Midterm Exam. Econ720. Fall 2023

Professor Lutz Hendricks. UNC.

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 75 minutes.
- The total number of points is 75.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Households Produce "Money" [65 points]

Demographics: Time is discrete and goes on forever. At each date t, N_t identical young are born. Each lives for 2 periods.

Preferences: $\mathcal{U}(c_t^y) + \beta \mathcal{U}\left(c_{t+1}^o\right)$ where $\mathcal{U}$ is well behaved.

Endowments:

- In period 1, old agents are endowed with M_1 units of an intrinsically useless object called "money."
- Each young is endowed with y units of consumption.

Technology:

- The endowment can only be eaten or used to produce new money: $N_t y = C_t + N_t z_t$ where C is aggregate consumption and $N_t z_t$ is the amount of goods used to make new money.
- "Money" is produced from goods according to

$$M_{t+1} = [M_t + N_t \mathcal{F}(M_t, z_t)] \times (1 - \delta) \tag{1}$$

  A young household can produce new money using goods z_t. The money produced in t may be sold in $t+1$ at price q_{t+1}. Any money held from one period to the next depreciates at rate δ. The production function $\mathcal{F}$ is strictly increasing and strictly concave $\left(\mathcal{F}_{zM} \equiv \frac{\partial \mathcal{F}}{\partial z \partial M} < 0\right)$.

Markets: There are competitive markets for goods (numeraire) and money (price q_t). Note that goods are the numeraire, not money.

Consider an equilibrium in sequence language.

Questions:

1. [20 points] Write down the household's young and old **budget constraints**. Remember that households spend z_t on producing money that they sell when old. Derive the lifetime budget constraint

$$y - c_t^y - z_t = \frac{c_{t+1}^o - q_{t+1}(1-\delta)\mathcal{F}(M_t, z_t)}{R_{t+1}} \tag{2}$$

   and **explain** what is says in words. Explain what the **real interest rate** facing the household is.

   Answer

   Young: $y = c_t^y + q_t x_t + z_t$, where x is the amount of money purchased.

   Old: $c_{t+1}^o = (1-\delta)[x_t + \mathcal{F}(M_t, z_t)]\, q_{t+1}$. The old bring the money purchased and the amount produced into the period. They sell the undepreciated amount.

The lifetime budget constraint is then given by

$$\frac{y - c_t^y - z_t}{q_t} = x_t = \frac{c_{t+1}^o}{q_{t+1}(1-\delta)} - \mathcal{F}(M_t, z_t) \tag{3}$$

It makes more sense when written as (2) where

$$R_{t+1} = \frac{q_{t+1}(1-\delta)}{q_t} \tag{4}$$

is the gross interest rate from buying and holding money. Now we have something of the form "present value of consumption = present value of income" where the present value of income consists of the endowment y plus the profit from producing money.

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2. [20 points] Derive and **explain** the household's **first order conditions**. Show that the standard Euler equation

$$\mathcal{U}'(c_t^y) = \beta \mathcal{U}'\left(c_{t+1}^o\right) R_{t+1} \tag{5}$$

holds. Also explain why

$$R_{t+1} = (1-\delta)\, q_{t+1} \mathcal{F}_z(M_t, z_t) \tag{6}$$

holds (where $\mathcal{F}_z \equiv \partial \mathcal{F}/\partial z$). Note that the household takes the M argument in $\mathcal{F}(M, z)$ as given (it is the aggregate money stock).

Answer ---

The household solves

$$\max_{x_t, z_t} \mathcal{U}(y - q_t x_t - z_t) + \beta \mathcal{U}((1-\delta)[x_t + \mathcal{F}(M_t, z_t)]\, q_{t+1}) \tag{7}$$

FOC: $\mathcal{U}'(c_t^y)\, q_t = \beta \mathcal{U}'\left(c_{t+1}^o\right)(1-\delta)\, q_{t+1}$ and $\mathcal{U}'(c_t^y) = \beta \mathcal{U}'\left(c_{t+1}^o\right)(1-\delta)\, q_{t+1} \mathcal{F}_z(M_t, z_t)$

Easier to interpret: We have a standard Euler equation of the form (5). In addition, we have a condition that maximizes the present value of income $R_{t+1} = (1-\delta)\, q_{t+1} \mathcal{F}_z(M_t, z_t)$. The household gives up one unit of consumption and either gains R_{t+1} units by buying money or $(1-\delta)\, q_{t+1} \mathcal{F}_z(M_t, z_t)$ units by producing it.

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3. [5 points] State the **market clearing** conditions.

Answer ---

Goods and money markets: the resource constraints.

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4. [10 points] Define a **competitive equilibrium**.

Answer

Objects:

- Household $c_t^y, c_{t+1}^o, x_t, z_t$
- Markets: M_t, q_t, R_t

Equations: Household (2 FOCs and 2 budget constraints); market clearing (2); identity for R; identity $M_t = N_t x_t$.

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5. [10 points] Consider a **steady state** with constant population where all real variables are constant over time. Show that inflation must be zero in such an equilibrium (q is constant over time). Provide intuition.

Answer

z must be constant over time. Suppose that M grows over time while q falls over time. Then $\mathcal{F}_z$ declines over time (because $\mathcal{F}_{zM} < 0$). But constant R requires constant $q\mathcal{F}_z$.

Intuition: Constant real money balances requires that the value of money falls over time if M grows over time. But money gets more and more expensive to produce (concavity of $\mathcal{F}$). Then the rate of return of making money falls over time (for constant z). Households cut back on z, but that causes money growth to slow down until it is zero.

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2 Short Question [10 points]

Consider an OLG production economy with standard assumptions, except that households live for T periods.

Population growth is n. Each household provides ℓ_a units of labor at age a.

The constant returns to scale production function is $F(K, L)$ where K is aggregate capital and L is aggregate labor input. Output is used for investment and consumption. Capital depreciates at rate δ. Households own the capital and rent it to a representative firm.

Write down the goods and labor market clearing conditions.

Answer

Let's define an aggregation function $\mathcal{A}(x, t) \equiv \sum_{a=1}^{T} x_{a,t} N_{a,t}$ where $N_{a,t} = N_t / (1+n)^{a-1}$ and $x_{a,t}$ is the choice of x of a household aged a in period t.

The goods market condition is $K_{t+1} + C_t = F(K_t, L_t) + (1-\delta) K_t$ where $C_t = \mathcal{A}(c)$ and $K_{t+1} = \mathcal{A}(s)$ where s is household saving (budget constraint $c_{a,t} + s_{a,t} = w_t \ell_a + R_t s_{t-1}$).

For the labor market: $L_t = \mathcal{A}\,(\ell)$.

End of exam.