Midterm Exam. Econ720. Fall 2014

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- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 OLG with Money and Trees

Demographics: N young are born in t. Each lives for 2 periods.

Endowments: Each person receives endowments of e_1 when young and e_2 when old (in units of the good). Assume that $e_1 > e_2$. The initial old hold M units of fiat money (pieces of paper) and 1 tree (total, no per person).

Preferences: $u\left(c_t^y\right) + u\left(c_{t+1}^o\right)$ with $u\left(c\right) = c^{1-\sigma}/\left(1-\sigma\right)$ and $0 < \sigma \leq 1$. Note that households do not discount old consumption.

Technology: The tree yields d units of the good in each period. Goods can only be eaten, not stored.

Markets: Money is the numeraire. Goods are traded at price p_t. Trees are traded at price $q_t p_t$.

Questions:

1. State the household problem.
2. Derive the saving function
$$s\left(R\right) = \frac{e_1 - e_2 R^{-1/\sigma}}{1 + R^{1-1/\sigma}} \tag{1}$$
where R is the return on money and trees. Note that the saving function is increasing in R when $\sigma < 1$.
3. Define an equilibirum with valued fiat money.
4. Derive an equation for the offer curve. Recall that this is a difference equation linking m_{t+1} to m_t . Hint: Start from $m_t = s\left(R_{t+1}\right)$. Then think about how R_{t+1} relates to m_t and m_{t+1}. For simplicity, set $\sigma = 1$.
5. Define a steady state with valued fiat money.
6. Does it exist? What is the intuition?
7. For the steady state where fiat money is *not* valued, solve for R. For simplicity, set $\sigma = 1$.
8. For the steady state without trees, solve for R and s. The value of σ does not matter here.

End of exam.

2 Answer: OLG with Money and Trees

Based on Sargent and Ljunqvist, exercises 8.1 and 8.3.

1. Household: $\max u\left(c_t^y\right) + u\left(c_{t+1}^o\right)$ subject to $c_t^y + m_t + \alpha_t q_t = e_1$ and $c_{t+1}^o = e_2 + m_t/\pi_{t+1} + \alpha_t\left(d + q_{t+1}\right)$.

2. Saving function: From the standard Euler equation, where $s\left(R_{t+1}\right) = e_1 - c_t^y = m_t + \alpha_t q_t$.

3. Equilibrium:

   Objects: $\{c_t^y, c_t^o, \alpha_t, m_t, q_t, p_t, R_t\}$

   Equations:

   (a) household: Euler equation and 2 budget constraints (portfolio composition is indeterminate)
   (b) goods market: $N\left(c_t^y + c_t^o\right) = N\left(e_1 + e_2\right) + d$
   (c) money market: $m_t = M/\left(N p_t\right)$
   (d) tree market: $\alpha_t = 1/N$
   (e) identities: $\pi_{t+1} = p_{t+1}/p_t$; $R_{t+1} = p_t/p_{t+1} = \left(d + q_{t+1}\right)/q_t$.

4. Offer curve: Note that $R_{t+1} = m_{t+1}/m_t$. Substitute that into

$$m_t = s\left(R_{t+1}\right) = \frac{e_1 - e_2\left(m_{t+1}/m_t\right)^{-1}}{1 + \left(m_{t+1}/m_t\right)^{1-1}} \tag{2}$$

   Solving yields $m_{t+1} = e_2/\left(e_1/m_t - 2\right)$ which is increasing.

5. Steady state: $\{c^y, c^o, \alpha, q, m, \pi, R\}$ that solve:

   (a) household, goods market, money market, tree market: unchanged
   (b) identities: $R = 1 = 1 + d/q$.

6. A stationary equilibrium does not exist. Intuition: Trees are always valued. But there cannot be a constant q that yields $R = 1$. But only $R = 1$ is consistent with constant money balances.

7. Stationary equilibrium without money: We still need q to be constant, so $R = 1 + d/q > 1$. We also have $s\left(R\right) = q$. With log utility ($\sigma = 1$): $s\left(R\right) = \left(e_1 - e_2/R\right)/2 = q = d/\left(R - 1\right)$. That can be solved for $R > 1$.

8. Stationary equilibrium without trees: We still need p to be constant, so $R = 1$. Then $s\left(R\right) = \left(e_1 - e_2\right)/2 = m$. The rest is just plugging into budget constraints.

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