Growth Through Product Creation
Part 2

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Pareto Efficient Allocation

Efficiency

Two distortions prevent efficiency of equilibrium:

1. Monopoly pricing $\implies$ high profits $\implies$ too much innovation.
2. "Aggregate demand externality":
   innovation $\implies$ smaller markets $\implies$ too little innovation

Planner's Problem

Resource constraint:

$$Y = C + X + Z \tag{1}$$

$$Y - X = C + \dot{N}/\eta \tag{2}$$

Solve in two stages:

1. Given N, find optimal static allocation $x(v,t)$.
   - That is: maximize $Y - X$ which is available for consumption and investment.
   - An odd feature of the model: goods are produced from goods without delay.
2. Given the reduced from production function from #1, find optimal Z.

Static Allocation

Given N, choose $x(v,t)$ to maximize $Y - X$:

$$\max (1-\beta)^{-1} L^{\beta} \int_0^{N_t} x(v,t)^{1-\beta} dv - \int_0^{N_t} \psi x(v,t) dv \quad (3)$$

L is fixed.

First-order condition

$$L^{\beta} x^{-\beta} = \psi \quad (4)$$

with $\psi = 1 - \beta$:

$$x = (1-\beta)^{-1/\beta} \ L \quad (5)$$

The planner's x is larger than the equilibrium x (Intuition?)

Static Allocation

Next: find $Y - X$.

$$X = \underbrace{\psi N x}_{\text{symmetry}} = \underbrace{(1-\beta)}_{\psi} N \underbrace{(1-\beta)^{-1/\beta} L}_{x} \tag{6}$$

Reduced form production function:

$$\begin{aligned} Y_t &= (1-\beta)^{-1} L^{\beta} N [(1-\beta)^{1-1/\beta} L]^{1-\beta} && (7) \\ &= (1-\beta)^{-1/\beta} L N_t && (8) \end{aligned}$$

Net output

$$\begin{aligned} Y - X &= (1-\beta)^{-1/\beta} L N - (1-\beta)^{1-1/\beta} L N \\ &= (1-\beta)^{-1/\beta} \beta \; L \; N && (9) \end{aligned}$$

Planner: Dynamic Optimization

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta} - 1}{1-\theta} dt$$

subject to

$$\begin{aligned} \dot{N} &= \eta Z \\ Y &= (1-\beta)^{-1/\beta} \ \beta \ L \, N = C + Z \end{aligned}$$

Or

$$\begin{aligned} \dot{N} &= A \, N - \eta C && (10) \\ A &= \eta \, (1-\beta)^{-1/\beta} \ \beta \ L && (11) \end{aligned}$$

Hamiltonian

$$H = \frac{C^{1-\theta} - 1}{1-\theta} + \mu \left[AN - \eta C\right] \tag{12}$$

FOC

$$\partial H / \partial C = C^{-\theta} - \mu \eta = 0 \tag{13}$$

$$\partial H / \partial N = \rho \mu - \dot{\mu} = \mu A \tag{14}$$

Optimal growth

The same as in an AK model with

$$A = \eta \left(1-\beta\right)^{-1/\beta} \ \beta \ L \tag{15}$$

we have

$$\dot{C}/C = \frac{A-\rho}{\theta} \tag{16}$$

Comparison with CE

- CE interest rate: $\eta\beta L$.
- Planner's "interest rate:" $(1-\beta)^{-1/\beta}\eta\beta L$.
- The planner chooses faster growth.
- Intuition:
  - CE under-utilizes the fruits of innovation: x is too low.
  - This reduces the value of innovation.

Policy Implications

- One might be tempted to reduce monopoly power.
- A policy that encourages competition (e.g. less patent protection, forcing lower p^x) reduces the static price distortion.
- But it also reduces growth: innovation is less valuable.
- Similar result for shorter patents.
- Policy trades off static efficiency and incentives for innovation.

Example:
Durable Intermediate Inputs

Environment

We study an example where intermediates are durable (the model has capital).

Unchanged relative to previous model:

- demographics
- preferences
- endowments
- final goods technology
- innovation technology

Technologies: Intermediates

- Upon invention, the inventor is endowed with $x_0 = 0$ units of $x(v)$.
- Additional units are accumulated according to

$$\dot{x}(v,t) = \omega I(v,t)^{\varphi} - \delta x(v,t) \tag{17}$$

- $0 < \varphi < 1$
- Diminishing returns imply smooth adjustment of x over time.
- Intermediates are *rented* to final goods firms at price $q(v,t)$.
- Total input of final goods: $X_t = \int_0^{N_t} I(v,t)\,dv$

Market arrangements

Markets:

- Final goods: price 1
- Labor: w_t
- Intermediate input rental: $q(v, t)$

Each intermediate input producer has a permanent monopoly for his variety.

Free entry into the market for innovation

Agents' Problems

Unchanged:

- Household
- Final goods firm
- Free entry of innovator

Changed:

- Intermediate goods firm

Intermediate input producer

Now a truly dynamic problem (v index suppressed)

$$V_t = \max \int_t^\infty e^{-r\tau}[R(x(\tau)) - I(\tau)]d\tau$$

subject to

$$\dot{x} = \omega I^\varphi - \delta x \tag{18}$$

Revenue

Final goods firm's demand (unchanged):

$$q(x) = L^{\beta} x^{-\beta} \tag{19}$$

Revenue:

$$R(x) = q(x)x \tag{20}$$

$$= L^{\beta} x^{1-\beta} \tag{21}$$

Marginal revenue:

$$R'(x) = (1-\beta) L^{\beta} x^{-\beta} \tag{22}$$

$$= (1-\beta) q(x) \tag{23}$$

Intermediate input producer

Hamiltonian:

$$H = R(x) - I + \mu\left[\omega I^{\varphi} - \delta x\right] \tag{24}$$

FOCs:

$$\begin{aligned}\partial H/\partial I &= -1 + \mu\omega\varphi I^{\varphi-1} = 0 \\ \dot{\mu} &= (r+\delta)\mu - R'(x)\end{aligned}$$

Intuition...

Solution: $\{I_t, x_t, \mu_t\}$ that solve 2 FOCs and law of motion for x.

Boundary conditions:

- $x(0)$ given,
- $\lim_{t\to\infty} e^{-rt}\mu_t x_t = 0$.

Free entry of innovators

Technology (unchanged):

$$\dot{N} = \eta Z \tag{25}$$

Free entry:

- Spend $1/\eta$ for period dt to obtain $dN = \eta/\eta \ dt$ new patents worth $V \ dt$.
- Equate cost and profits:

$$1/\eta = V \tag{26}$$

Equilibrium

Objects: $\{q(v,t), x(v,t), N_t, I(v,t), \mu(v,t), y_t, L_t, r_t, c_t, w_t\}$

Equilibrium conditions:

- Household: Euler (1)
- Final goods firm: 3
- Intermediate goods firm: 3
- Free entry:

$$1/\eta = V = \int e^{-rt} [R(x_t) - I_t] dt \tag{27}$$

  where R defined above
- Market clearing

Market clearing

1. Final goods: Resource constraint or $Y = C + NI + \dot{N}/\eta$.
2. Intermediates: implicit in notation.
3. Labor: $L = 1$.
4. Asset markets: suppressed (details not specified)

Case $\varphi = 1$

Assume that the same equilibrium conditions hold for $\varphi = 1$ (not obvious).

Then FOC for investment in x becomes

$$1 = \mu \omega \varphi I^{\varphi - 1} = \mu \omega \tag{28}$$

μ must be constant over time (assuming investment takes place at all times; not obvious).

Constant μ implies:

$$\dot{\mu} = (r + \delta)\,\mu - R'(x) = 0 \tag{29}$$

x must be constant over time.

Case $\varphi = 1$

Demand function implies (cf. (23)):

$$R'(x) = (1-\beta)\, q(x) \tag{30}$$

Therefore:

$$R'(x) = (1-\beta)\, q(x) = (r+\delta)\, \mu \tag{31}$$

where $\mu = 1/\omega$ so that

$$q = \frac{r+\delta}{(1-\beta)\, \omega} \tag{32}$$

Then we know x from the demand function (19)

$$x = L\,[q]^{-1/\beta} \tag{33}$$

With a linear technology, the best approach is to build all x in one shot, then keep x constant.

Symmetric equilibrium I

With $\varphi = 1$ there is a symmetric equilibrium because it does not take time to build up the stock of x.

Start from the Euler equation: $g(c) = (r - \rho)/\sigma$.

Free entry pins down r:

$$1/\eta = V = \int_0^\infty e^{-rt}\left[R(x_t) - I_t\right]dt - \underbrace{\frac{x}{\omega}}_{I_0} \tag{34}$$

Stationary x:

$$I_t = x\delta/\omega \tag{35}$$

From marginal revenue (31) we have:

$$R(x) = \frac{r+\delta}{(1-\beta)\,\omega}x \tag{36}$$

Symmetric equilibrium II

Therefore the integrand becomes:

$$R(x) - I = x \left[\frac{r+\delta}{\omega(1-\beta)} - \frac{\delta}{\omega} \right] \tag{37}$$

and free entry implies

$$1/\eta = V = \frac{R(x) - \frac{\delta}{\omega} x}{r} - \frac{x}{\omega} \tag{38}$$

or

$$1/\eta = \frac{1}{r} x \left[\frac{r+\delta}{\omega(1-\beta)} - \frac{\delta}{\omega} \right] - \frac{x}{\omega} \tag{39}$$

Demand for intermediates (33) gives x.

Now we have 3 equations in (q, r, x):

Symmetric equilibrium III

1. Demand for intermediates (33)
2. Marginal revenue: (31)
3. Free entry (39)

These could, in principle, be solved for the equilibrium values.

Reading

- Acemoglu (2009), ch. 13.
- Krusell (2014), ch. 9
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.

Krusell, P. (2014): "Real Macroeconomic Theory," Unpublished.

Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.