# R&D Models: Introduction

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# Issues

- We study models where **intentional innovation** drives productivity growth.
- We start by describing the demand block (common to essentially all models).
- Later we embed it into a GE model.

# Background

- Historians often view innovation as the result of research that is not profit driven.
- Economists treat innovation as producing goods that are sold in markets ("blueprints").
- There are historical examples of both types of innovation.
- How important are the 2 cases? – An open question.

# How to model innovation

- Current models are somewhat reduced form.
- The issue how existing knowledge feeds into future innovation is treated as a **knowledge spillover**.
- Knowledge is treated as a scalar - like capital.
- In fact, the only difference between blueprints and machines is **non-rivalry**:
  - blueprints can be used simultaneously in the production of several goods.

# How to model innovation

There are $N$ consumption goods (or intermediate inputs).

The goods are imperfect substitutes in preferences (or final output production).

- Therefore downward sloping demand curves

Approach 1: **Quality ladders**

- Each good can be made by many firms.
- Firms can invest to improve quality (equivalently: lower the cost) of 1 good.

Approach 2: **Increasing variety**

- Each firm can invest to create a new variety $(N \to N+1)$
- Then it becomes the monopolist for that variety

# The Demand Block

# Modeling the Demand Side

- The trick in all R&D models:
  a demand side that generates a **constant price elasticity** $\varepsilon_D$
- This makes the monopoly price essentially exogenous

$$p_M = MC/(1 - 1/\varepsilon_D) \tag{1}$$

# Dixit Stiglitz Model

- The world is static.
- There are $N$ consumption goods $c_i$ with prices $p_i$.
- Household income is $m$.

# Preferences

- Households aggregate the varieties using a CES aggregator

$$C = \left( \sum_{i=1}^{N} c_i^{\theta} \right)^{1/\theta} \tag{2}$$

- Utility is $u(C)$
- Elasticity of substitution $\varepsilon = 1/(1-\theta) > 1$.
- Then $\theta = (\varepsilon - 1)/\varepsilon > 0$.
- The trick: constant substitution elasticity implies constant price elasticity.

# Demand functions

The household solves:

$$\max u(C)$$

subject to

$$\sum_{i=1}^{N} p_i c_i = m \tag{3}$$

Given $m$, this is just a CES cost minimization problem.

# Demand functions

One way of thinking about the household problem:

For any given $C$, find the cost minimizing $c_i$:

$$\min_{c_i} \sum_{i=1}^{N} p_i c_i + \lambda \left[ C - \left( \sum_i c_i^{\theta} \right)^{1/\theta} \right] \tag{4}$$

FOC:

$$p_i = \lambda \left( \sum_i c_i^{\theta} \right)^{1/\theta - 1} c_i^{\theta - 1} \tag{5}$$

The implied demand function is of the form

$$c_i = X p_i^{1/(1-\theta)} = X p_i^{-\varepsilon} \tag{6}$$

with **constant price elasticity** $\varepsilon$.

# Ideal price index

Define the minmized cost of $C$ as

$$PC = \sum p_i c_i \tag{7}$$

The cost minimizing price index is

$$P = \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \tag{8}$$

This is just the CES unit cost function.

Details

# Love for variety

A key implication: simply having more varieties increases welfare.

Assume you have $\bar{C}$ units of "stuff" that can be made (1-for-1) into any variety:

$\sum_{i=1}^{N} c_i = \bar{C}$.

Consider the symmetric case: $c_i = \bar{C}/N$.

Then

$$\begin{aligned} C &= \left( \sum_{i=1}^{N} [\bar{C}/N]^{\theta} \right)^{1/\theta} \\ &= \left( N \ \left[\bar{C}/N\right]^{\theta} \right)^{1/\theta} && (9) \\ &= N^{(1-\theta)/\theta} \ \bar{C} && (10) \end{aligned}$$

Spreading $\bar{C}$ over more varieties $(N)$ increases utility.

# Household summary

- Assume a Dixit-Stiglitz composite consumption good in preferences.
- Then demand is isoelastic.
  - the elasticity is determined by the elasticity of substitution across varieties in $C$.
- The cost of the optimal bundle $C$ is given by the CES minimized cost $P$.
- More varieties increase utility.

# Firms

- Each firm has a monopoly over a variety $i$.
- The demand elasticity is $\varepsilon$.
- Optimal monopoly pricing implies a constant markup over marginal cost:

$$p_i = \frac{\psi}{1 - 1/\varepsilon} \tag{11}$$

- Assumption: The firm is small enough to neglect its effect on $C$ and $P$.

# Equilibrium

Assume symmetry.

Price index:

$$\begin{aligned} P &= \left(\sum p_i^{1-\varepsilon}\right)^{1/(1-\varepsilon)} \\ &= N^{\frac{1}{1-\varepsilon}} \frac{\psi}{1-1/\varepsilon} \end{aligned}$$

Recall that the elasticity $\varepsilon$ is $> 1$.

Then increasing variety $N$ decreases the cost of $C$ (love for variety again).

# Equilibrium: Profits

$$\begin{aligned}\pi_i &= c_i\,(p_i - \psi) \\ &= C\;P^{\varepsilon}\;p_i^{-\varepsilon}\;(p_i - \psi) \\ &= C\;N^{\varepsilon/(1-\varepsilon)}\;\frac{\varepsilon}{\varepsilon - 1}\psi \end{aligned} \tag{12}$$

**More varieties can increase profits**:

- Direct effect: $P$ falls - more competitors erode profits.
- "Aggregate demand externality": $C$ may rise (depends on preferences)
  - Higher $N$ raises marginal utility for a given variety.
  - Innovators impose pecuniary externality on competitors.

# Continuum of varieties

- Nothing changes when $i$ is continuous.
- Replace all $\sum$ with $\int$.

# Reading

- Acemoglu (2009), ch. 12.
- Romer (2011), ch. 3.1-3.4.
- Jones (2005)

# Ideal price index I

Proof:

$$\min \sum_i p_i c_i + \lambda \left[ \left( \sum_j c_j^\theta \right)^{1/\theta} - C \right] \tag{13}$$

FOC:

$$\begin{aligned} p_i &= \lambda \left( \sum_j c_j^\theta \right)^{(1/\theta)-1} c_i^{\theta-1} \qquad (14) \\ &= \lambda C^{1-\theta} c_i^{\theta-1} \qquad (15) \end{aligned}$$

Solve for $\lambda$:

$$c_i = (\lambda / p_i)^{1/(1-\theta)} C \tag{16}$$

# Ideal price index II

$$\left(\sum c_i^\theta\right)^{1/\theta} = C\lambda^{1/(1-\theta)}\left(\sum p_i^{\theta/(1-\theta)}\right)^{1/\theta} \tag{17}$$

$$\lambda = \left(\sum p_i^{\theta/(1-\theta)}\right)^{(1-\theta)/\theta} \tag{18}$$

Substitute and simplify.

The demand functions $c_i/C = (p_i/P)^{-\varepsilon}$ emerge.

QED

# Digression: An Alternative Derivation

By definition:

$$PC = \sum p_i c_i \tag{19}$$

We need to express $C$ and $\sum p_i c_i$ as functions of prices to solve for $P$.

First-order conditions determine relative demands:

$$c_i / c_1 = p_i^{-\varepsilon} / p_1^{-\varepsilon} \tag{20}$$

Sub into expression for

$$\begin{aligned} \sum p_i c_i &= c_1 \sum p_i \left( c_i / c_1 \right) \\ &= c_1 p_1^{\varepsilon} \sum p_i^{1-\varepsilon} \end{aligned}$$

# Alternative Derivation

Sub the same into expression for

$$
\begin{aligned}
C &= c_1 \left( \sum (c_i/c_1)^{(\varepsilon-1)/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\
&= c_1 \left( \sum (p_i/p_1)^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)} \\
&= c_1 p_1^{\varepsilon} \ \left( \sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}
\end{aligned}
$$

Take the ratio:

$$
P = \frac{PC}{C} = \frac{c_1 p_1^{\varepsilon}}{c_1 p_1^{\varepsilon}} \frac{\sum p_i^{1-\varepsilon}}{\left( \sum p_i^{1-\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}}
$$

Simplify to get the solution for $P$.

# Alternative Derivation

The demand functions take the form

$$c_i/C = (p_i/P)^{-\varepsilon} \tag{21}$$

Proof:

$$p_i c_i = p_i c_1 \ (p_i/p_1)^{-\varepsilon}$$

$$\begin{aligned} \sum p_i c_i &= PC = c_1 p_1^{\varepsilon} \ \sum p_i^{1-\varepsilon} \\ &= c_1 p_1^{\varepsilon} \ P^{1-\varepsilon} \end{aligned}$$

$$PC \ P^{\varepsilon-1} = c_1 p_1^{\varepsilon}$$

Rearrange. QED.

# References I


Acemoglu, D. (2009): *Introduction to modern economic growth*, MIT Press.

Jones, C. I. (2005): "Growth and ideas," *Handbook of economic growth*, 1, 1063–1111.

Romer, D. (2011): *Advanced macroeconomics*, McGraw-Hill/Irwin.