Problem Set 6: Innovation and Growth

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1 Stochastic patent duration[1]

Consider a version of the "Expanding Variety of Goods" model in which innovators' monopoly power diminishes over time. Otherwise the model is standard.

Demographics: There is a single representative household.

Endowments: The household is endowed with L units of labor, which can only be used for work.

Preferences:

$$U = \int_0^\infty e^{-\rho t} \frac{c(t)^{1-\theta} - 1}{1-\theta} dt. \tag{1}$$

Technology:

- Final goods are produced from labor and intermediate inputs according to

$$Y = (1-\beta)^{-1} L^\beta \int_0^N x(v)^{1-\beta} dv \tag{2}$$

where $0 < \beta < 1$, Y is output, L is labor input, $x(v)$ is the input of the vth type of the intermediate good, and N is the number of varieties.

- Intermediates: It takes ψ units of the final good to make an intermediate good.
- Innovation: It costs $1/\eta$ units of the final good to create a new type of intermediate good.

Market arrangements:

- The final goods sector is perfectly competitive.
- Intermediate goods producers hold monopolies.
- There is free entry for innovators.
- Households own all firms in the economy.

Patents: Upon innovation, the innovator receives a patent. If intermediate good v is currently monopolized, it becomes competitive in the next instant dT with probability $\delta \cdot dT$, where $\delta \geq 0$. Thus, if good j is invented at time t, the probability of it still being monopolized at some future date $v \geq t$ is $e^{-\delta \cdot (v-t)}$.

Notation: Denote by N_1, the number of intermediate goods produced by monopolists and by N_2 the number that is produced competitively. $N = N_1 + N_2$.

Note that the household problem is the same as in the model discussed in class. It is characterized by the Euler equation

$$g(C) = \frac{r-\rho}{\theta} \tag{3}$$

[1] Due to Matt Doyle.

Questions:

1. Solve the problem of the final goods producer. Note that it faces different prices for competitive versus monopolized goods.

   A: The final goods producer solves exactly the same problem as in the baseline model that we solved in class. While prices differ across goods, the producer takes these as given. The demand functions are

$$x(v,t) = L\ p^x(v,t)^{-1/\beta} \tag{4}$$

2. Solve the problem of a monopolist intermediate input producer.

   A: This is also unchanged from the baseline model. Therefore,

$$p^x = \psi/(1-\beta) \tag{5}$$

3. Show that the value of a new blueprint is now given by

$$V = \frac{\pi}{r+\delta} \tag{6}$$

   Here, we are assuming (and verifying later) that $\dot{V}(v) = 0$ while the patent lives. Explain why (6) makes intuitive sense.

   A: The generic solution is

$$rV(v) = \pi(v) + \dot{V}(v) - \delta V(v) \tag{7}$$

   The new term here is $\delta V(v)$ which reflects the fact that the monopolist loses the patent (and its value) with flow probability δ. With $\dot{V} = 0$, we have (6). Losing the patent (just like mortality in the perpetual youth model) effectively increases the discount rate.

4. Show that free entry implies

$$r = \left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta} \eta\beta L - \delta \tag{8}$$

   A: Apply the demand function with the optimal price to obtain profits (unchanged relative to the baseline model):

$$\pi = \left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta} \beta L \tag{9}$$

   Finally, note that free entry requires $\eta V = 1$.

5. Derive the equilibrium growth rate. Which patent duration (or δ) maximizes growth? Does this also maximize welfare (you need to guess here, unless you want to follow the hint at the end of the question and solve for consumption).

   A: Simply substitute r into the Euler equation to obtain

$$g(C) = \frac{1}{\theta}\left[\left(\frac{\psi}{1-\beta}\right)^{-(1-\beta)/\beta} \eta\beta L - \delta - \rho\right] \tag{10}$$

   The growth maximizing patent duration is $\delta = 0$. There is no expectation that this maximizes welfare.

6. Consider the balanced growth path. Show that

$$\frac{N_1}{N_2} = \frac{g}{\delta} \tag{11}$$

Hint: Write out differential equation for N_1 and N_2.

A: For competitive varieties, obviously $\dot{N}_2 = \delta N_1$. For monopolistic varieties, $\dot{N}_1 = \eta Z - \delta N_1$. Balanced growth requires that N_1 and N_2 grow at the same rate. Then

$$g\left(N_2\right) = g = \delta N_1 / N_2 \tag{12}$$

7. Define a competitive equilibrium.

A: Objects $\{C, Y, Z, x\left(v\right), \pi\left(v\right), V\left(v\right), N_1, N_2, w, p, r\}$ that satisfy:

   (a) Household Euler equation.
   (b) Final goods firm demand functions and definition of Y.
   (c) Intermediate goods pricing rules and definitions of π and V.
   (d) Free entry.
   (e) Market clearing for goods (resource constraint), intermediates (implicit), labor (implicit).
   (f) Law of motion for N_2.

With symmetry, it is easy, but tedious, to show that

$$Y = AL\left(N_1\omega + N_2\right) \tag{13}$$

and

$$X = A\left(1-\beta\right)\left(N_1\omega + N_2\right) \tag{14}$$

where $A \equiv \left(1-\beta\right)L\psi^{-(1-\beta)/\beta}$ and $\omega \equiv \left(1-\beta\right)^{1/\beta}$. From, this one can derive balanced growth consumption, but this is also quite tedious, so I am not asking you to do this.