}$$

Minimum wage observed: $\bar{w}$, given by

$$\bar{w} = c + \frac{\beta\lambda}{1 - \beta(1-\alpha)} \int_{\bar{w}}^{B} (w - \bar{w})\, dF(w) \tag{17}$$

# Deriving Wage Dispersion

Assume $1+r=1/\beta$, so that

$$r+\alpha=\frac{1-\beta(1-\alpha)}{\beta} \tag{18}$$

Then

$$\bar{w}=c+\frac{\lambda}{r+\alpha}\int_{\bar{w}}^{B}(w-\bar{w})\,dF(w) \tag{19}$$

Note that we can break the integral into 2 terms:

$$\int_{\bar{w}}^{B}(w-\bar{w})\,dF(w)=(1-F(\bar{w}))\left[w_{mean}-\bar{w}\right] \tag{20}$$

# Deriving Wage Dispersion

Define the job finding rate as

$$H = \lambda \left(1 - F\left(\bar{w}\right)\right) \tag{21}$$

Then

$$\bar{w} = c + \frac{H \times \left(w_{mean} - \bar{w}\right)}{r + \alpha} \tag{22}$$

Divide by $w_{mean}$ to obtain

$$\mu \equiv \frac{w_{mean}}{\bar{w}} = \frac{1 + \frac{H}{r+\alpha}}{\frac{c}{w_{mean}} + \frac{H}{r+\alpha}} \tag{23}$$

# How Large is Wage Dispersion?

With monthly data, roughly:

- job finding rate $H = 0.43$ (that's the key moment)
- separation rate $\alpha = 0.03$
- interest rate $r = 0.004$
- replacement rate $c/w_{mean} \approx 0.4$

Then mean-min wage ratio

$$\mu \approx 1.05 \tag{24}$$

Data: $\mu > 2$.

# Why So Small?

Intuition:

- job finding rate is high
- waiting for a better job does not take long
- wage benefits are large
- cannot generate substantial wage dispersion

# Possible Solutions

1. On the job search
    - No need to wait for the perfect job before accepting.
2. Job ladders
    - While employed, workers learn or find better matches.

- Stochastic worker productivity

# Stochastic Wages

# Model With Stochastic Wages

Based on Rogerson et al. (2005).

**Timing**:

Enter the period either as

- unemployed: value $V_U$ or as
- employed: value $V(w)$.

If **unemployed**:

- earn $c$ today
- draw a wage offer $w'$ for next period with probability $\alpha$
- if accept: get $V(w)$ tomorrow
- if reject: get $V_U$ tomorrow

# Timing

If **employed**:

- earn $w$ today and eat it
- draw a new wage $w'$ for tomorrow with probability $\lambda$.
- if accept: $V(w')$
- if reject: unemployed tomorrow

All wage offers are drawn from the same distribution:

$F(W) = \Pr(w' \leq W)$ with support $[0, B]$.

# Value of a wage offer

Consider an unemployed (or employed) worker who is about to receive a wage offer.

His value is

$$\hat{Q} = \int \max\left\{V(w'), V_U\right\} dF(w') \tag{25}$$

Independent of current $w$ (in case of employed)

- because the current $w$ is replaced by the new offer.

Call the reservation wage $\bar{w}$.

- it is the same for employed or unemployed

# Value of a wage offer

$$\hat{Q} = \int \max\{V(w'), V_U\}\, dF(w') \tag{26}$$

$$= \int \max\{V(w') - V_U, 0\}\, dF(w') + V_U \tag{27}$$

$$= \underbrace{\int_{\bar{w}}^{B} \{V(w') - V_U\}\, dF(w')}_{Q} + V_U \tag{28}$$

In words:

- you always get at least $V_U$ (because you can always take that option)
- if $w' > \bar{w}$, you also get a surplus $Q$

# Unemployed Worker

Before receiving offer

$$V_U = c + \beta \left[\alpha \hat{Q} + (1-\alpha) V_U\right] \tag{29}$$
$$= c + \beta \left[\alpha (Q + V_U) + (1-\alpha) V_U\right] \tag{30}$$
$$= c + \beta \alpha Q + \beta V_U \tag{31}$$

Get $c$ today.

With probability $\alpha$ get to choose between work and unemployment tomorrow.

Therefore

$$(1-\beta) V_U = c + \beta \alpha Q \tag{32}$$

# Employed Worker

Bellman equation for a worker with wage $w$:

$$V(w) = w + \beta \left[\lambda \hat{Q} + (1 - \lambda) V(w)\right] \tag{33}$$

Get $w$ today.

With probability $\lambda$, face the same choice as an unemployed worker with an offer.

Simplify:

$$V(w) = w + \beta\lambda (Q + V_U) + \beta (1 - \lambda) V(w) \tag{34}$$

# Reservation Wage

Evaluate $V(w)$ at $w = \bar{w}$ and use $V(\bar{w}) = V_U$:

$$V(\bar{w}) = \bar{w} + \beta\lambda(Q + V_U) + \beta(1-\lambda)V_U \tag{35}$$

Therefore

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{36}$$

# Reservation Wage

We now have

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{37}$$

$$= c + \beta\alpha Q \tag{38}$$

Perhaps easier:

$$\bar{w} - c = \beta(\alpha - \lambda)Q \tag{39}$$

# Reservation Wage

If $\alpha = \lambda$: $\bar{w} = c$.

- Accept any job that pays more than unemployment benefits
- The reason is that the continuation value does not depend on employment status.

If $\alpha > \lambda$: $\bar{w} > c$.

- Being unemployed has a search value. So the agent holds out for better wage offers.

# Reservation Wage

Add and subtract $V_U - V(w)$ in equation for $V(w)$:

$$(1-\beta)V(w) = w + \beta\lambda Q + \beta\lambda[V_U - V(w)] \tag{40}$$

Substitute out $Q$ from equation for reservation wage

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{41}$$

to obtain

$$(1-\beta)[V(w) - V_U] = w - \bar{w} + \beta\lambda[V_U - V(w)] \tag{42}$$

Solve for

$$V(w) - V_U = \frac{w - \bar{w}}{1 - \beta + \beta\lambda} \tag{43}$$

If we specified the distribution $F$, we could use this to evaluate $Q$ and solve for everything else.

# Applications

Life-cycle earnings profiles and occupational mobility:

- Kambourov and Manovskii (2009, 2008)

Business cycle models that match labor market facts:

- Jovanovic (1987)

# What is missing?

- Not satisfactory: The job finding rate / wage offer distribution should be endogenous.
  - Think about analyzing policies...
- Matching and search models address this.
  - by introducing endogenous supply of jobs
  - and wage bargaining.

# Reading

- Macroeconomics, ch. 20.9
- Ljungqvist and Sargent (2004), ch. 6.3
- Williamson (2006), "Notes on macroeconomic theory," ch. 7, works out a similar model with exogenous job separations.

# References I


Jovanovic, B. (1987): "Work, Rest, and Search: Unemployment, Turnover, and the Cycle," *Journal of Labor Economics*, 131–148.

Kambourov, G. and I. Manovskii (2008): "RISING OCCUPATIONAL AND INDUSTRY MOBILITY IN THE UNITED STATES: 1968–97*," *International Economic Review*, 49, 41–79.

——— (2009): "Occupational mobility and wage inequality," *The Review of Economic Studies*, 76, 731–759.

Ljungqvist, L. and T. J. Sargent (2004): *Recursive macroeconomic theory*, 2nd ed.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature*, 43, 959–988.