Wealth Distribution: Motivation and Baseline Model

Prof. Lutz Hendricks

Econ720

November 22, 2019

The Question

We study heterogeneous agent models of the wealth distribution.

Theoretical objective:

- learn how to define equilibrium
- how to take such models to the data
- also think a bit about computing equilibrium

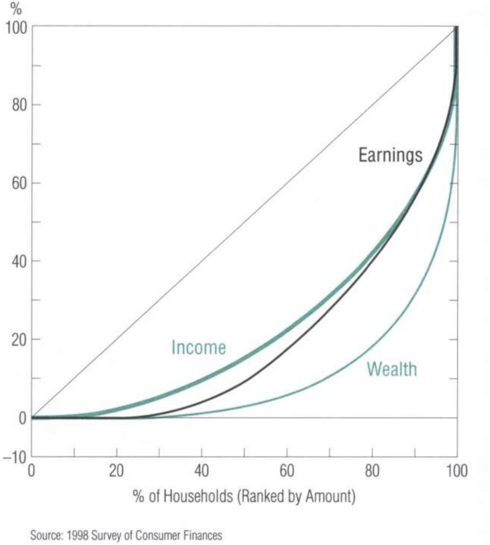
Applied objective:

- introduction to an important literature (wealth distribution)
- a newer literature: taxing top incomes

Data: U.S. Wealth Distribution

- Top 1% hold 28% of total wealth
- Top 5% hold half of total wealth
- Bottom 40% hold essentially nothing
- Gini: 0.72

Lorenz curves



Source: Rodríguez et al. (2002)

Baseline model: Huggett (1996)

- We start with a classic paper: Huggett (1996)
- The question
  - to what extent can a standard life-cycle model with idiosyncratic earnings risk account for the observed concentration of wealth?
- Model ingredients:
  - uninsured shocks
  - finite lives
  - ex ante identical agents (Bewley model)

Demographics / Preferences

- Demographics
  - in each period 1 unit mass of agents are born
  - they live at most N periods
  - exogenous survival probabilities s_j
- Preferences

$$\mathbb{E}\sum_{t=1}^{N}\beta^t(\prod_{j=1}^{t}s_j)u(c_t) \tag{1}$$

Endowments / Technologies

Endowments

- an agent of age t is endowed with $e(z,t)$ units of work time (experience efficiency profile)
- z is a Markov productivity shock

Technology

$$Y = AK^{\alpha}L^{1-\alpha} \tag{2}$$

Markets

- labor rental (wage w)
- capital rental (interest rate r)
- good (price 1)
- risk free bonds (interest rate r)

Government

- taxes income rate rate τ
- social security tax θ pays old age transfers b
- lump-sum transfers T redistribute accidental bequests

Key model ingredients

This is the simplest extension of a standard growth model that makes sense.

Finite lives

- age is important for wealth (inequality)

Stochastic deaths

- otherwise the old dissave too much

Earnings heterogeneity

- how far can we go with only this?

No aggregate uncertainty

- the only source of uncertainty is the idiosyncratic z shock

Household Problem

Individual state: $x = (a, z)$

Bellman

$$V(x,t) = \max_{c,a'} u(c) + \beta s_{t+1} \mathbb{E} V(a', z', t+1) \tag{3}$$

subject to

$$c + a' = a(1 + r[1-\tau]) + (1 - \theta - \tau) e(z,t) w + T + b_t \tag{4}$$

$$a \geq \underline{\mathsf{a}} \tag{5}$$

Terminal value: $V(x, N+1) = 0$

Equilibrium

Focus on stationary equilibria.

Aggregate state:

- joint distribution of (a, z) for each age t
- density for age t: $\psi_t(B)$ where B is a set of states

Transition function: $P(x, t, B) = \Pr(x' \in B | x, t)$.

Stationarity condition

Stationarity of distribution requires

$$\psi_t(B) = \int_X P(x, t-1, B) d\psi_{t-1}(x) \tag{6}$$

In words:

- today's distribution for age $t-1$ is ψ_{t-1}
- agents make choices that induce transitions described by P
- then tomorrow's distribution for age t is ψ_t (for the same ψ)

Stationary Equlibrium

Objects:

- household: $c(x,t), a' = g(x,t), V(x,t)$
- prices: r, w
- policies: τ, θ, b_t, T, G
- aggregates: K, L

All of these are functions of the aggregate state, but that is a constant, so we don't need to worry about it.

Equilibrium conditions

- households "maximize"
- firm first-order conditions
- government budget constraint

$$G = \tau(rK + wL) \tag{7}$$

- social security budget constraint

$$\theta wL = b \sum_{t=R}^{N} \mu_t \tag{8}$$

  where μ_t is the mass of persons aged t
- market clearing
- stationarity of the aggregate state

Market Clearing

Goods

$$F(K,L)+(1-\delta)K = G + \sum_t \mu_t \int_X [c(x,t)+g(x,t)]d\psi_t(x) \tag{9}$$

Capital

$$K = \sum_t \mu_t \int_X a(x,t)d\psi_t(x) \tag{10}$$

Labor

$$L = \sum_t \mu_t \int_X e(z,t)d\psi_t(x) \tag{11}$$

Perspective

Why did Huggett choose this model?

He was aiming for the simplest, most standard model as a benchmark.

The goal is not to fit the empirical distribution, but starting to understand what it might take to fit it.

Key ingredients of the model:

- finite lives: because a chunk of wealth heterogeneity comes from cross-age variation
- a single source of heterogeneity: earnings shocks (clearly important)

Calibration

How to quantify the model's implications?

General approach:

- set model parameters
- simulate many households
- compute statistics from simulated histories (wealth distribution, ...)
- search over parameters until model moments "match" data moments

Setting model parameters

Set some parameters based on outside evidence

- e.g. capital share in production function $= 1/3$
- tax rates
- stochastic process for earnings

The remaining parameters can be set through

1. calibration
2. estimation

Estimation

Roughly speaking:

- add "error terms" to the model equations
- add covariates to the model equations (e.g. utility depends on family size, marital status, ...)
- simulate households observed in the data (with their covariates)
- search over model parameters that optimize the "fit" of the model somehow

Note: in "micro" models, error terms and covariates are built into the model from the start.

Example: MLE

- maximize the likelihood of the error terms

Calibration

1. Set calibration targets
   - data moments that seem informative about the calibrated parameters
   - e.g.: discount factor affects K/Y
2. Simulate model and compute the same moments (e.g. K/Y)
3. Find parameters that minimize the "distance" between model moments and data moments.

The model contains no error terms or covariates.

Calibration

Simplest case: exactly identified

- the number of calibrated parameters matches the number of moments
- the model matches the moments exactly

More common these days: overidentified

- number of targets > number of calibrated parameters
- the minimize a "distance" between data moments and model moments

Which Approach Is Better?

Researchers disagree.

Both approaches are widely used.

Estimation is always used in micro.

Both methods are used in macro.

Some papers are in between.

- especially those that use indirect inference or simulated method of moments

Benefits of calibration

1. can target moments that matter
   prevent "incidental moments" from driving results
2. more transparent
   clearer intuition about data features that drive results
3. computationally less expensive than estimation
   though not always; can use Indirect Inference with identity weighting matrix (Fan et al., 2018)
4. can combine data moments drawn from different datasets
   that also works for some estimation methods

Benefits of estimation

1. Discipline
   cannot choose moments that matter (in some estimation methods)
   cannot choose how to weight those moments in the distance function
2. Parameter estimates have standard errors
   but they don't account for model uncertainty
3. The role of covariates and the stochastic processes governing "shocks" are explicit

Huggett's calibration

Fixed based on outside evidence:

- preference parameters: discount factor, risk aversion
- technology parameters: capital share, depreciation
- demographics: retirement age, survival rates
- taxes
- credit limit (0 or $-w$)
- age-productivity profile

Labor endowment process: AR(1)

- some parameters based on outside evidence (shock variance, persistence)
- some parameters calibrated to match earnings Ginis (ages 20 and 65 and overall)

The model is exactly identified.

Why these choices?

This is an old paper. Computing the model was expensive. Hence, few parameters are calibrated.

Exactly identified models were popular following Kydland and Prescott (1982)

Where possible, parameters are taken from micro evidence (e.g., preferences)

This imposes discipline and saves computational costs.

But one has to be careful about aggregation (Keane and Rogerson, 2012).

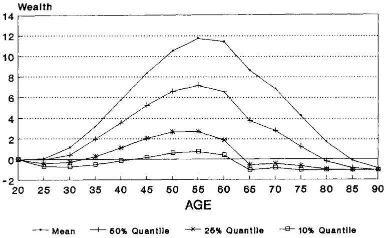
The data moments chosen (earnings Ginis)

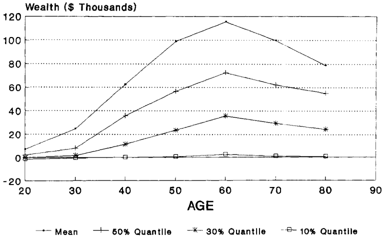
- are intuitively informative about the calibrated parameters
- must be matched for the experiment to make sense

But note: only a few of many possible data moments are considered.

Is this a good model?

One approach: show that the model fits non-targeted moments.





US Economy

Comparison of age-wealth profiles.

Main Result

Fraction held by top	1%	5%	20%	Gini	% neg. wealth
Huggett (1996)	10.8	32.4	68.9	0.70	19%
U.S. data	**34.7**	57.8	81.7	0.80	11%

The model has too many households without wealth.

Still, wealth inequality is lower than in the data.

Models of this kind fail to account for wealth concentration at the top

The paper spawned a **large literature** that tries to generate enough rich households.

What Goes Wrong?

1. The rich do not have an **incentive to save**
   Possible solutions: entrepreneurship, bequests
   Quadrini (1999), Cagetti and Nardi (2006)
2. The only **source of income** is earnings
   The rich don't earn enough to accumulate as much wealth as in the data
   Possible solutions: entrepreneurship, bequests
3. Earnings and wealth are too highly **correlated**
   Hendricks (2007)

A bird's eye view of the literature

The challenge: the top 1% hold 1/3 of total wealth

- the literature is (overly) fixated on matching that number

Huggett (1996):

- standard ingredients of a life-cycle model do not get close to 1/3
- challenge: get the rich to save a lot

Now the literature spent a lot of time trying to get the top 1%.

Getting the top 1%

Bequests: De Nardi (2004)

- the rich save to give to their kids
- this literature is thin on data (especially on the distribution of inheritances among the rich)

Entrepreneurship: Quadrini (1999)

- some people have great business ideas
- they need assets for collateral in their businesses
- this literature is also thin on data (just cross-sectional moments)

Getting the top 1%

A reduced form: Castaneda et al. (2003)

- panel data understate the incomes of the rich
- one can invent an earnings process that matches cross section data and allows a model with bequests to match how rich the top 1% are
- many papers still use this approach, even though better data are now available (De Nardi et al., 2018).

References I

Cagetti, M. and M. D. Nardi (2006): "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 114, 835–870.

Castaneda, A., J. Diaz-Gimenez, and J. V. Rios-Rull (2003): "Accounting for the US earnings and wealth inequality," *Journal of political economy*, 111, 818–857.

De Nardi, M. (2004): "Wealth inequality and intergenerational links," *The Review of Economic Studies*, 71, 743–768.

De Nardi, M., G. Fella, and G. P. Pardo (2018): "Nonlinear Household Earnings Dynamics, Self-insurance, and Welfare," Tech. Rep. w24326, National Bureau of Economic Research, Cambridge, MA.

Fan, X., A. Seshadri, and C. Taber (2018): "Estimation of a Life-Cycle Model with Human Capital, Labor Supply and Retirement," .

References II

Hendricks, L. (2007): "Retirement Wealth and Lifetime Earnings," *International Economic Review*, 48, pp. 421–456.

Huggett, M. (1996): "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 38, 469–494.

Keane, M. and R. Rogerson (2012): "Micro and macro labor supply elasticities: A reassessment of conventional wisdom," *Journal of Economic Literature*, 50, 464–476.

Kydland, F. E. and E. C. Prescott (1982): "Time to build and aggregate fluctuations," *Econometrica: Journal of the Econometric Society*, 1345–1370.

Quadrini, V. (1999): "The Importance of Entrepreneurship for Wealth Concentration and Mobility," *Review of Income and Wealth*, 45, 1–19.

References III

Rodríguez, S. B., J. Díaz-Giménez, V. Quadrini, and J.-V. Ríos-Rull (2002): "Updated Facts on the US Distributions of Earnings, Income, and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 26, 2–35.