1 Ben-Porath Model

We study the decision problem of an infinitely lived agent in discrete time. At \( t = 0 \), the agent is endowed with \( h_0 \) units of human capital. In each period, he can invest \( l_t \) units of time, so that human capital evolves according to

\[
\begin{align*}
    h_{t+1} &= (1 - \delta) h_t + F(h_t l_t) \\
    F(hl) &= (hl)^\alpha
\end{align*}
\]

with \( 0 < \alpha, \delta < 1 \). The objective is to maximize the present value of lifetime earnings, given by

\[
    Y = \sum_{t=0}^{\infty} R^{-t} w_t h_t (1 - l_t)
\]

where \( R > 0 \) is taken as given.

Questions:

1. Write down the agent’s Dynamic Program.
2. Derive and interpret the first-order condition for \( l \).
3. Derive \( V'(h) = w + (1 - \delta) R^{-1} V'(h') \).
4. Derive and interpret \( V'(h) = w \frac{R}{R + \delta} \) where \( R = 1 + r \).
5. How do the wage and the interest rate affect steady state \( h \) and \( l \)?

2 Education Costs

Consider the following version of a standard growth model with human capital. The planner solves

\[
\begin{align*}
    \max \sum_{t=1}^{\infty} \beta^t u (c_t) \\
    \text{s.t.} \\
    k_{t+1} &= (1 - \delta) k_t + x_{kt} \\
    h_{t+1} &= (1 - \delta) h_t + x_{ht} \\
    c_t + x_{kt} + \eta x_{ht} &= f(k_t, h_t)
\end{align*}
\]
with $k_1$ and $h_1$ given. Here $c$ is consumption, $k$ is physical capital, $h$ is human capital, and $\eta$ is a constant representing education costs. Assume that the production function is Cobb-Douglas:

$$f(k, h) = z k^\alpha h^\varepsilon$$

where $z$ is a constant technology parameter and $\alpha + \varepsilon < 1$.

Questions:

1. Derive the first-order condition for the planner’s problem using Dynamic Programming. Define a solution in sequence language and in functional language.

2. Solve for the steady state levels of $k/h$ and $k$.

3. Characterize the impact of cross-country differences in education costs ($\eta$) on output per worker in steady state. In particular, calculate the ratio of outputs per worker for two countries that only differ in their $\eta$'s.