Problem Set 7: Asset Pricing

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1 Lucas Fruit Trees With Crashes

Demographics: There is a single, representative household who lives forever.

Preferences: $U = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $u(c) = c^{1-\sigma} / (1-\sigma)$.

Endowments: The agent is endowed at $t = 0$ with 1 tree. In each period, the tree yields stochastic consumption d_t, which cannot be stored. d_t evolves as follows:

- If $d_t = d_{t-1}$, then $d_{t+1} = d_t$ forever after.
- If $d_t \neq d_{t-1}$, then $d_{t+1} = \gamma d_t$ with probability π and $d_{t+1} = d_t$ with probability $1 - \pi$. $\gamma > 1$.

In words: d grows at rate $\gamma - 1$ until some random event occurs (with probability $1 - \pi$), at which point growth stops forever.

Markets: There are competitive markets for consumption (numeraire) and trees (price p_t). Assume that p_t is *cum dividend*, meaning that d_t accrues to the household who buys the tree in t and holds it into $t + 1$.

Questions:

1. State the household's dynamic program.
2. Derive the Euler equation.
3. Define a recursive competitive equilibrium. Key: what is the state vector?
4. Characterize the stochastic process of p_t. Is p_t a Markov process? Hint: there are 2 phases: before and after dividends have stopped growing. Assume that p/d is constant during the phase with growth.
5. What happens to the stock market when the economy stops growing? Does it crash? Under what condition?

2 Answer: Lucas Fruit Trees With Crashes

[Based on a question due to Rodolfo Manuelli]

1. The household problem is standard, except for the stock price being *cum dividend*:
$$V(k, S) = \max u(c) + \beta \mathbb{E} \{V(k', S')\} \tag{1}$$
where S is the aggregate state, subject to
$$p(k' - k) + c = dk' \tag{2}$$

2. Euler:

$$p_t = d_t + \beta \mathbb{E} \left\{ \frac{u'(c_{t+1})}{u'(c_t)} p_{t+1} \right\} \tag{3}$$

3. The distribution of d_{t+1} depdends on d_t and d_{t-1}. These form the state vector.
   Objects: $V(k; S)$, $\kappa(k, S)$, $p(S)$
   Equilibrium conditions:
   - V and κ solve the household problem
   - goods market clearing: $c(k, S) = d$
   - asset market clearing: $\kappa(k, S) = 1$
   - law of motion of the aggregate state (exogenous)

4. Stock prices:
   Once growth has stopped: $c = d' = d$, so that $p - d = \beta p \implies p = d/(1-\beta)$.
   While growth continues:

$$p_g = d + \beta \pi \gamma^{\sigma} p_g' + \beta (1-\pi) d/(1-\beta) \tag{4}$$

   Divide by d:

$$p_g/d = 1 + \beta \pi \gamma^{1-\sigma} p_g'/d' + \beta (1-\pi)/(1-\beta) \tag{5}$$

   or:

$$p_g/d = \frac{1 + \beta (1-\pi)/(1-\beta)}{1 - \beta \pi \gamma^{1-\sigma}} \tag{6}$$

   The price is Markov in the state $S = (d, d_{-1})$.

5. The growth slowdown leads to a stock market crash, if $p_g/d < p/d$ or

$$\frac{1 + \beta (1-\pi)/(1-\beta)}{1 - \beta \pi \gamma^{1-\sigma}} < \frac{1}{1-\beta} \tag{7}$$

   or

$$\beta \pi > \beta \pi \gamma^{1-\sigma} \tag{8}$$

   Whether the growth slowdown crashes the stock market depends on the sign of $1 - \sigma$ (the curvature of preferences or the relative strengths of income and substitution effects).