Growth Through Product Creation

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Issues

We study models where **intentional innovation** drives productivity growth.

Background:

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.
- How important are the 2 cases? – An open question.

How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
  - blueprints can be used simultaneously in the production of several goods.

How to model innovation

There are N consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- Therefore downward sloping demand curves

Approach 1: **Quality ladders**

- Each good can be made by many firms.
- Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: **Increasing variety**

- Each firm can invest to create a new variety $(N \to N+1)$
- Then it becomes the monopolist for that variety

2. A Model of Increasing Varieties

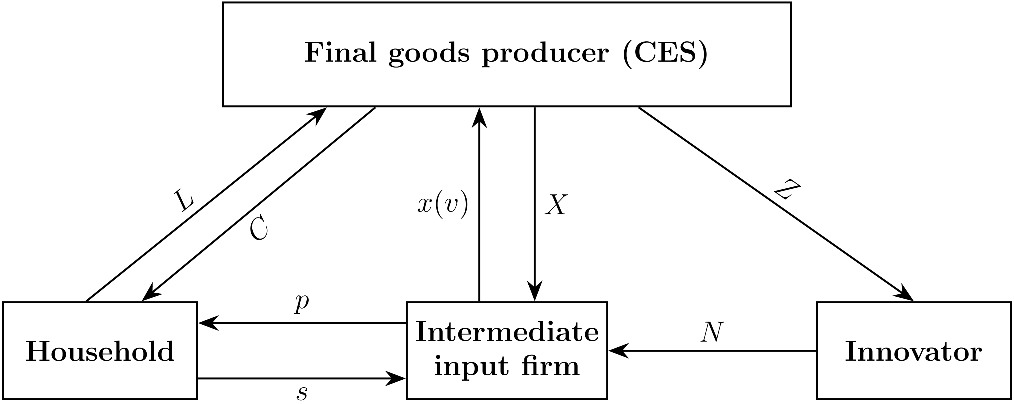
Setup

Agents:

1. A representative **household** supplies labor to firms
2. **Final** goods firms use labor and intermediate inputs
3. **Intermediate** inputs are produced from final goods
4. **Innovators**
   create new intermediates from final goods
   receive permanent monopolies

Note: Now that models get more complicated, it really pays off to be pedantic about details.

Model structure



The Story Line

Innovators

- buy goods from the final goods firm (Z).
- invent a new variety j
- receive a permanent patent for good j

Intermediate goods producers

- buy the patent from the innovator
- forever make x_j using the final good (X)
- sell it as monopolist to the final goods firm
- profits go to households

Demographics and Preferences

Demographics:

- A representative household.

Preferences:

$$\int_0^\infty e^{-\rho t} \, \frac{C_t^{1-\theta} - 1}{1-\theta} dt \tag{1}$$

- C: the final good

Technology: Final Goods

Resource constraint:

$$C_t + X_t + Z_t = Y_t$$

Final goods Y are used for

- Z: R&D investment.
- X: Inputs into the production of intermediates x.
- C: consumption

Technology: Final Goods

Production of **final goods** from intermediates and labor:

$$Y_t = \underbrace{(1-\beta)^{-1}}_{\text{normalization}} \mathcal{Q}^{1-\beta} L^{\beta} \tag{2}$$

where

$$\mathcal{Q} \equiv \left[\int_0^N x(v)^{1-\beta}\, dv\right]^{\frac{1}{1-\beta}} \tag{3}$$

is a CES aggregator of intermediate inputs x.

Simplify:

$$Y_t = (1-\beta)^{-1} \left[\int_0^{N_t} x(v,t)^{1-\beta}\, dv\right] L^{\beta} \tag{4}$$

Technology: Final Goods

$$Y_t = (1-\beta)^{-1} \left[\int_0^{N_t} x(v,t)^{1-\beta} \, dv \right] L^{\beta} \quad (5)$$

Properties:

1. Constant returns to purchased factors $x(v)$ and L
2. Increasing returns to all factors (including N_t)
3. Possibly constant returns to reproducible factors $x(v)$ and N
   But need parameters to take on just the right values ...

Demand Elasticity

This is the key trick of the model:

- the CES aggregator $\mathscr{Q}$ implies a **constant price elasticity** of demand for x
- assume that the marginal cost of x is constant
- if the suppliers of x are monopolists, their prices are a fixed markup over marginal costs

Then the price of x becomes a known function of parameters.

Technology: Intermediate Inputs

Each unit of x requires ψ units of Y.

- This is the constant marginal cost of x assumption.

The total amount of goods used to make intermediates is

$$X = \psi \int_0^{N_t} x(v,t)\,dv \tag{6}$$

Intermediate inputs fully depreciate in use.

Technology: Innovation

Investing the final good yields a flow of new patents:

$$\dot{N} = \eta Z_t \tag{7}$$

Think of this as the aggregate (deterministic) outcome of the (stochastic) innovation efforts of many firms.

Market arrangements

- Final goods and labor markets are competitive.
- Intermediates are sold by **monopolists** (the innovators).
  - Monopolies are permanent.
  - What the monopolists do with their profits is not clear.
- Free entry into innovation
  - ensures zero present value of profits
- The household owns the innovating firms.
- Asset markets are complicated
  - there is often no need to spell out the details

Notes

Production is cyclical:

- today's Y is used to make X which makes Y
- the alternative: durable X (more complicated)
- implication: the efficient allocation maximizes $Y - X = C + Z$

The only long-lived object is a patent

- this keeps the model simple

Assuming that intermediates are made from final goods fixes marginal costs (and prices)

3. Solving Each Agent's Problem

Final goods producers

- Maximize period profits by choosing L and $x(v,t)$.
- Take prices $p(v,t)$ as given.
- Normalize the price Y to 1.
- Profits

$$Y_t - w_t L_t - \int_0^{N_t} p^x(v,t)\ x(v,t)\,dv \tag{8}$$

where

$$Y_t = (1-\beta)^{-1}\left[\int_0^{N_t} x(v,t)^{1-\beta}\,dv\right]\ L^{\beta} \tag{9}$$

Final goods producers

FOCs:

- $\partial Y/\partial x(v) = L^{\beta} x(v)^{-\beta} = p^{x}(v)$
- $\partial Y/\partial L = \beta Y/L = w$

Demand function:

$$x(v,t) = L\, p^{x}(v,t)^{-1/\beta} \tag{10}$$

Note the constant price elasticity $1/\beta$.

Solution to the firm's problem: $L_t, x(v,t)$ that satisfy the "2" first-order conditions.

Intermediate input producers

Problem after inventing a variety.

x is produced at constant marginal cost ψ.

Maximize present value of profits

$$V(v,t) = \int_t^\infty e^{-rs} \pi(v,s)\, ds \tag{11}$$

Instantaneous profits are

$$\pi(v,t) = (p^x(v,t) - \psi)\, x(v,t) \tag{12}$$

where $x(v,t) = Lp^x(v,t)^{-1/\beta}$

This is a sequence of static problems

Intermediate input producers

- First order condition (standard monopoly pricing formula):

$$p^x = \frac{\psi}{1-\beta} = \frac{MC}{1 - 1/\text{price elasticity}} \tag{13}$$

- Profits are

$$\pi(v,t) = \psi \frac{\beta}{1-\beta} x(v,t) \tag{14}$$

- Solution: A constant p^x.

Innovators

Technology:

$$\dot{N} = \eta Z_t \tag{15}$$

Spend 1 unit of the good for one unit of time at total cost 1.

Obtain η new patents, each valued (initially) at $V(v,t)$.

Innovate as long as

$$\eta V(v,t) \geq 1 \tag{16}$$

In equilibrium, free entry makes (16) hold with equality.

Household

- The household holds shares of all intermediate input firms.
- Each firm produces a stream of profits.
- New firms issue new shares.
- But: the details don't matter to the household.
- There simply is an asset with rate of return r.
- Euler equation is standard:

$$g(C) = \frac{r - \rho}{\theta} \tag{17}$$

- Invoke Walras' law - so you never have to write down the budget constraint!

4. Equilibrium

Objects:

- allocation: $C_t, X_t, Z_t, x(v,t), V(v,t), N_t$
- prices: $p^x(v,t), r(t), w(t)$.

Conditions:

- "Everybody maximizes." (see above)
- Markets clear.
  1. Goods: resource constraint.
  2. Shares: omitted b/c I did not write out the household budget constraint.
  3. Intermediates: implicit in notation.
- Innovation effort satisfies a **free entry** condition: present value of profits equals 0.

Symmetric Equilibrium

We assume (and then show) that all varieties v share the same x, V and p^x.

Intuition:

- p^x: monopoly pricing with a constant elasticity
- x: varieties enter final goods production symmetrically
- V: the age of a variety does not matter
  (no stock of x to build; permanent patents)

Simplifications

Normalize marginal cost $\psi = 1 - \beta$

- so that $p^x = 1$.
- Why can I do that?

Focus on balanced growth paths.

5. Equilibrium: Characterization

There is an algorithm ...

- The growth rate follows from the Euler equation: $g(C) = (r - \rho)/\theta$.
- We get r from free entry by innovators: present value of profits = cost of creating a variety.

Equilibrium: Characterization

Free entry will determine the interest rate

Innovators enter until marginal cost equals value of patents created:

$$\eta V(v,t) = 1 \tag{18}$$

Then V is constant over time.

- This assumes that innovation takes place.

With balanced growth and constant profits (to be shown):

$$V = \int_0^\infty e^{-rt} \pi dt = \pi / r \tag{19}$$

Profits

With a fixed markup, profits are a multiple of revenues.

We derived (cf (14)):

$$\pi(t) = \psi \frac{\beta}{1-\beta} x(t) \tag{20}$$

$$= \beta x(t) \tag{21}$$

Demand for intermediates (cf (10)):

$$x(t) = L\, p^x(t)^{-1/\beta}$$

$$= L$$

Profits: $\pi = \beta L$.

Free Entry

Free entry:

$$\eta V = \eta \pi / r = \eta \beta L / r = 1 \tag{22}$$

This is the closed form solution for

$$r = \eta \beta L \tag{23}$$

Balanced **growth** rate then follows from the Euler equation.

$$g(C) = \frac{\eta \beta L - \rho}{\theta} \tag{24}$$

Equilibrium: Characterization

Production function for final goods with $x = L$:

$$Y = \frac{1}{1-\beta} N_t L \tag{25}$$

Wage (from firm's FOC):

$$w_t = \beta \frac{Y_t}{L_t} = \frac{\beta}{1-\beta} N_t \tag{26}$$

Total expenditure on intermediates:

$$X_t = \psi N_t x_t = (1-\beta) N_t L \tag{27}$$

Summary of Equilibrium

Prices and quantities of intermediate inputs are constant.

- the model is rigged to deliver this
- for tractability

Growth comes from rising N

No Transition Dynamics

The equilibrium looks like an AK model with production function

$$\begin{aligned} Y_t - X_t &= \left[\frac{L}{1-\beta} - (1-\beta) \right] N_t \\ \dot{N}_t &= \eta \; s_z \; (Y_t - X_t) \end{aligned}$$

Intuition:

- Period profits π are constant at βL.
- At any moment we need $\eta V = 1$.
- V is the present value of (constant) profits.
- Constant V is only possible with constant r.
- Then at least consumption growth must be constant.

Scale Effects

$$g(C) = \frac{\eta\beta L - \rho}{\theta}$$

Larger economies (L) grow faster.

Population growth implies exploding income growth (!)

Mechanical reason:

- Innovation technology is linear in goods.
- Larger economy $\rightarrow$ higher Y $\rightarrow$ higher Z $\rightarrow$ faster growth.

We will return to this later.

Reading

- Macroeconomics, ch. 13.5
- Acemoglu (2009), ch. 13.
- Krusell (2014), ch. 9
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

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