

Bewley Models

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Bewley Models

- ▶ For many applications we need models with **heterogeneous agents**.
- ▶ In Bewley models, agents are ex ante identical.
- ▶ They are ex post heterogeneous because they are hit by idiosyncratic shocks.
- ▶ Incomplete markets prevents sharing these risks.

Endowment Economy

An Endowment Economy

- ▶ Demographics:

- ▶ There is a unit measure of households.
- ▶ Each lives forever.

- ▶ Preferences:

$$E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

- ▶ Technology:

- ▶ Households receive random endowments $y_t \in Y$ (finite).
- ▶ Transition matrix: $\pi(y'|y)$.

No aggregate uncertainty

- ▶ Assume a "law of large numbers."
- ▶ Let $\Pi(y)$ be the stationary distribution of y .
- ▶ Assume that the fraction of households with endowment y is $\Pi(y)$.
- ▶ The aggregate endowment \bar{y} is constant over time.
- ▶ With complete markets, households would not face any uncertainty.

Household problem

- ▶ Flow budget constraint:

$$a' = y + (1+r)a - c \quad (2)$$

- ▶ Borrowing constraint:

$$a' \geq -b \quad (3)$$

Household problem

- ▶ Focus on a **stationary equilibrium**.
 - ▶ Meaning: Aggregates & prices are constant over time.
- ▶ State vector: (a, y) .
- ▶ Given: r .
- ▶ Bellman equation:

$$v(a, y) = \max u(c) + \beta \sum_{y'} \pi(y'|y) v(a', y') \quad (4)$$

s.t. budget constraint and borrowing constraint.

Household problem I

The borrowing constraint may bind. We need Kuhn-Tucker.

Bellman equation

$$v(a, y) = \max u(y + (1+r)a - a') \quad (5)$$

$$+ \beta \sum_{y'} \pi(y'|y) v(a', y') + \mu(a' + b) \quad (6)$$

First-order conditions:

$$u'(c) = \beta \sum \pi(y'|y) \frac{\partial v(a', y')}{\partial a'} + \mu \quad (7)$$

$$\partial v / \partial a = u'(c) (1+r) \quad (8)$$

$$\mu(a' + b) = 0 \quad (9)$$

Household problem

- ▶ Euler equation

$$u'(c) \geq E\beta(1+r)u'(c') \quad (10)$$

with equality if $a' > -b$.

- ▶ Solution: $v(a,y), c(a,y), a'(a,y)$ that satisfy the usual conditions.

How does the household behave?

Assume (for the moment) that $y \sim iid$

Then (obviously?), consumption and saving depend on "cash on hand"

$$x = y + (1 + r)a \quad (11)$$

Also (obviously?), a' is increasing in x

If x is sufficiently high: Choose $a' > -b$ and satisfy standard Euler equation.

If x is below a cutoff, set $a' = -b$ and "violate" the Euler equation.

The borrowing constraint depresses current consumption.

Stationary Recursive Competitive Equilibrium

- ▶ Aggregate state: The joint distribution of assets and endowments: $\Phi(a, y)$.
 - ▶ This is needed to compute aggregates.
- ▶ Objects:
 - ▶ Household: $v(a, y), c(a, y), a'(a, y)$.
 - ▶ $\Phi(a, y)$.
 - ▶ Price function: $r(\Phi)$.
- ▶ Equilibrium conditions:
 - ▶ Household: see above.
 - ▶ Market clearing.
 - ▶ Time invariance of Φ .

Stationary Recursive Competitive Equilibrium

Market clearing

- ▶ Goods:

$$\int \int c(a, y) \Phi(da, dy) = \int y \Pi(dy) = \bar{y} \quad (12)$$

- ▶ Bonds:

$$\int \int a'(a, y) \Phi(da, dy) = 0 \quad (13)$$

Time invariance of the distribution

- ▶ Informally, household choices determine tomorrow's distribution ϕ' .
- ▶ The policy function $a'(a,y)$ implies a law of motion for ϕ .
- ▶ In stationary equilibrium, the law of motion must imply $\phi' = \phi$.

Law of motion for the distribution

- ▶ Define a transition function $Q((a, y), (A, Y))$.
- ▶ Its value is the probability (mass) of households in state (a, y) today that end up in $(a', y') \in (A, Y)$ tomorrow.

$$Q((a, y), (A, Y)) = \begin{cases} \sum_{y' \in Y} \pi(y'|y) & \text{if } a'(a, y) \in A \\ 0 & \text{otherwise} \end{cases}$$

- ▶ This is because a' is deterministic.

Law of motion for the distribution

- ▶ Law of motion

$$\Phi'(A, Y) = \int \int Q((a, y), (A, Y)) \Phi(da, dy) \quad (14)$$

- ▶ In words:
 - ▶ $\Phi'(A, Y)$: What is the mass of households in the set of states (A, Y) tomorrow?
 - ▶ For any (a, y) , this is given by $Q((a, y), (A, Y))$.
 - ▶ Sum over all (a, y) to get the total mass.
- ▶ Stationarity then means: $\Phi'(A, Y) = \Phi(A, Y)$ for all (A, Y) .

Non-stationary Equilibrium

- ▶ The equilibrium concept generalizes easily to economies where Φ evolves over time.
- ▶ Household:
 - ▶ Add the aggregate state Φ to the household's state: $v(a, y, \Phi)$ and $a'(a, y, \Phi)$.
 - ▶ The household takes prices as functions of the aggregate state: $r(\Phi)$.
 - ▶ The household knows the law of motion for Φ : $\Phi' = H(\Phi)$.
- ▶ Equilibrium:
 - ▶ Drop stationarity of Φ .

Where is this useful?

- ▶ Models of the wealth distribution:
 - ▶ Krusell and Smith (1998)
- ▶ Models of business cycles with heterogeneous agents:
 - ▶ Castaneda et al. (1998)

Reading

- ▶ Acemoglu (2009), ch. 17.4.
- ▶ Krueger, "Macroeconomic Theory," ch. 10.

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- Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.
- Castaneda, A., J. Diaz-Giménez, and J.-V. Ríos-Rull (1998): “Exploring the income distribution business cycle dynamics,” *Journal of Monetary economics*, 42, 93–130.
- Krusell, P. and J. Smith, Anthony A. (1998): “Income and Wealth Heterogeneity in the Macroeconomy,” *The Journal of Political Economy*, 106, 867–896.