Knowledge Spillovers and Scale Effects

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Issues

- What happens when innovation takes labor (a non-reproducible factor)?
- Then we need a knowledge spillover to sustain growth.
- It takes some tricks to prevent the model from exhibiting explosive growth.

Knowledge Spillovers

Ideas Produced From Labor

The previous model had endogenous growth because ideas were produced with constant return from a **reproducible factor**: ideas (embodied in goods).

If ideas are produced from (non-reproducible) labor: there is no sustained growth.

Example

Assume $\dot{N}_t = \eta \; Z_t^{\alpha} \; L_{Rt}^{1-\alpha}$. Show that the balanced growth rate is 0 unless $\alpha = 1$.

Knowledge Spillovers

We need a mechanism that offsets diminishing returns to ideas in the production of ideas.

Knowledge spillover: N appears in the innovation production function for N.

This is an **externality**: firms do not pay for the N input.

This is possible because N is non-rival.

The idea: "standing on the shoulders of giants"

Problem: A **knife-edge** parameter assumption is needed for endogenous growth.

- Some parameters must sum to 1.
- This is always true because we need **constant returns to reproducible factors**.

Knowledge spillover model

Keep everything the same, except the production of ideas:

$$\dot{N}_t = \eta N_t L_{Rt} \tag{1}$$

We show later: linearity in N is required for endogenous growth.

Labor now has 2 uses:

- produce goods: L_E
- produce ideas: L_R

Resource constraint:

$$L = L_{Rt} + L_{Et} \tag{2}$$

Note: this does not change the problems of household, final goods firms, or intermediate input firms.

Balanced growth rate

Euler equation is still: $g(C) = (r - \rho)/\theta$.

Interest rate is determined by free entry: $V = \pi/r$.

But now the cost of creating a new patent is different:

$$\eta N_t V_t = w_t \tag{3}$$

- hire a unit of labor and produce a flow of ηN_t patents per "period"

Balanced growth rate

Wage rate (unchanged):

$$w_t = \frac{\beta}{1-\beta} N_t \tag{4}$$

Profits earned by monopolists (unchanged):

$$\pi = \beta L_E \tag{5}$$

Sub wage rate into free entry:

$$\eta N_t \frac{\beta L_E}{r} = w = \frac{\beta}{1-\beta} N_t \tag{6}$$

$\Longrightarrow$

$$r^* = (1-\beta)\,\eta L_E^* \tag{7}$$

Intuition ...

Balanced growth rate

Euler equation (unchanged):

$$g^* = g(C) = \frac{(1-\beta)\eta L_E^* - \rho}{\theta} \tag{8}$$

Almost done - just need to find L_E.

Balanced growth requires

$$g(C) = g(Y) = g(N) \tag{9}$$

Ideas production function:

$$g(N) = \eta L_R^* = \eta (L - L_E^*) \tag{10}$$

Balanced growth

Solve for the growth rate.

$$\begin{aligned} g(C) &= \frac{(1-\beta)L_E^* - \rho}{\theta} \\ &= \eta(1-L_E^*) \end{aligned}$$

Intuition ...

$\Longrightarrow$

$$L_E^* = \frac{\theta\eta L + \rho}{(1-\beta)\eta + \theta\eta} \tag{11}$$

Scale effects: larger economies grow faster.

With population growth, output growth explodes.

Growth without scale effects

- The previous models do not have balanced growth paths when there is population growth.
- The reason is the scale effect:
  - Larger population $\rightarrow$ more R&D $\rightarrow$ faster growth.
- Diminishing returns to reproducible factors avoid the scale effect, but also kill endogenous growth.

Growth without scale effects

To avoid scale effects, modify the model as follows.

Innovation:

$$\dot{N}_t = \eta N_t^{\phi} L_{Rt} \tag{12}$$

$$0 < \phi \leq 1 \tag{13}$$

Demographics:

$$L_t = e^{nt} \tag{14}$$

$$= L_{Rt} + L_{Et} \tag{15}$$

Balanced growth

From the innovation technology:

$$g(N) = \eta \; N_t^{\phi-1} \; L_{Rt} \tag{16}$$

Constant growth requires constant $N^{\phi-1}L_R$ and

$$g(N) = \frac{n}{1-\phi} \tag{17}$$

The growth rate is "**semi-endogenous**:" endogenous, but not responding to changes in agents' choice variables.

There are still scale effects:

- Larger economies tend towards higher levels of output per person.

Avoiding scale effects

It is possible to write down models that have endogenous growth, but no scale effects (growth does not increase with L).

The idea: Prevent innovator profits from increasing with L.

One approach: the number of products increases with L exactly so that the market size for each variety remains the same (Young, 1998).

Avoiding scale effects requires knife-edge assumptions like this.

Reading

- Acemoglu (2009), ch. 13.
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

References I

Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

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Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.

Young, A. (1998): "Growth without scale effects," *The Journal of Political Economy*, 106, 41.