

Problem Set 1: OLG Models

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1 OLG with Arrow-Debreu

Consider the standard OLG production economy with Arrow-Debreu trading.

Demographics: N_t young are born in t . Each lives for 2 periods.

Endowments: Each young has 1 unit of work time. The initial old each have K_0/N_{-1} units of capital.

Preferences: $u(c_t^y) + \beta u(c_{t+1}^o)$.

Technology: $F(K_t, L_t) + (1 - \delta)K_t = K_{t+1} + N_t c_t^y + N_{t-1} c_t^o$.

Markets:

- Trading takes place in $t = 0$.
- Goods are traded at price p_t . Capital is rented (to firms) at rate q_t (in units of account). Labor is rented at rate w_t (in units of account).

Questions:

1. Explain why the following is the correct budget constraint:

$$w_t + q_{t+1}s_{t+1} + (1 - \delta)p_{t+1}s_{t+1} = p_t c_t^y + p_{t+1}c_{t+1}^o + p_t s_{t+1} \quad (1)$$

2. Derive the household's first-order conditions.
3. Define a solution to the household problem.
4. What is the real interest rate in this economy?
5. Interpret the condition $p_t = (1 - \delta)p_{t+1} + q_{t+1}$.
6. State the firm's first-order conditions. Watch your units!
7. Define a competitive equilibrium.
8. Why don't we lose an equation due to Walras' law?
9. Where is the numeraire?
10. Define a steady state.
11. Under what condition do the Welfare theorems hold / fail? Recall that the Welfare theorems require $\lim_{t \rightarrow \infty} p_t = 0$.

2 OLG Model with Assets

Demographics: There are two types of households, indexed by h . In each period, a mass of 0.5 households is born of each type. Each person lives for 2 periods.

Endowments: Households receive endowments (e^y, e^o) when young and old, respectively.

Preferences: $\ln(c_{h,t}^y) + \beta_h \ln(c_{h,t+1}^o)$.

Technologies: Endowments can only be eaten.

Markets: Households trade goods and one period bonds that are issued and purchased by households.

Questions:

1. Define a solution to the household problem. Solve for the household's bond supply function.
2. Solve for the equilibrium bond interest rate.
3. Your solution for R should reveal the following features: (i) If old endowments are larger, R is higher. (ii) If β_h increases, R decreases. (iii) R is time invariant. Provide intuition for these features.
4. Now add a durable good to the economy. It is in fixed supply, K . It pays a dividend d per period (in units of consumption goods). Households trade "shares" of this good in an asset market at price p_t , measured in units of consumption goods. Define a competitive equilibrium for this economy. Hint: Explain why $R_{t+1} = (d + p_{t+1})/p_t$ must hold.
5. Why do you find that the number of equations equals the number of objects to be determined? Usually, we find that we have one additional equation, which is redundant by Walras' law.
6. Derive an equation that determines the equilibrium price sequence p_t .