McCall Model

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Motivation

- We would like to study basic labor market data:
  - unemployment and its duration
  - wage heterogeneity among seemingly identical workers
  - job to job transitions
  - how do policies affect those variables?
- Frictionless models of the labor market cannot talk about these issues.
- We need models in which workers must **search for jobs**.

Search Models

- Unemployment is a productive activity:
  workers search for a new job.
- Types of models:
  1. Decision theoretic (McCall model).
  2. Matching: A matching function creates new jobs.
  3. Search: Random encounters and bargaining.

McCall Model

- A partial equilibrium model of a worker searching for a job.
- The worker lives forever, in discrete time.
- Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t y_t$$

- y_t is income.
- When employed: $y = w$.
- When unemployed: $y = c$.

Timing

Enter the period as unemployed worker.

- Consume c.

With probability λ:

- Draw a wage offer w from the distribution $F(W) = \Pr(w \leq W)$.
- Support: $[0, B]$.
- Choose whether to accept or reject.
- If accept:
  - work forever at wage w with lifetime income $\frac{w}{1-\beta}$.
  - starting next period

- If no offer accepted: start over next period.

Employed

Work forever at wage w.

Value is present value of wages

$$v(w) = \frac{w}{1-\beta} \tag{1}$$

Unemployed

$$V_U = c + \beta(1-\lambda)V_U + \beta\lambda\int_0^B \max\{v(w), V_U\}\,dF(w) \tag{2}$$

Reservation wage satisfies

$$v(\bar{w}) = V_U \tag{3}$$

Accept all offers that are better than being unemployed tomorrow.

Reservation wage property

Add and subtract $\beta\lambda V_U$:

$$V_U = c + \beta(1-\lambda)V_U + \beta\lambda V_U + \beta\lambda \int_0^B \max\{v(w) - V_U, 0\}\, dF(w) \tag{4}$$

$$= c + \beta V_U + \beta\lambda \int_{\bar{w}}^B [v(w) - V_U]\, dF(w) \tag{5}$$

Replace $V_U = \bar{w}/(1-\beta)$ and $v(w) = w/(1-\beta)$

$$\bar{w} = c + \frac{\beta\lambda}{1-\beta} \int_{\bar{w}}^B (w - \bar{w})\, dF(w) \tag{6}$$

Reservation wage

$$\bar{w} = c + \frac{\beta\lambda}{1-\beta}\int_{\bar{w}}^{B}(w-\bar{w})\,dF(w) \tag{7}$$

In words:

- the surplus from working now $(\bar{w}-c)$ equals
- the surplus from searching: the expected lifetime wage gain from perhaps finding a better job

Implications: Unemployment Benefits

What is the effect of more generous unemployment benefits (higher c)?

Optimality: $c = \bar{w} -$ expected surplus or

$$c = \bar{w} - \frac{\beta \lambda}{1-\beta} \int_{\bar{w}}^{B} (w - \bar{w})\, dF(w) \tag{8}$$

Expected surplus shrinks when $\bar{w}$ rises.

RHS increases in $\bar{w}$.

Higher $c \rightarrow$ higher reservation wage $\rightarrow$ longer unemployment.

More dispersed wage offers

- Result: A mean preserving spread in the wage offer distribution raises the reservation wage and ex ante utility.
- Intuition:
  - Making bad wage offers worse is costless - they are rejected anyway.
  - Making good wage offers better is valuable.
- Proof: Ljunqvist & Sargent.

Extension: Job separations

Each period the worker is fired with probability α.

A fired worker must wait 1 period before drawing a new wage.

Now we have 3 states the worker can be in:

1. unemployed, waiting for a wage offer: v_U
2. unemployed with a wage offer: $v(w)$
3. employed: $v_E(w)$

Value functions

Value when unemployed without offer is unchanged:

$$v_U = c + \beta V_U + \beta\lambda \int_{\bar{w}}^{B} [v(w) - V_U]\, dF(w)$$

Value when unemployed with an offer also unchanged:

$$v(w) = \max\{v_E(w), v_U\}$$

Value when employed at wage w:

$$\begin{aligned} v_E(w) &= w + \beta(1-\alpha)v_E(w) + \beta\alpha v_U \\ &= \frac{w + \beta\alpha V_U}{1 - \beta(1-\alpha)} \end{aligned}$$

Reservation wage

Recall

$$v(w) = \frac{w + \beta \alpha V_U}{1 - \beta(1 - \alpha)} \tag{9}$$

and

$$V_U = v(\bar{w}) = \frac{\bar{w} + \beta \alpha V_U}{1 - \beta(1 - \alpha)} \tag{10}$$

$$\bar{w} = c + \frac{\beta \lambda}{1 - \beta(1 - \alpha)} \int_{\bar{w}}^{B} (w - \bar{w})\, dF(w) \tag{11}$$

The reservation wage falls when α rises.

- If jobs do not last as long, there is no point holding out for the perfect offer.

3. Frictional Wage Dispersion

Frictional Wage Dispersion

A puzzle: The McCall model implies **very small wage dispersion**.

Intuition:

- Unemployment spells tend to be short.
- Wage offers must arrive frequently.
- Waiting for a better job is cheap, but highly profitable.

Reference: Macroeconomics, ch. 20.9.

Deriving Wage Dispersion

We will look at the ratio of mean wage to minimum (accepted) wage.

Mean accepted wage:

$$w_{mean} = \frac{\int_{\bar{w}}^{B} w \times dF(w)}{1 - F(\bar{w})} \tag{12}$$

Minimum wage observed: $\bar{w}$, given by

$$\bar{w} = c + \frac{\beta \lambda}{1 - \beta (1 - \alpha)} \int_{\bar{w}}^{B} (w - \bar{w})\, dF(w) \tag{13}$$

Deriving Wage Dispersion

Assume $1+r=1/\beta$, so that

$$r+\alpha=\frac{1-\beta(1-\alpha)}{\beta} \tag{14}$$

Then

$$\bar{w}=c+\frac{\lambda}{r+\alpha}\int_{\bar{w}}^{B}(w-\bar{w})\,dF(w) \tag{15}$$

Note that we can break the integral into 2 terms:

$$\int_{\bar{w}}^{B}(w-\bar{w})\,dF(w)=(1-F(\bar{w}))\left[w_{mean}-\bar{w}\right] \tag{16}$$

Deriving Wage Dispersion

Define the job finding rate as

$$H = \lambda \left(1 - F\left(\bar{w}\right)\right) \tag{17}$$

Then

$$\bar{w} = c + \frac{H \times \left(w_{mean} - \bar{w}\right)}{r + \alpha} \tag{18}$$

Divide by w_{mean} to obtain

$$\mu \equiv \frac{w_{mean}}{\bar{w}} = \frac{1 + \frac{H}{r+\alpha}}{\frac{c}{w_{mean}} + \frac{H}{r+\alpha}} \tag{19}$$

How Large is Wage Dispersion?

With monthly data, roughly:

- job finding rate $H = 0.43$ (that's the key moment)
- separation rate $\alpha = 0.03$
- interest rate $r = 0.004$
- replacement rate $c/w_{mean} \approx 0.4$

Then mean-min wage ratio

$$\mu \approx 1.05 \tag{20}$$

Data: $\mu > 2$.

Why So Small?

Intuition:

- job finding rate is high
- waiting for a better job does not take long
- wage benefits are large
- cannot generate substantial wage dispersion

Possible Solutions

1. On the job search
    - No need to wait for the perfect job before accepting.
2. Job ladders
    - While employed, workers learn or find better matches.

- Stochastic worker productivity

Stochastic Wages

Model With Stochastic Wages

Based on Rogerson et al. (2005).

Timing:

Enter the period either as

- unemployed: value V_U or as
- employed: value $V(w)$.

If **unemployed**:

- earn c today
- draw a wage offer w' for next period with probability α
- if accept: get $V(w)$ tomorrow
- if reject: get V_U tomorrow

Timing

If **employed**:

- earn w today and eat it
- draw a new wage w' for tomorrow with probability λ.
- if accept: $V(w')$
- if reject (or no offer): unemployed tomorrow

All wage offers are drawn from the same distribution:

$F(W) = \Pr(w' \leq W)$ with support $[0, B]$.

Value of a wage offer

Consider an unemployed (or employed) worker who is about to receive a wage offer.

His value is

$$\hat{Q} = \int \max\left\{V(w'), V_U\right\} dF(w') \tag{21}$$

Independent of current w (in case of employed)

- because that offer is lost

Call the reservation wage $\bar{w}$.

- it is the same for employed or unemployed

Value of a wage offer

$$\hat{Q} = \int \max\left\{V(w'), V_U\right\} dF(w') \tag{22}$$

$$= \int \max\left\{V\left(w'\right) - V_U, 0\right\} dF\left(w'\right) + V_U \tag{23}$$

$$= \underbrace{\int_{\bar{w}}^{B} \left\{V\left(w'\right) - V_U\right\} dF\left(w'\right)}_{Q} + V_U \tag{24}$$

In words:

- you always get at least V_U (because you can always take that option)
- if $w' > \bar{w}$, you also get a surplus Q

Unemployed Worker

Before receiving offer

$$V_U = c + \beta \left[\alpha \hat{Q} + (1-\alpha) V_U\right] \tag{25}$$
$$= c + \beta \left[\alpha (Q + V_U) + (1-\alpha) V_U\right] \tag{26}$$
$$= c + \beta \alpha Q + \beta V_U \tag{27}$$

Get c today.

With probability α get to choose between work and unemployment tomorrow.

Therefore

$$(1-\beta) V_U = c + \beta \alpha Q \tag{28}$$

Employed Worker

Bellman equation for a worker with wage w:

$$V(w) = w + \beta \left[\lambda \hat{Q} + (1 - \lambda) V(w) \right] \tag{29}$$

Get w today.

With probability λ, face the same choice as an unemployed worker with offer w'.

Simplify:

$$V(w) = w + \beta \lambda (Q + V_U) + \beta (1 - \lambda) V(w) \tag{30}$$

Reservation Wage

Evaluate $V(w)$ at $w = \bar{w}$ and use $V(\bar{w}) = V_U$:

$$V(\bar{w}) = \bar{w} + \beta\lambda(Q + V_U) + \beta(1-\lambda)V_U \tag{31}$$

Therefore

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{32}$$

Reservation Wage

We now have

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \quad (33)$$
$$= c + \beta\alpha Q \quad (34)$$

Perhaps easier:

$$\bar{w} - c = \beta(\alpha - \lambda)Q \quad (35)$$

Reservation Wage

If $\alpha = \lambda$: $\bar{w} = c$.

- Accept any job that pays more than unemployment benefits
- The reason is that the continuation value does not depend on employment status.

If $\alpha > \lambda$: $\bar{w} > c$.

- Being unemployed has a search value. So the agent holds out for better wage offers.

Reservation Wage

Add and subtract $V_U - V(w)$ in equation for $V(w)$:

$$(1-\beta)V(w) = w + \beta\lambda Q + \beta\lambda[V_U - V(w)] \tag{36}$$

Substitute out Q from equation for reservation wage

$$(1-\beta)V_U = \bar{w} + \beta\lambda Q \tag{37}$$

to obtain

$$(1-\beta)[V(w) - V_U] = w - \bar{w} + \beta\lambda[V_U - V(w)] \tag{38}$$

Solve for

$$V(w) - V_U = \frac{w - \bar{w}}{1 - \beta + \beta\lambda} \tag{39}$$

If we specified the distribution F, we could use this to evaluate Q and solve for everything else.

Applications

Life-cycle earnings profiles and occupational mobility:

- Kambourov and Manovskii (2009, 2008)

Business cycle models that match labor market facts:

- Jovanovic (1987)

What is missing?

- Not satisfactory: The job finding rate / wage offer distribution should be endogenous.
  - Think about analyzing policies...
- Matching and search models address this.
  - by introducing endogenous supply of jobs
  - and wage bargaining.

Reading

- Macroeconomics, ch. 20.9
- Ljungqvist and Sargent (2004), ch. 6.3
- Krusell (2014), ch. 11
- Williamson (2006), "Notes on macroeconomic theory," ch. 7, works out a similar model with exogenous job separations.

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