Money in the Utility Function

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Econ720

July 29, 2019
Money in the utility function

- A shortcut for getting money valued in equilibrium: assume that households gain utility from holding money.
- "Sidrauski" model.
- Benefits: Tractability.
- Drawbacks: Arbitrary specification of utility affects results.
The Economic Environment

- Much of the model is a standard growth model.
- The government prints paper (costlessly).
- Households gain utility from holding paper.
Environment

- Demographics: 1 representative household
- Endowments:
  - 1 unit of work time at each instant
  - $k_0$ units of the good
  - $M_0$ bits of paper
- Technology:
  - $F(K, L) - \delta K = c + \dot{K}$
Environment

- Preferences:
  \[ \int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \]  
  \[ m_t = \frac{M_t}{p_t} \]

- Government:
  - prints \( \dot{M}_t \) and hands it to households

- Markets:
  - goods, labor, capital rental, money
Households solve:

\[
\max \int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt
\]

subject to \(k_0, m_0\) given and

\[
p(c + \dot{k}) + \dot{M} = p(w + rk + x)
\]

\(x\) are lump-sum transfers (of money).
Budget constraint in real terms

\[ \dot{k} + \dot{M}/p = w + rk + x - c \]  \hspace{1cm} (4)

Note that

\[ \dot{m} = \dot{M}/p - (M/p^2)\dot{p} \]
\[ = \dot{M}/p - m\pi \]

where \( \pi \) is the inflation rate \((\pi = \dot{p}/p)\).

Therefore

\[ \dot{k} + \dot{m} = w + rk + x - c - \pi m \]  \hspace{1cm} (5)
We seem to have 2 state variables \((k, m)\) but only one law of motion.

The reason: the correct state variable is wealth: \(A = k + m\).

To transform the budget constraint into a law of motion for \(A\), write it as

\[
\dot{A} = w + rA + x - c - (r + \pi)m
\]  

(6)

Every unit of wealth held in money reduces income by the nominal interest rate \((r + \pi)\).
An Equivalence

- The household problem is exactly the same as in a real two-good economy.
- Money is like a consumption good with price $r + \pi$. 
Solving the household problem

\[ H = u(c, m) + \lambda [w + rA + x - c - (r + \pi)m] \quad (7) \]

FOC:

\[ u_c = \lambda \]
\[ u_m = \lambda (r + \pi) \]
\[ \dot{\lambda} = (\rho - r)\lambda \]

TVC:

\[ \lim_{t \to \infty} e^{-\rho t} \lambda_t A_t = 0 \]
Household optimality

Static condition:

\[ u_c = u_m / (r + \pi) \]  \hspace{1cm} (8)

Intuition?

Intertemporal condition:

\[ \frac{\dot{\lambda}}{\lambda} = g(u_c) = -(r - \rho) \]  \hspace{1cm} (9)

where \( g(z) \equiv \frac{\dot{z}}{z} \) denotes a growth rate.
If the utility function is separable,

\[ u(c, m) = v(c) + \bar{v}(m) \quad (10) \]

then

\[ u_c = v'(c) \quad (11) \]

and

\[ g(u_c) = v''(c) \frac{\dot{c}}{v'(c)} = -\sigma g_c \quad (12) \]

Then a very common expression emerges:

\[ g(c) = \frac{(r - \rho)}{\sigma} \quad (13) \]
Firms solve the standard static profit maximization problem:

\[ r = f'(k) - \delta \quad (14) \]
\[ w = f(k) - f'(k)k \quad (15) \]
The government grows the money supply at the constant rate $\mu = g(M)$.

Implied lump-sum transfers are

$$x = \frac{\dot{M}}{p} = \mu m$$

(16)
Market clearing

- Money and factor market clearing are implicit in the notation.
- Goods market clearing is feasibility:

\[
\dot{k} + c = f(k) - \delta k
\]  
(17)
Equilibrium

An equilibrium is a set of functions of time that satisfy

These are 9 variables and 10 equations. The boundary conditions are initial values for $M$ and $k$ and the TVC.
Characterization

- We reduce the CE to 4 equations in \((c, p, k, m)\).

- Household first-order conditions:
  \[
  g(u_c[c, m]) = -(f'(k) - \delta - \rho) \\
  u_c(c, m) = \frac{u_m(c, m)}{(f'(k) - \delta + \pi)}
  \]

- Goods market clearing:
  \[
  \dot{k} + c = f(k) - \delta k
  \]

- Money growth rule:
  \[
  \dot{m} = (\mu - \pi)m
  \]
Monetary Neutrality

Assume: the utility function is additively separable

$$u(c,m) = \bar{u}(c) + v(m)$$  \hspace{1cm} (18)

Then money has absolutely no effect on the real sector.

The evolution of $c$ and $k$ is determined by the Euler equation and the goods market clearing condition alone.

Intuition?
Steady state

In steady state $c, k, m$ are constant.

The Euler equation then determines the steady state capital stock:

$$ r = f'(k) - \delta = \rho \quad (19) $$

Goods market clearing then yields consumption:

$$ c = f(k) - \delta k $$

Constant real balances require $\pi = \mu$.

The static optimality condition yields an implicit equation for $m$:

$$ u_m(c_{SS}, m_{SS}) = (\rho + \mu)u_c(c_{SS}, m_{SS}) \quad (20) $$

$$ \Rightarrow \quad m_{SS} = m^d(c_{SS}, \rho + \mu) \quad (21) $$
Super-neutral money

- Changes in money growth ($\mu$) only affect the inflation rate, but not real variables ($k_{ss}, c_{ss}$).
- Intuition: inflation does not alter the intertemporal tradeoff between consumption today and tomorrow.
- Inflation only affects the relative levels of goods and money consumed
Inflation and welfare

What is the effect of inflation on real money balances? Differentiate (20) to obtain

$$u_{mm} dm = (\rho + \mu) u_{cm} dm + u_c d\mu$$  \hspace{1cm} (22)

$$\Rightarrow$$

$$dm/d\mu = u_c / [u_{mm} - (\rho + \mu) u_{cm}]$$  \hspace{1cm} (23)

Unless money and consumption are too strong complements ($u_{cm}$ large and positive), higher inflation is associated with lower real money balances and thus lower steady state utility.
The Friedman Rule

- Which money growth rate maximizes steady state utility?
- Since $\mu$ does not affect $c_{ss}$, we only need to know how to maximize $m_{ss}$.
- If we set $\rho + \mu = 0$, then $u_m = 0$, which is the best we can do: satiate the household with money.
- If $u_m > 0$ even asymptotically, the problem does not have a solution.
- The intuition is quite general:
  - If money provides some kind of benefit, the best we can do is to make it costless to hold money.
  - That will be the case when money pays the same rate of return as capital (the Friedman rule).
Is This a Good Theory of Money?

Pros:
- tractable

Cons:
- the value of money is assumed
  therefore: no non-monetary equilibrium / hyperinflation
- money is not used in transactions
  it’s really a consumption good
Where Is this Used?

Models of the financial sector, where the details why households hold money play a minor role

Reading

- Blanchard and Fischer (1989), ch. 4.5