Heterogeneous Agent Models

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Bewley Models

For many applications, we need models with **heterogeneous agents**.

In **Bewley models**, agents are ex ante identical.
They are ex post heterogeneous because they are hit by idiosyncratic shocks.
Incomplete markets prevents sharing these risks.

In this section, we study a simple endowment economy.
The goal is to get the mechanics down in a simple setting.

Endowment Economy

An Endowment Economy

- Demographics:
  - There is a unit measure of households.
  - Each lives forever.
- Preferences:

$$E\sum_{t=0}^{\infty}\beta^{t}u(c_{t}) \tag{1}$$

- Technology:
  - Households receive random endowments $y_t \in Y$ (finite).
  - Transition matrix: $\pi(y'|y)$.
- Markets:
  - Goods (numeraire), one period bonds (interest rate r)

No aggregate uncertainty

- Assume a "law of large numbers."
- Let $\Pi(y)$ be the stationary distribution of y.
- Assume that the fraction of households with endowment y is $\Pi(y)$.
- The aggregate endowment $\bar{y}$ is constant over time.
- With complete markets, households would not face any uncertainty.

Household problem

Flow budget constraint:

$$a' = y + (1+r)a - c \tag{2}$$

Borrowing constraint:

$$a' \geq -b \tag{3}$$

Household problem

- Focus on a **stationary equilibrium**.
  - Meaning: Aggregates & prices are constant over time.
  - But individual households move around in the distribution.
- State vector: (a, y).
- Given: r.
- Bellman equation:

$$v(a,y) = \max u(c) + \beta \sum_{y'} \pi(y'|y) \; v(a',y') \tag{4}$$

s.t. budget constraint and borrowing constraint.

Household problem

The borrowing constraint may bind. We need Kuhn-Tucker.

Bellman equation

$$v(a,y) = \max u\left(y+(1+r)a-a'\right) \quad (5)$$

$$+\beta\sum_{y'}\pi\left(y'|y\right)\ v\left(a',y'\right)+\mu(a'+b) \quad (6)$$

First-order conditions:

$$u'(c) = \beta\sum\pi\left(y'|y\right)\frac{\partial v\left(a',y'\right)}{\partial a'}+\mu \quad (7)$$

$$\partial v/\partial a = u'(c)\ (1+r) \quad (8)$$

$$\mu(a'+b) = 0 \quad (9)$$

Household problem

Euler equation

$$u'(c) \geq E\beta(1+r)u'(c') \tag{10}$$

with equality if $a' > -b$.

Solution:

- $v(a,y), c(a,y), a'(a,y)$ that satisfy the usual conditions.

How does the household behave?

Assume (for the moment) that $y \sim iid$

Then (obviously?), consumption and saving depend on "cash on hand"

$$x = y + (1+r)a \tag{11}$$

Also (obviously?), a' is increasing in x

If x is sufficiently high:

- Choose $a' > -b$ and satisfy standard Euler equation.

If x is below a cutoff:

- Set $a' = -b$ and "violate" the Euler equation.

The borrowing constraint depresses current consumption.

- Precautionary saving

Recursive competitive equilibrium

Key feature of RCE

Everything in the economy is a function of the aggregate state S.
Agents also have individual states.

Agents form expectations using the law of motion for S: $S' = G(S)$

- E.g., to form expectations over future interest rate, use the law of motion for k and the price function $q = f'(k)$.

A fixed point problem:

- Agents' policy functions depends on the laws of motion.
- The laws of motion depend on agents' policy functions.

Stationary RCE

- Aggregate state:
  - The joint distribution of assets and endowments: $\Phi(a, y)$.
  - This is needed to compute aggregates.
- Objects:
  - Household: $v(a,y), c(a,y), a'(a,y)$.
  - $\Phi(a,y)$.
  - Price function: $r(\Phi)$.
- Equilibrium conditions:
  - Household: see above.
  - Market clearing.
  - Time invariance of Φ.

Market Clearing

Goods:

$$C = \int \int c(a,y)\,\Phi(da,dy) = \int y\Pi(dy) = \bar{y} \tag{12}$$

Bonds:

$$\int \int a'(a,y)\,\Phi(da,dy) = 0 \tag{13}$$

Time invariance of the distribution

Informally, household choices determine tomorrow's distribution Φ'.

The policy function $a'(a,y)$ implies a law of motion for Φ.

In stationary equilibrium, the law of motion must imply $\Phi' = \Phi$.

Law of motion for the distribution

Define a **transition function** $Q((a,y),(A,Y))$.

Its value is the probability that

- a household in state (a,y) today
- ends up in $(a',y') \in (A,Y)$ tomorrow.

$$Q((a,y),(A,Y)) = \begin{cases} \sum_{y' \in Y} \pi(y'|y) & \textit{if } a'(a,y) \in A \\ 0 & \text{otherwise} \end{cases}$$

In words:

- Need to draw $y' \in Y$ and
- Need to choose saving in A

Law of motion for the distribution

Law of motion

$$\Phi'(A,Y) = \int\int Q((a,y),(A,Y))\ \Phi(da,dy) \tag{14}$$

In words:

- $\Phi'(A,Y)$: What is the mass of households in the set of states (A,Y) tomorrow?
- For any (a,y), this is given by $Q((a,y),(A,Y))$.
- Sum over all (a,y) to get the total mass.

Stationarity then means: $\Phi'(A,Y) = \Phi(A,Y)$ for all (A,Y).

Quantitative Implications

A similar model was calibrated by Aiyagari (1994).

Main insights:

1. A simple bond yields a lot of **self-insurance**.
   Households build up enough wealth to stay away from the borrowing constraint most of the time.
   Consumption is quite smooth.
2. Most households have the same, low **MPC**
   This follows from self-insurance.
3. Wealth is more concentrated than earnings.
   But not nearly as concentrated as in the data.

But it's important that households live forever...

Non-stationary RCE

- Now Φ evolves over time.
- Household:
  - Add the aggregate state Φ to the household's state: $v(a, y, \Phi)$ and $a'(a, y, \Phi)$.
  - The household takes prices as functions of the aggregate state: $r(\Phi)$.
  - The household knows the law of motion for Φ: $\Phi' = H(\Phi)$.
- Equilibrium:
  - Drop stationarity of Φ.
  - Replace with a **consistency condition**.

Household Problem

$$v(a,y,\Phi) = \max u\left(y+(1+r(\Phi))a-a'\right) \tag{15}$$

$$+\beta\sum_{y'}\pi\left(y'|y\right)\ v\left(a',y',H(\Phi)\right)+\mu(a'+b) \tag{16}$$

Consistency Condition

The idea:

- Household decisions imply a distribution Φ'
- The decisions are based on the expectation $\Phi' = H(\Phi)$
- Both need to be consistent with each other.

$$\Phi'(A,Y) = \int\int Q((a,y),(A,Y)) \; \Phi(da,dy) = H(\Phi)(A,Y) \tag{17}$$

Reading

- Acemoglu (2009), ch. 17.4.
- Macroeconomics, ch. 11.4
- Krueger, "Macroeconomic Theory," ch. 10.

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

Aiyagari, S. R. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109, 659–684.