Wealth Distribution: Stochastic Aging

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Econ720

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We have seen that the stochastic life-cycle model goes a long way towards accounting for U.S. wealth inequality. But fails to account for the concentration of wealth within the top 5% or 1% of the population.

We study two candidate solutions (in one paper)

1. bequests (see also Nardi 2015; De Nardi and Yang 2014)
2. alternative labor earning processes
Stochastic aging

A computational problem: the curse of dimensionality. The household problem must be solved for all possible combinations of states.

Approximation: put states on a grid.

With many states, the grid gets very large.

Stochastic aging collapses the age dimension into a few phases (e.g. work and retirement)

Key reference: Castaneda et al. (2003)
Main innovations relative to Huggett (1996):

- Households are altruistic (additional source of wealth and motive for saving).
- Earnings process is chosen to match SCF data on earnings and wealth inequality.
- Social Security system modeled in more detail (to give high retirement incomes to low earnings households; helps account for low wealth observations).
- Progressive income tax system (found important for wealth distribution).
- Stochastic aging.

Main finding: The model accounts for distribution of earnings and wealth.
Environment

There is a continuum of families. Each family consists of non-overlapping individuals. In each period, a person:

- draws a stochastic labor endowment $e$,
- chooses consumption and saving,
- retires with some probability,
- dies with some probability.

New individuals inherit assets and labor endowments from their parents.
Household problem

State variables:

- "age": working or retired (there is no symbol for age).
- labor endowment $e$.
- wealth $a$.

The exogenous states are collected in $s = (\text{age}, e)$. $s_t$ evolves according to a transition matrix $\Gamma$. 
Household problem

\[ \max E \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell - l_t) | s_0 \right\} \]

subject to the budget constraint

\[ c + z = y - \tau(y) + a \quad (1) \]

\[ y = ar + wes l(s, a) + \omega(s) \quad (2) \]

\[ a'(z) = \begin{cases} z & \text{if survive} \\ (1 - \tau_E(z))z & \text{if death} \end{cases} \quad (3) \]

\( \omega(s) \): retirement benefits
Remarks

Households are modeled as infinitely lived.

- This is a reduced form for a sequence of non-overlapping individuals linked by altruistic bequests.
- There is no separate age state variable.

Labor endowments are drawn from $S = \varepsilon \cup R$.

- $e \in \varepsilon$ means "working".
- $e \in R$ means retired.
Stochastic aging

Individuals are born as working \((e \in \epsilon, \omega = 0)\).
In each period, they draw a new \(e\).
If \(e \in \mathbb{R}\), the household retired.
If retired and household draws \(e \in \epsilon\), he dies and is replaced by a child.

Benefits:

- Small state vector: \((s, a)\).
- Value function must be computed for only 2 "ages"

Drawbacks:

- Some households have very long or short working lives.
- Hard to match life-cycle features (age-earnings profile, mortality rates)
Dynamic program

\[ v(s, a) = \max \, u(c, \ell - l) + \beta \sum_{s' \in S} \Gamma_{ss'} \, v(s', a'(z)) \]  \hspace{1cm} (4) \]

\[ c + z = y - \tau(y) + a \]  \hspace{1cm} (5) \]

\[ y = a \, r + e(s) \, lw + \omega(s) \]  \hspace{1cm} (6) \]

\[ a'(z) = \begin{cases} (1 - \tau_E(z)) \, z & \text{if } s \in \mathcal{R} \text{ and } s' \in \mathcal{E} \\ z & \text{otherwise} \end{cases} \]  \hspace{1cm} (7) \]
Other model agents

Firms maximize period profits.

- Production technology is \( F(K, L) \).

Government

- Taxes bequests at rate \( \tau_{E}(z) \), where \( z \) is the bequest amount.
- Taxes income at rate \( \tau(y) \).
- Provides retirement transfers to households.
- Balances the budget in each period: \( G_t + Tr_t = T_t \).
Steady state

Objects:

- Policy functions: \( c(s, a), z(s, a), l(s, a) \).
- Government policies: \( \tau(y), \tau_E(z), \omega(s), G \).
- A stationary probability distribution over household types: \( x \).
- Aggregate quantities: \( K, L, T, Tr \).
Steady state

These satisfy:

- Policy functions are optimal decision rules.
- Factor market clearing: $K = \int a \, dx$, $L = \int e(s) \, l(s,a) \, dx$.
- Goods market clearing:
  \[ F(K, L) + (1 - \delta) \, K = G + \int [c(s,a) + z(s,a)] \, dx. \]
- Firm’s first-order conditions.
- Government budget constraints.
- Measure of households is stationary.
Calibration

(We omit details)

Income and estate tax schedule mimic U.S. progressive tax system.

Labor endowments are drawn from a Markov chain

Transition matrix matches:

- points on the Lorenz curves for earnings and wealth \((\Gamma_{\varepsilon\varepsilon}, e(s))\).
- intergenerational persistence of labor endowments \((\Gamma_{\Re\varepsilon})\).
- length of working lives \((p_{\varepsilon, \rho})\).
- life expectancy \((p_{\rho, \rho})\).

Total number of parameters: 39 (unusually large [for macro])
Other calibration targets

Various features of U.S. tax schedules.
Aggregate ratios: \( K/Y, I/Y, G/Y, Tr/Y, l/\ell \)
Ratio of standard deviations for \( c \) and \( l \).
Average length of work life: 45 years.
Average length of retirement: 18 years.
Average earnings middle age / young: 1.3
Intergenerational correlation of log lifetime earnings: 0.4
Results

Model economy matches cross-sectional earnings distribution very well.

Wealth distribution match is good, not perfect.

<table>
<thead>
<tr>
<th>ECONOMY</th>
<th>GINI</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
<th>Fourth</th>
<th>Fifth</th>
<th>90th–95th</th>
<th>95th–99th</th>
<th>99th–100th</th>
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<tbody>
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<td>81.97</td>
<td>16.97</td>
<td>18.21</td>
<td>29.85</td>
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</table>
Assessment

The model successfully replicates the cross-sectional distribution of wealth.
No departure from standard theory is needed.
Key features for the model’s success:

- Intended bequests permit households to accumulate wealth over longer time periods.
- Earnings process consistent with cross-sectional SCF data.
Earnings process

Calibration does not use information on persistence of earnings. The earnings process is "cooked" to match the wealth distribution. The lower 3 earnings states "look like" something estimated from the PSID (though persistence is very high). The top earnings state is totally transitory.

<table>
<thead>
<tr>
<th>TABLE 5</th>
<th>Relative Endowments of Efficiency Labor Units, e(s), and the Stationary Distribution of Working-Age Households, $\gamma^*_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s = 1</td>
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<tr>
<td>e(s)</td>
<td>1.00</td>
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<tr>
<td>$\gamma^*_\varepsilon$ (%)</td>
<td>61.11</td>
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</tbody>
</table>
The top earnings level is very large

<table>
<thead>
<tr>
<th>From $s$</th>
<th>$s' = 1$</th>
<th>$s' = 2$</th>
<th>$s' = 3$</th>
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<td>$s = 4$</td>
<td>10.66</td>
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<td>6.11</td>
<td>80.51</td>
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</tbody>
</table>

Intuition:

- households win the lottery once every 25 years
- lottery winners save everything because the top state is so transitory
Reservations

The paper shoes that it is possible to write down a standard life-cycle model that matches wealth concentration based on an earnings process with the right amount of cross-sectional inequality.

It does not show that a life-cycle model generates the right wealth distribution when a "realistic" earnings process is imposed.

Could one fix this?

- why not combine info on the process for the bottom 99% from the PSID with info for the cross-sectional distribution for everyone from SCF?
- one solution: De Nardi et al. (2016) using administrative data
A literature that studies taxation of top earners. Also uses the Castaneda trick of the high top earnings state. Krueger and Kindermann (2014), Guner et al. (2016)

There are alternative approaches.

- entrepreneurship: Brüggemann (2017)


