# Aggregation

#### Prof. Lutz Hendricks

Econ720

October 27, 2022

We have assumed a representative household.

How restrictive is this assumption?

If households are not identical, do they "aggregate" into a representative household?

Recall the Perpetual Youth model:

there was a representative household, but the Euler equation was different from that of an individual.

# Example with Heterogeneity

# Example with Heterogeneity

- Consider a Cass-Koopmans model with two types of households, *i* = 1,2.
- Demographics:
  - The population of each type is constant  $(N^i)$ .
- Preferences are identical:  $\int_0^\infty e^{-\rho t} \frac{c^{1-\sigma}-1}{1-\sigma} dt$ .
- Endowments:
  - Each household starts with capital  $k_0^i$ .
  - Each has one unit of type *i* time at any moment.

Example with Heterogeneity



$$Y_t = K_t^{\theta} \left[ (L_t^1)^{1-\theta} + (L_t^2)^{1-\theta} \right]$$
  
=  $\dot{K}_t + \delta K_t + C_t.$ 

#### ▶ Note: Each household supplies a different type of labor.

# Household

- ► The household problem is entirely standard.
- Solution is  $k_t^i$  and  $c_t^i$  which satisfy Euler equation

$$g(c_t^i) = (r - \rho) / \sigma \tag{1}$$

and budget constraint:

$$\dot{k}^i = rk^i + w^i - c^i \tag{2}$$



### Firm

► Factor prices equal marginal products.

• 
$$q = F_k$$
 and  $w^i = F_{L^i}$ .

# Equilibrium

A CE consists of functions of time  $c^i, k^i, w^i, r, q, K, L^i$  that satisfy

- 2x2 household conditions
- 3 firm first order conditions
- Factor market clearing:  $K = \sum k^i L^i$  and  $L^i = N^i$
- Goods market clearing:  $F(K, L^1, L^2) \delta K = \dot{K} + \sum L^i c^i$
- Identity:  $r = q \delta$

### Representative Household

- We now show that the entire economy behaves as if a representative household chose consumption.
- From lifetime budget constraint: present value of consumption = present value of income + initial assets

$$c_0^i \Pi_0 = k_0^i + PV_0\left(w^i\right)$$

where

$$\Pi_{0} = \int_{0}^{\infty} \exp\left(\int_{0}^{t} \left[g\left(c_{\tau}\right) - r_{\tau}\right] d\tau\right)$$

### Representative Household

Aggregate consumption

$$C_{0} = \sum_{i} L_{i} c_{0}^{i} = \sum_{i} L_{i} \left( k_{0}^{i} + PV_{0} \left( w^{i} \right) \right) / \Pi_{0}$$
(3)  
=  $K_{0} / \Pi_{0} + PV_{0} \left( \sum_{i} w^{i} L_{i} \right) / \Pi_{0}$ (4)

The level is what a household who owns all capital and labor would choose.

### Representative Household

The growth rate of aggregate consumption obeys the individual Euler equation:

$$g(C_t) = \frac{\sum_i L_i \dot{c}_t^i}{\sum_i L_i c_t^i} = \sum_i \frac{L_i c_t^i}{\sum_i L_i c_t^i} g(c_t^i) = g(c_t^i)$$
(5)

Why is this true?

Because the marginal propensity to consume out of capital / labor income is the same for all households.

This would fail if utility were not iso-elastic.

Then  $g(c_t^i) = (r_t - \rho) / \sigma(c_t^i)$  is not independent of the level of  $c_t^i$ 

# Steady State

The same results are easier to see in steady state. A steady state is: the same objects (but as scalars):  $c^{i},k^{i},w^{i},r,q,K,L^{i}$ .

These satisfy, in sequential order:

- Labor inputs are exogenous.
- $F_K = \rho + \delta$  determines *K*.
- $\blacktriangleright$   $r = \rho$ .
- $w^i = (1 \theta) (K/L^i)^{\theta}$  determines  $w^i$ .

# Steady State

We then have an additional 3 equations:

1. capital market clearing:

$$K = \sum k^i L^i \tag{6}$$

2. household budget constraints with  $\dot{k}^i = 0$ :

$$c^{i} = \rho \, k^{i} + w^{i} \tag{7}$$

The 3 equations are supposed to determine 4 variables:  $c^i, k^i$ .

# Steady State

- The steady state is not unique.
- Any  $k^i$  that sum to K are a steady state.
- For any k<sup>i</sup> pair we pick, the budget constraints tell us the corresponding steady state consumption levels.

# Why is the steady state not unique?

- Both households have the same marginal propensity to consume: ρ.
- Redistribute a bit of k<sup>1</sup> to k<sup>2</sup>. Aggregate C is unchanged. All markets clear.
- Effectively, the households behave as if they were one a representative household.
- This is good: when it works, we don't have to explicitly model heterogeneous households.

# The Representative Household

### The representative household

How hard is it to get a representative household?

One perspective:

Any aggregate demand curve is consistent with optimal behavior by a set of households.

#### Theorem

(Debreu-Mantel-Sonnenschein) Let  $\varepsilon > 0$  be a scalar and  $N < \infty$  be a positive integer. Consider a set of prices  $P_{\varepsilon} = \{p \in \mathbb{R}^{N}_{+} : p_{j}/p_{j'} \ge \varepsilon \forall j, j'\}$  and any continuous function  $x : P_{\varepsilon} \to \mathbb{R}^{N}_{+}$  that satisfies Walras' Law and is homogeneous of degree 0. Then there exists an exchange economy with N commodities and  $H < \infty$  households, where the aggregate demand is given by x(p) over the set  $P_{\varepsilon}$ .

# Why is aggregation so hard?

- ► The problem is income effects.
- Changing prices effectively redistributes income across households.
- If the income elasticities of various goods are very different, demand curves could be upward sloping over some intervals.
- But there is hope if income effects are not too strong.

### Gorman aggregation

#### Theorem

(Gorman aggregation) Consider an economy with a finite number N of commodities and a set H of households. Suppose that the preferences of household  $i \in H$  can be represented by an indirect utility function of the form

 $v^{i}(p, y^{i}) = a^{i}(p) + b(p)y^{i}$ 

then these preferences can represented by those of a representative household with indirect utility

$$v(p,y) = \int a^{i}(p) di + b(p) y$$

where y is aggregate income.

### Gorman aggregation

- Key feature of Gorman preferences:
  - All households have the same constant propensity to consume out of income.
- This is why redistributing income does not change consumption.
- Then aggregate income is sufficient to figure out demand.

# **CES** Preferences

The growth model has CES preferences:

$$\sum_{k=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma}$$

- CES preferences are consistent with balanced growth.
- This is because the marginal propensity to consume is constant on the balanced growth path.
- This is why redistribution does not change aggregate consumption.

### Implications

Exact aggregation is rare.

How worried should we be?

One faction of economists views representative agent models as toy models.

Another faction is more pragmatic:

- start with a simple model
- check whether heterogeneity makes a quantitatively significant difference

# Application: Labor Supply Elasticity

# Application: Labor supply elasticity

How responsive are hours worked to wages? Micro literature:

- weak correlation of hours and wages in panel data
- labor supply elasticities are near 0

Macro literature:

- over the business cycle, small wage fluctuations lead to large movements in hours
- labor supply elasticity must be large

How to reconcile?

### Model

Household maximizes

$$\sum_{a=1}^{T} \beta^{a} \left[ \frac{c^{1-1/\eta}}{1-1/\eta} - \alpha \frac{h^{1+1/\gamma}}{1+1/\gamma} \right]$$
(8)

Present value budget constraint

$$\sum_{a=1}^{T} \beta^{a} c_{a} = \sum_{a=1}^{T} \beta^{a} (1-\tau) e_{a} h_{a} w + z$$
(9)

Assumptions:

- interest rate = discount rate
- z: lump sum transfer that rebates labor income tax revenue
- *e<sub>a</sub>*: productivity

How could one estimate the labor supply elasticity?

First order conditions:

$$U_c = c_a^{-1/\eta} = \lambda \tag{10}$$

$$-U_h = \alpha h_a^{1/\gamma} = (1 - \tau) \lambda e_a w \tag{11}$$

where  $\lambda$  is the marginal utility of wealth (Lagrange multiplier). Estimation equation:

$$\ln h_a = b(\lambda) + \gamma \ln e_a \tag{12}$$

where

- $b(\lambda)$  depends on parameters and  $\lambda$
- in the regression, e<sub>a</sub> can be replaced by the observed wage per hour

Equations of the form

$$\Delta \ln h_{it} = \gamma \Delta \ln \left( w_{it} \left( 1 - \tau_{it} \right) \right) + X_{it} \beta + \varepsilon_{it}$$
(13)

have been estimated many times in the micro literature.

Consensus result: the labor supply elasticity  $(\gamma)$  is near 0. MaCurdy (1983): a 10% permanent wage change implies a 0.8% change in hours.

- 1. The labor supply elasticity is a preference parameter.
- 2. If preferences are age invariant, the labor supply elasticity is the same for all ages.
- 3. Then the aggregate labor supply elasticity is the same as the individual one.
- 4. The labor supply elasticity is small.

### Aggregation

Now consider the same model with a nonconvexity in the mapping of hours to efficiency

- the idea: there is a fixed cost of working
- accounts for the fact that many work full time

Earnings are now

$$(1-\tau)we_a\left(h-\bar{h}\right) \tag{14}$$

Implication: there is an extensive margin

 workers who would choose low hours in the standard model now choose 0 hours.

Fact: most empirical variation in hours happens along the extensive margin

### Macro results

Rogerson and Wallenius (2009) calibrate such a model Results:

- 1. The estimated micro labor supply elasticity is only about half the size of  $\gamma$
- 2. The aggregate labor supply elasticity is large: a 20% increase in the tax implies a 75% decrease in labor supply

Intuition:

- small elasticity at the intensive margin (estimated by micro elasticities),
- but large elasticity at the extensive margin.
- also large changes in retirement ages

# Reading

- Acemoglu (2009), ch. 5.
- The labor supply elasticity material is based on Keane and Rogerson (2012)

# References I

- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Keane, M. and R. Rogerson (2012): "Micro and macro labor supply elasticities: A reassessment of conventional wisdom," *Journal of Economic Literature*, 50, 464–476.
- MaCurdy, T. E. (1983): "A simple scheme for estimating an intertemporal model of labor supply and consumption in the presence of taxes and uncertainty," *International Economic Review*, 265–289.
- Rogerson, R. and J. Wallenius (2009): "Micro and macro elasticities in a life cycle model with taxes," *Journal of Economic theory*, 144, 2277–2292.