Midterm Exam. Econ720. Fall 2019

Professor Lutz Hendricks. UNC.

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- The total number of points is 100.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Arrow Debreu endowment economy

Demographics: Time is discrete and goes on forever. There are two types of households, both of measure 1, indexed by $j \in \{1, 2\}$.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_{j,t})$ with $0 < \beta < 1$. Assume $u(c) = c^{1-\sigma}/(1-\sigma)$ where needed.

Endowments: In odd periods, households are endowed with $e_{j,t} = d_j \geq 0$ goods. In even periods, the endowments are $f_j \geq 0$.

Technology: Endowments can only be eaten: $\sum_j e_{j,t} = \sum_j c_{j,t}$.

Markets: Trading takes place at date 0, Arrow-Debreu style.

Questions:

1. [12 points] Define the solution to the household problem.
2. [8 points] Define a competitive equilibrium.
3. [9 points] Show that the ratio of marginal utilities of the two households is constant over time. Explain the intuition.
4. [4 points] Is this outcome optimal? Explain.
5. [9 points] Derive the sequence of Arrow-Debreu prices as a function of endowments. Explain the intuition for their properties.
6. [5 points] If we considered a sequential trading equilibrium that delivers the same allocation, what would be sequence of interest rates? Explain. You do not need to set up the sequential trading economy to answer this question.

1.1 Answers

1. Each household has the Euler equation

$$u'(c_{j,t}) = \beta u'(c_{j,t+1}) \, p_t / p_{t+1} \tag{1}$$

   and budget constraint

$$\sum_t p_t (e_{j,t} - c_{j,t}) = 0 \tag{2}$$

2. A CE consists of objects $\{c_{j,t}, p_t\}$ that satisfy:
   (a) households choose the $c_{j,t}$ given the p_t;
   (b) the goods market clears (same as RC).

3. Simply take the ratio of the households' Euler equation. Intuition: this really restates the Euler equation. Each household allocates total income over time to equate MRS with relative prices. So the MRS is the same for all households.

4. This is optimal. Consider a deviation. Then one could redistribute at t and $t+1$ to make both households better off.

5. With CRRA utility, the ratio of consumption $c_{2,t}/c_{1,t}$ is constant over time. Hence, $c_{j,t}$ is a constant fraction of the total endowment $E_t = \sum_j e_{j,t}$. Then $c_{j,t+1}/c_{j,t} = E_{t+1}/E_t$. Then $MRS(t, t+1) = (E_{t+1}/E_t)^{-\sigma} \beta$. From the Euler equation, this is the price ratio p_{t+1}/p_t. Normalize $p_0 = 1$.

6. With sequential trading, we would have the same Euler equation, except that p_t/p_{t+1} would be replaced with $1 + r_{t+1}$.

2 Land and Human Capital

Demographics: A unit mass of infinitely lived, identical households.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$ with $0 < \beta < 1$.

Endowments: At the start of period 0, the household is endowed with h_0 units of human capital, and $l_0 = L$ units of land. Land is in fixed supply.

Technologies:

- Human capital is accumulated according to $h_{t+1} = (1-\delta) h_t + A x_t$ where x_t denotes goods devoted to learning.
- Goods are produced from land and human capital according to $F(L_t, H_t) = L_t^{\alpha} H_t^{1-\alpha} = c_t + x_t$.

Markets: There are competitive markets for goods (numeraire), land rental (price q_t), labor rental (wage w_t), and land purchases (price p_t).

Questions:

1. [5 points] Write down the household's dynamic program. Hint: the budget constraint is $w_t h_t + q_t l_t + p_t l_t = c_t + x_t + p_t l_{t+1}$.
2. [13 points] Write down the first-order and envelope conditions.
3. [7 points] Derive and interpret
$$u'(c) = \beta u'(c') \frac{p' + q'}{p} \tag{3}$$
$$= A\beta [w' u'(c') + \beta w'' u'(c'') + ...] \tag{4}$$
4. [11 points] Define a competitive equilibrium.
5. [17 points] Go as far as you can towards characterizing the steady state. For full credit, you should solve for the steady state values of c, h, x, w in closed form. It is fine to write the solution for one variable as a function of the closed form solution of another (e.g., solve for w and then write c in terms of parameters and w).

2.1 Answers

1. Household:

   States: land and human capital. Bellman equation
$$V(l, h) = \max_{l', x} u(wh + ql - p(l' - l) - x) + \beta V(l', (1-\delta) h + Ax) \tag{5}$$

2. FOCs and envelope:

   (a) FOCs:

$$u'(c) = \beta V_h(.') A \tag{6}$$
$$u'(c) p = \beta V_l(.') \tag{7}$$

   In words: Giving up a unit of consumption buys A units of h' or $1/p$ units of land.

   (b) Envelope:

$$V_l = u'(c)(p+q) \tag{8}$$
$$V_h = u'(c) w + \beta V_h(.')(1-\delta) \tag{9}$$

3. Derivation is easy. Interpretation:

   (a) The first condition is the standard asset pricing equation.

   (b) The second conditions says: give up one unit of consumption to buy one unit of human capital. This yields a "dividend" stream of $w_{t+j}(1-\delta)^j$ which can be eaten.

4. Equilibrium: $\{c_t, h_{t,} x_{t,} l_t, L_{t,} H_{t,} p_t, q_t, w_t\}$ that satisfy:

   (a) household: 2 Euler equations, budget constraint, law of motion for h.

   (b) firm: 2 standard first-order conditions

   (c) market clearing: goods (RC), land rental and purchase ($l_t = L_t = L$), labor ($H_t = h_t$)

   We have 9 objects and 9 (independent) equations.

5. Steady state:

   (a) Constant h requires $Ax = \delta h$.

   (b) Then $c = F(L,h) - \delta/A \times h$ or $c/h = F(L/h, 1) - \delta/A$. Define $F(L/h, 1) = f(L/h)$

   (c) Constant consumption requires $\beta(p+q)/p = 1$ or $q/p = 1/\beta - 1$.

   (d) Firm: $q = f'(L/h)$ and $w = f(L/h) - f'(L/h) L/h = (1-\alpha)(L/h)^\alpha$.

   (e) Second FOC: $1 = A \sum_{j=1}^{\infty} \beta^j (1-\delta)^{j-1} w$.

   (f) Now we can unravel the entire steady state. Solve the expression from (e) for the wage. From that, recover L/h and therefore h from the firm's FOC. Then we know $x = \delta/A \times h$ and c from the resource constraint.

End of exam.