R&D Models: Introduction

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We study models where **intentional innovation** drives productivity growth.

We start by describing the demand block (common to essentially all models).

Later we embed it into a GE model.
Background

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.
- How important are the 2 cases? – An open question.
How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:  
  - blueprints can be used simultaneously in the production of several goods.
How to model innovation

There are $N$ consumption goods (or intermediate inputs). The goods are imperfect substitutes in preferences (or final output production).

▶ Therefore downward sloping demand curves

Approach 1: Quality ladders

▶ Each good can be made by many firms.
▶ Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: Increasing variety

▶ Each firm can invest to create a new variety ($N \rightarrow N + 1$)
▶ Then it becomes the monopolist for that variety
The Demand Block
Modeling the Demand Side

- The trick in all R&D models: a demand side that generates a constant price elasticity $\varepsilon_D$
- This makes the monopoly price essentially exogenous

$$p_M = \frac{MC}{(1 - 1/\varepsilon_D)}$$ (1)
Dixit Stiglitz Model

- The world is static.
- There are $N$ consumption goods $c_i$ with prices $p_i$.
- Household income is $m$. 
Preferences

- Households aggregate the varieties using a CES aggregator

\[ C = \left( \sum_{i=1}^{N} c_i^\theta \right)^{1/\theta} \]  

- Utility is \( u(C) \)

- Elasticity of substitution \( \varepsilon = 1/(1 - \theta) > 1. \)

- Then \( \theta = (\varepsilon - 1)/\varepsilon > 0. \)

- The trick: constant substitution elasticity implies constant price elasticity.
Demand functions

The household solves:

\[
\max u(C)
\]

subject to

\[
\sum_{i=1}^{N} p_i c_i = m \quad \text{(3)}
\]

Given \( m \), this is just a CES cost minimization problem.
Demand functions

One way of thinking about the household problem:
For any given $C$, find the cost minimizing $c_i$:

$$\min_{c_i} \sum_{i=1}^{N} p_i c_i + \lambda \left[ C - \left( \sum_i c_i^\theta \right)^{1/\theta} \right] \quad (4)$$

FOC:

$$p_i = \lambda \left( \sum_i c_i^\theta \right)^{1/\theta-1} c_i^{\theta-1} \quad (5)$$

The implied demand function is of the form

$$c_i = X p_i^{1/(1-\theta)} = X p_i^{-\varepsilon} \quad (6)$$

with constant price elasticity $\varepsilon$. 
Define the minimized cost of $C$ as

$$PC = \sum p_i c_i$$

The cost minimizing price index is

$$P = (\sum p_i^{1-\varepsilon})^{1/(1-\varepsilon)}$$

This is just the CES unit cost function.
Love for variety

A key implication: simply having more varieties increases welfare.

Assume you have $\bar{C}$ units of “stuff” that can be made (1-for-1) into any variety:

$$\sum_{i=1}^{N} c_i = \bar{C}.$$ 

Consider the symmetric case: $c_i = \bar{C}/N$.

Then

$$C = \left( \sum_{i=1}^{N} [\bar{C}/N]^\theta \right)^{1/\theta}$$

$$= \left( N \ [\bar{C}/N]^\theta \right)^{1/\theta}$$

$$= N^{(1-\theta)/\theta} \bar{C}$$

(9)  

(10)

Spreading $\bar{C}$ over more varieties ($N$) increases utility.
Assume a Dixit-Stiglitz composite consumption good in preferences.

Then demand is isoelastic.
- the elasticity is determined by the elasticity of substitution across varieties in $C$.

The cost of the optimal bundle $C$ is given by the CES minimized cost $P$.

More varieties increase utility.
Each firm has a monopoly over a variety $i$. The demand elasticity is $\varepsilon$. Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\varepsilon}$$

Assumption: The firm is small enough to neglect its effect on $C$ and $P$. 


Equilibrium

Assume symmetry.

Price index:

\[ P = \left( \sum p_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} = N^{1-\varepsilon} \frac{\psi}{1 - 1/\varepsilon} \]

Recall that the elasticity \( \varepsilon \) is \( > 1 \).

Then increasing variety \( N \) decreases the cost of \( C \) (love for variety again).
Equilibrium: Profits

\[ \pi_i = c_i (p_i - \psi) = C P^\varepsilon p_i^{-\varepsilon} (p_i - \psi) = C N^{\varepsilon/(1-\varepsilon)} \frac{\varepsilon}{\varepsilon - 1} \psi \] (12)

More varieties can increase profits:

- Direct effect: \( P \) falls - more competitors erode profits.
- "Aggregate demand externality": \( C \) may rise (depends on preferences)
  - Higher \( N \) raises marginal utility for a given variety.
  - Innovators impose pecuniary externality on competitors.
Continuum of varieties

- Nothing changes when $i$ is continuous.
- Replace all $\Sigma$ with $\int$. 
Reading

- Acemoglu (2009), ch. 12.
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)
Ideal price index I

Proof:

\[
\min \sum_i p_i c_i + \lambda \left[ \left( \sum_j c_j^\theta \right)^{1/\theta} - C \right]
\]  \hspace{1cm} (13)

FOC:

\[
p_i = \lambda \left( \sum_j c_j^\theta \right)^{(1/\theta)-1} c_i^{\theta-1}
\]  \hspace{1cm} (14)

\[
= \lambda C^{1-\theta} c_i^{\theta-1}
\]  \hspace{1cm} (15)

Solve for \( \lambda \):

\[
c_i = (\lambda / p_i)^{1/(1-\theta)} C
\]  \hspace{1cm} (16)
Ideal price index II

\[
\left( \sum c_i^\theta \right)^{1/\theta} = C \lambda^{1/(1-\theta)} \left( \sum p_i^{\theta/(1-\theta)} \right)^{1/\theta} \tag{17}
\]

\[
\lambda = \left( \sum p_i^{\theta/(1-\theta)} \right)^{(1-\theta)/\theta} \tag{18}
\]

Substitute and simplify.

The demand functions \( c_i/C = (p_i/P)^{-\varepsilon} \) emerge.

QED
Digression: An Alternative Derivation

By definition:

\[ PC = \sum p_i c_i \] (19)

We need to express \( C \) and \( \sum p_i c_i \) as functions of prices to solve for \( P \).

First-order conditions determine relative demands:

\[ \frac{c_i}{c_1} = \frac{p_i^{-\varepsilon}}{p_1^{-\varepsilon}} \] (20)

Sub into expression for \( \sum p_i c_i = c_1 \sum p_i \left( \frac{c_i}{c_1} \right) \)

\[ = c_1 p_1^{\varepsilon} \sum p_i^{1-\varepsilon} \]
Alternative Derivation

Sub the same into expression for $C$

\[
C = c_1 \left( \sum \frac{c_i}{c_1} \right)^{(\varepsilon-1)/\varepsilon} \frac{\varepsilon}{(\varepsilon-1)}
\]
\[
= c_1 \left( \sum \frac{p_i}{p_1} \right)^{1-\varepsilon} \frac{\varepsilon}{(\varepsilon-1)}
\]
\[
= c_1 p_1^{\varepsilon} \left( \sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}
\]

Take the ratio:

\[
P = \frac{PC}{C} = \frac{c_1 p_1^{\varepsilon}}{c_1 p_1^{\varepsilon}} \frac{\sum p_i^{1-\varepsilon}}{\left( \sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}}
\]

Simplify to get the solution for $P$. 

Alternative Derivation

The demand functions take the form

\[ \frac{c_i}{C} = \left( \frac{p_i}{P} \right)^{-\varepsilon} \]  

(21)

Proof:

\[ p_i c_i = p_i c_1 \left( \frac{p_i}{p_1} \right)^{-\varepsilon} \]

\[ \sum p_i c_i = PC = c_1 p_1^{\varepsilon} \sum p_i^{1-\varepsilon} \]

\[ = c_1 p_1^{\varepsilon} P^{1-\varepsilon} \]

\[ PC P^{\varepsilon-1} = c_1 p_1^{\varepsilon} \]

Rearrange. QED.
