1 Lucas Fruit Trees With Crashes

Demographics: There is a single, representative household who lives forever.

Preferences: $U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $u(c) = c^{1-\sigma}/(1-\sigma)$.

Endowments: The agent is endowed at $t=0$ with 1 tree. In each period, the tree yields stochastic consumption $d_t$, which cannot be stored. $d_t$ evolves as follows:

- If $d_t = d_{t-1}$, then $d_{t+1} = d_t$ forever after.
- If $d_t \neq d_{t-1}$, then $d_{t+1} = \gamma d_t$ with probability $\pi$ and $d_{t+1} = d_t$ with probability $1-\pi$. $\gamma > 1$.

In words: $d$ grows at rate $\gamma - 1$ until some random event occurs (with probability $1-\pi$), at which point growth stops forever.

Markets: There are competitive markets for consumption (numeraire) and trees (price $p_t$). Assume that $p_t$ is *cum dividend*, meaning that $d_t$ accrues to the household who buys the tree in $t$ and holds it into $t+1$.

Questions:

1. State the household’s dynamic program.

2. Derive the Euler equation.

3. Define a recursive competitive equilibrium. Key: what is the state vector?

4. Characterize the stochastic process of $p_t$. Is $p_t$ a Markov process? Hint: there are 2 phases: before and after dividends have stopped growing. Assume that $p/d$ is constant during the phase with growth.

5. What happens to the stock market when the economy stops growing? Does it crash? Under what condition?