Growth Through Product Creation

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Issues

We study models where **intentional innovation** drives productivity growth.

Background:

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.
- How important are the 2 cases? – An open question.

How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
  - blueprints can be used simultaneously in the production of several goods.

How to model innovation

There are N consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- Therefore downward sloping demand curves

Approach 1: **Quality ladders**

- Each good can be made by many firms.
- Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: **Increasing variety**

- Each firm can invest to create a new variety $(N \to N+1)$
- Then it becomes the monopolist for that variety

A Model of Product Innovation

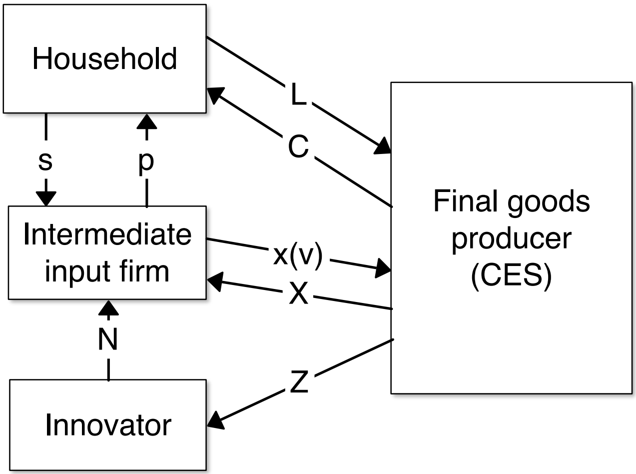
A Model of Product Innovation

Agents:

1. A representative **household** supplies labor to firms
2. **Final** goods firms use labor and intermediate inputs
3. **Intermediate** inputs are produced from final goods
4. **Innovators**
   create new intermediates from final goods
   receive permanent monopolies

Note: Now that models get more complicated, it really pays off to be pedantic about details.

Model structure



The Story Line

Innovators

- buy goods from the final goods firm (Z).
- invent a new variety j
- receive a permanent patent for good j

Intermediate goods producers

- buy the patent from the innovator
- forever make x_j using the final good (X)
- sell it as monopolist to the final goods firm
- profits go to households

Demographics and Preferences

Demographics:

- A representative household.

Preferences:

$$\int_0^\infty e^{-\rho t} \ \frac{C_t^{1-\theta} - 1}{1-\theta} dt \tag{1}$$

- C: the final good

Technology: Final Goods

Resource constraint:

$$C_t + X_t + Z_t = Y_t$$

Final goods Y are used for

- Z: R&D investment.
- X: Inputs into the production of intermediates x.
- C: consumption

Technology: Final Goods

Production of **final goods** from intermediates and labor:

$$Y_t = (1-\beta)^{-1} \left[\int_0^{N_t} x(v,t)^{1-\beta} \, dv \right] L^{\beta} \tag{2}$$

Write $\left[\int x^{1-\beta} dv\right]^{\frac{1-\beta}{1-\beta}}$ to see that this is a CES aggregator of x.

This is the key trick of the model:

- the CES aggregator implies a constant price elasticity of demand for x
- if the suppliers of x are monopolists, their prices are a fixed markup over marginal costs

Technology: Intermediate Inputs

Each unit of x requires ψ units of Y.

The total amount of goods used to make intermediates is

$$X = \psi \int_0^{N_t} x(v,t)\, dv \tag{3}$$

Intermediate inputs fully depreciate in use.

Technology: Innovation

Investing the final good yields a flow of new patents:

$$\dot{N} = \eta Z_t \tag{4}$$

Think of this as the aggregate (deterministic) outcome of the (stochastic) innovation efforts of many firms.

Market arrangements

- Final goods and labor markets are competitive.
- Intermediates are sold by **monopolists** (the innovators).
  - Monopolies are permanent.
  - What the monopolists do with their profits is not clear.
- Free entry into innovation
  - ensures zero present value of profits
- The household owns the innovating firms.
- Asset markets are complicated
  - there is often no need to spell out the details

Notes

Production is cyclical:

- today's Y is used to make X which makes Y
- the alternative: durable X (more complicated)
- implication: the efficient allocation maximizes $Y - X = C + Z$

The only long-lived object is a patent

- this keeps the model simple

Assuming that intermediates are made from final goods fixes marginal costs (and prices)

Solving Each Agent's Problem

Final goods producers

- Maximize period profits by choosing L and $x(v,t)$.
- Take prices $p(v,t)$ as given.
- Normalize the price Y to 1.
- Profits

$$Y_t - w_t L_t - \int_0^{N_t} p^x(v,t)\; x(v,t)\, dv \tag{5}$$

where

$$Y_t = (1-\beta)^{-1} \left[\int_0^{N_t} x(v,t)^{1-\beta}\, dv \right] L^{\beta} \tag{6}$$

Final goods producers

FOCs:

- $\partial Y / \partial x(v) = L^{\beta} x(v)^{-\beta} = p^{x}(v)$
- $\partial Y / \partial L = \beta Y / L = w$

Demand function:

$$x(v,t) = L\, p^{x}(v,t)^{-1/\beta} \tag{7}$$

Note the constant price elasticity $1/\beta$.

Solution to the firm's problem: $L_t, x(v,t)$ that satisfy the "2" first-order conditions.

Intermediate input producers

Problem after inventing a variety.

x is produced at constant marginal cost ψ.

Maximize present value of profits

$$V(v,t) = \int_t^{\infty} e^{-rs} \pi(v,s)\, ds \tag{8}$$

Instantaneous profits are

$$\pi(v,t) = (p^x(v,t) - \psi)\; x(v,t) \tag{9}$$

where $x(v,t) = Lp^x(v,t)^{-1/\beta}$

This is a sequence of static problems

Intermediate input producers

- First order condition (standard monopoly pricing formula):

$$p^x = \psi/(1-\beta) \tag{10}$$

- Profits are

$$\pi(v,t) = \psi \frac{\beta}{1-\beta} x(v,t) \tag{11}$$

- Solution: A constant p^x.

Household

- The household holds shares of all intermediate input firms.
- Each firm produces a stream of profits.
- New firms issue new shares.
- But: the details don't matter to the household.
- There simply is an asset with rate of return r.
- Euler equation is standard:

$$g(C) = \frac{r-\rho}{\theta} \tag{12}$$

- Invoke Walras' law - so you never have to write down the budget constraint!

Equilibrium

- Objects: $C_t, X_t, Z_t, x(v,t), V(v,t), N_t$ and prices $p^x(v,t), r(t), w(t)$.
- Conditions:
  - "Everybody maximizes." (see above)
  - Markets clear.
    1. Goods: resource constraint.
    2. Shares: omitted b/c I did not write out the household budget constraint.
    3. Intermediates: implicit in notation.
  - Innovation effort satisfies a **free entry** condition: present value of profits equals 0.

Symmetric Equilibrium

We assume (and then show) that all varieties v share the same x, V and p^x.

Intuition:

- p^x: monopoly pricing with a constant elasticity
- x: varieties enter final goods production symmetrically
- V: the age of a variety does not matter
  (no stock of x to build; permanent patents)

Simplifications

Normalize marginal cost $\psi = 1 - \beta$

- so that $p^x = 1$.
- Why can I do that?

Focus on balanced growth paths.

Equilibrium: Characterization

There is an algorithm ...

- The growth rate follows from the Euler equation: $g(C) = (r - \rho)/\theta$.
- We get r from free entry by innovators: present value of profits = cost of creating a variety.

Equilibrium: Characterization

Free entry will determine the interest rate

Spend 1 to obtain η new patents, each valued (initially) at $V(v,t)$

$$\eta V(v,t) = 1 \tag{13}$$

- Then V is constant over time.
- This assumes that innovation takes place.

With balanced growth and constant profits (to be shown):

$$V = \pi / r \tag{14}$$

Profits

With a fixed markup, profits are a multiple of revenues:

$$\pi(t) = \psi \frac{\beta}{1-\beta} x(t) \tag{15}$$

$$= \beta x(t) \tag{16}$$

Demand for intermediates:

$$x(t) = L\, p^x(t)^{-1/\beta}$$

$$= L$$

Profits: $\pi = \beta L$.

Free Entry

- Free entry:

$$\eta V = \eta \beta L / r = 1 \tag{17}$$

- This is the closed form solution for r.
- Balanced **growth** rate then follows from the Euler equation.

$$g(C) = \frac{\eta \beta L - \rho}{\theta} \tag{18}$$

Equilibrium: Characterization

Production function for final goods with $x = L$:

$$Y = \frac{N_t L}{1-\beta} \tag{19}$$

Wage (from firm's FOC):

$$w_t = \beta \frac{Y_t}{L_t} = \frac{\beta}{1-\beta} N_t \tag{20}$$

Total expenditure on intermediates:

$$X_t = \psi N_t x_t = (1-\beta) N_t L \tag{21}$$

Summary of Equilibrium

Prices and quantities of intermediate inputs are constant.

- the model is rigged to deliver this
- for tractability

Growth comes from rising N

No Transition Dynamics

The equilibrium looks like an AK model with production function

$$\begin{aligned} Y_t &= \frac{L}{1-\beta} N_t \\ \dot{N}_t &= \eta \; s_z \; Y_t \end{aligned}$$

Intuition:

- Period profits π are constant at βL.
- At any moment we need $\eta V = 1$.
- V is the present value of (constant) profits.
- Constant V is only possible with constant r.
- Intuition: There is a reduced form AK structure.

Scale Effects

$$g(C) = \frac{\eta \beta L - \rho}{\theta}$$

Larger economies (L) grow faster.

Population growth implies exploding income growth (!)

Mechanical reason:

- Innovation technology is linear in goods.
- Larger economy $\rightarrow$ higher Y $\rightarrow$ higher Z $\rightarrow$ faster growth.

We will return to this later.

Pareto Efficient Allocation

Efficiency

Two distortions prevent efficiency of equilibrium:

1. Monopoly pricing $\implies$ high profits $\implies$ too much innovation.
2. "Aggregate demand externality":
   innovation $\implies$ smaller markets $\implies$ too little innovation

Planner's Problem

Solve in two stages:

1. Given N, find optimal static allocation $x(v,t)$.
   - That is: maximize $Y - X$ which is available for consumption and investment.
   - An odd feature of the model: goods are produced from goods without delay.
2. Given the reduced from production function from #1, find optimal Z.

Static Allocation

Given N, choose $x(v,t)$ to maximize $Y - X$:

$$\max (1-\beta)^{-1} L^{\beta} \int_0^{N_t} x(v,t)^{1-\beta} dv - \int_0^{N_t} \psi x(v,t) dv \tag{22}$$

First-order condition

$$L^{\beta} x^{-\beta} = \psi \tag{23}$$

with $\psi = 1 - \beta$:

$$x = (1-\beta)^{-1/\beta} \; L \tag{24}$$

The planner's x is larger than the equilibrium x (Intuition?)

Static Allocation

Next: find $Y - X$.

$$X = \psi N x = (1-\beta) N (1-\beta)^{-1/\beta} L \tag{25}$$

Reduced form production function:

$$\begin{aligned} Y_t &= (1-\beta)^{-1} L^{\beta} N [(1-\beta)^{1-1/\beta} L]^{1-\beta} & (26) \\ &= (1-\beta)^{-1/\beta} L N_t & (27) \end{aligned}$$

Net output

$$\begin{aligned} Y - X &= (1-\beta)^{-1/\beta} L N - (1-\beta)^{1-1/\beta} L N \\ &= (1-\beta)^{-1/\beta} \beta \; L \; N & (28) \end{aligned}$$

Planner: Dynamic Optimization

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

subject to

$$\begin{aligned} \dot{N} &= \eta Z \\ Y &= (1-\beta)^{-1/\beta} \; \beta \; L \, N = C + Z \end{aligned}$$

Or

$$\begin{aligned} \dot{N} &= A \; N - \eta C && (29) \\ A &= \eta \, (1-\beta)^{-1/\beta} \; \beta \; L && (30) \end{aligned}$$

Hamiltonian

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \mu \left[AN - \eta C\right] \tag{31}$$

FOC

$$\partial H/\partial C = C^{-\theta} - \mu\eta = 0 \tag{32}$$

$$\partial H/\partial N = \rho\mu - \dot{\mu} = \mu A \tag{33}$$

Optimal growth

The same as in an AK model with

$$A = \eta\,(1-\beta)^{-1/\beta}\ \beta\ L \tag{34}$$

we have

$$\dot{C}/C = \frac{A-\rho}{\theta} \tag{35}$$

Comparison with CE

- CE interest rate: $\eta\beta L$.
- Planner's "interest rate:" $(1-\beta)^{-1/\beta}\eta\beta L$.
- The planner chooses faster growth.
- Intuition:
  - CE under-utilizes the fruits of innovation: x is too low.
  - This reduces the value of innovation.

Policy Implications

- One might be tempted to reduce monopoly power.
- A policy that encourages competition (e.g. less patent protection, forcing lower p^x) reduces the static price distortion.
- But it also reduces growth: innovation is less valuable.
- Similar result for shorter patents.
- Policy trades off static efficiency and incentives for innovation.

Final Example:
Durable Intermediate Inputs

Environment

We study a final example where intermediates are durable (the model has capital).

Unchanged relative to previous model:

- demographics
- preferences
- endowments
- final goods technology
- innovation technology

Technologies: Intermediates

- Upon invention, the inventor is endowed with x_0 units of $x(v)$.
- Additional units are accumulated according to

$$\dot{x}(v,t) = \omega I(v,t)^{\varphi} - \delta x(v,t) \tag{36}$$

- $0 < \varphi < 1$
- Diminishing returns imply smooth adjustment of x over time.
- Intermediates are *rented* to final goods firms at price $q(v,t)$.
- Total input of final goods: $X_t = \int_0^{N_t} I(v,t)\,dv$

Market arrangements

Markets:

- Final goods: price 1
- Labor: w_t
- Intermediate input rental: $q(v,t)$

Each intermediate input producer has a permanent monopoly for his variety.

Free entry into the market for innovation

Agents' Problems

Unchanged:

- Household
- Final goods firm
- Free entry of innovator

Changed:

- Intermediate goods firm

Intermediate input producer

Now a truly dynamic problem (v index suppressed)

$$V_t = \max \int_t^\infty e^{-r\tau}[R(x(\tau)) - I(\tau)]d\tau$$

subject to

$$\dot{x} = \omega I^\varphi - \delta x \tag{37}$$

Revenue

Final goods firm's demand (unchanged):

$$q(x) = L^{\beta} x^{-\beta} \tag{38}$$

Revenue:

$$R(x) = q(x)x \tag{39}$$

$$= L^{\beta} x^{1-\beta} \tag{40}$$

Marginal revenue:

$$R'(x) = (1-\beta) L^{\beta} x^{-\beta} \tag{41}$$

$$= (1-\beta) q(x) \tag{42}$$

Intermediate input producer

Hamiltonian:

$$H = R(x) - I + \mu\left[\omega I^{\varphi} - \delta x\right] \tag{43}$$

FOCs:

$$\begin{aligned} \partial H/\partial I &= -1 + \mu\omega\varphi I^{\varphi-1} = 0 \\ \dot{\mu} &= (r+\delta)\mu - R'(x) \end{aligned}$$

Intuition...

Solution: $\{I_t, x_t, \mu_t\}$ that solve 2 FOCs and law of motion for x.

Boundary conditions:

- $x(0) = 0$ given,
- $\lim_{t\to\infty} e^{-rt}\mu_t x_t = 0$.

Free entry of innovators

Technology (unchanged):

$$\dot{N} = \eta Z \tag{44}$$

Free entry:

- Spend $1/\eta$ for period dt to obtain $dN = \eta/\eta \; dt$ new patents worth $V \; dt$.
- Equate cost and profits:

$$1/\eta = V \tag{45}$$

Equilibrium

Objects: $\{q(v,t), x(v,t), N_t, I(v,t), \mu(v,t), y_t, L_t, r_t, c_t, w_t\}$

Equilibrium conditions:

- Household: Euler (1)
- Final goods firm: 3
- Intermediate goods firm: 3
- Free entry:

$$1/\eta = V = \int e^{-rt}[R(x_t) - I_t]\,dt \tag{46}$$

  where R defined above

- Market clearing

Market clearing

1. Final goods: Resource constraint or $Y = C + NI + \dot{N}/\eta$.
2. Intermediates: implicit in notation.
3. Labor: $L = 1$.
4. Asset markets: suppressed (details not specified)

Case $\varphi = 1$

Assume that the same equilibrium conditions hold for $\varphi = 1$ (not obvious).

Then FOC for investment in x becomes

$$1 = \mu \omega \varphi I^{\varphi - 1} = \mu \omega \tag{47}$$

μ must be constant over time (assuming investment takes place at all times; not obvious).

Constant μ implies:

$$\dot{\mu} = (r + \delta)\mu - R'(x) = 0 \tag{48}$$

x must be constant over time.

Case $\varphi = 1$

Demand function implies (cf. (42)):

$$R'(x) = (1-\beta)\, q(x) \tag{49}$$

Therefore:

$$R'(x) = (1-\beta)\, q(x) = (r+\delta)\, \mu \tag{50}$$

where $\mu = 1/\omega$ so that

$$q = \frac{r+\delta}{(1-\beta)\,\omega} \tag{51}$$

Then we know x from the demand function (38)

$$x = L\,[q]^{-1/\beta} \tag{52}$$

With a linear technology, the best approach is to build all x in one shot, then keep x constant.

Symmetric equilibrium I

With $\varphi = 1$ there is a symmetric equilibrium because it does not take time to build up the stock of x.

Start from the Euler equation: $g(c) = (r - \rho)/\sigma$.

Free entry pins down r:

$$1/\eta = V = \int_0^\infty e^{-rt} [R(x_t) - I_t]\, dt - \underbrace{\frac{x - x_0}{\omega}}_{I_0} \qquad (53)$$

Assume $x_0 = 0$.

Stationary x:

$$I_t = x\delta/\omega \qquad (54)$$

Symmetric equilibrium II

From marginal revenue (50) we have:

$$R(x) = \frac{r+\delta}{(1-\beta)\,\omega}x \tag{55}$$

Therefore the integrand becomes:

$$R(x) - I = x\left[\frac{r+\delta}{\omega(1-\beta)} - \frac{\delta}{\omega}\right] \tag{56}$$

and free entry implies

$$1/\eta = V = \frac{R(x) - \frac{\delta}{\omega}x}{r} - \frac{x}{\omega} \tag{57}$$

or

$$1/\eta = \frac{1}{r}x\left[\frac{r+\delta}{\omega(1-\beta)} - \frac{\delta}{\omega}\right] - \frac{x}{\omega} \tag{58}$$

Symmetric equilibrium III

Demand for intermediates (52) gives x.

Now we have 3 equations in (q, r, x):

1. Demand for intermediates (52)
2. Marginal revenue: (50)
3. Free entry (58)

These could, in principle, be solved for the equilibrium values.

Reading

- Acemoglu (2009), ch. 13.
- Krusell (2014), ch. 9
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

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