Midterm Exam. Econ720. Spring 2010

Professor Lutz Hendricks

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 1:15 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

---

1 Two types of households [35 points]

Consider a standard OLG model with two types of households.

Demographics: Each period N_i households of type $i \in \{1, 2\}$ are born. There is no population growth. Households live for 2 periods.

Endowments: Households are endowed with one unit of work time when young.

Preferences: Household utilities are given by $u_i(c_t^y) + \beta u_i(c_{t+1}^o)$. $u_i(c) = \frac{c^{1-\sigma_i}}{1-\sigma_i}$.

Technology: The resource constraint is $K_{t+1} = F(K_t, L_t) + (1-\delta)K_t - C_t$ where upper case variables denote aggregates. F has constant returns to scale and satisfies Inada conditions.

Markets: Households work for firms when young, earning a competitive wage w_t. Households rent capital to firms, earning the rental price q_t.

Questions:

1. Derive the decision rules for both household types.
2. Define a competitive equilibrium.
3. Derive an implicit equation for the law of motion of the aggregate capital stock.

2 Habit formation [65 points]

We add habit formation to the standard growth model in continuous time.

Demographics: There is a single, infinitely lived representative household.

Preferences: The household maximizes $\int_0^\infty e^{-\rho t} u(c_t, z_t) dt$ where c is consumption and

$$z_t = \int_{-\infty}^{t} e^{-\gamma(t-j)} c_j dj \tag{1}$$

is a weighted average of lagged consumption. Assume $0 < \rho < 1$ and $0 < \gamma < 1$.

Endowments: At date 0 the household is endowed with k_0 units of the good.

Technology: The household can produce new goods using only goods. The resource constraint is $\dot{k} = f(k) - \delta k - c$.

Markets: This is a Robinson Crusoe economy. You may think of identical households trading goods or a single planner choosing the entire allocation.

Questions:

1. What are the household state variables? Write down their laws of motion. Hint: (Leibniz's rule):

$$\frac{d}{dt} \int_a^t f(x_j, t) dj = f(x_t) + \int_a^t [\partial f(x_j, t)/\partial t] dj \tag{2}$$

   If you cannot solve this part, assume that $\dot{z} = ac - bz$ where a and b are some positive constants.

2. Write down the current value Hamiltonian and the first order conditions.

3. Define a solution to the household problem.

4. Define a steady state.

5. How does the presence of habit formation affect steady state consumption and capital? Explain the intuition.

3 Answers

3.1 Answer: Two types of households[1]

1. Decision rule: Household with log utility: $k' = \frac{\beta}{1-\beta} w$. Household with CRRA: $k' = w \left\{ 1 + \beta^{-1/\sigma} (R')^{1-1/\sigma} \right\}$.

2. Competitive equilibrium: $\left\{ c^y_{i,t}, c^o_{i,t}, k_{i,t}, K_t, L_t, w_t, R_t, q_t \right\}$ that satisfy:

1. Household: foc and 2 budget constraints. As usual.
2. Firms: 2 focs
3. Market clearing: capital, goods, labor.
4. Identity: $R = 1 - \delta + q$.

3. Law of motion: $k' = \sum_i (N_i/N) k'_i$. Sub in laws of motion and sub out prices.

3.2 Answer: Habit formation

1. State variables: State variables are k and z. The law of motion for k is given. That for z is $\dot{z} = c - \gamma z$.

2. Hamiltonian:

$$H = u(c, z) + \lambda \left(f(k) - \delta k - c \right) + \mu \left(c - \gamma z' \right) \tag{3}$$

First order conditions:

$$c: \;\; u_c - \lambda + \mu = 0 \tag{4}$$
$$k: \;\; -\dot{\lambda} + \rho\lambda = \lambda \left(f'(k) - \delta \right) \tag{5}$$
$$z: \;\; -\dot{\mu} + \rho\mu = u_z - \mu\gamma \tag{6}$$

3. Solution: Functions of time $(c, z, k, \lambda\mu)$ that solve: 3 first-order conditions, 2 laws of motion, boundary conditions (k_0, z_0 given and TVC).

4. Steady state: Determined recursively: $g(u_c) = 0$ solves for k_{ss}: $f'(k_{ss}) - \delta = \rho$. $\dot{k} = 0$ solves for consumption. $\dot{z} = 0$ solves for $z = c/\gamma$.

[1] Based on a question due to Tony Smith

5. Effect of habit formation: Habit formation has no effect. Steady state values of k, c, z are independent of $u()$. Intuition: With constant z marginal utility is shifted by a constant. The intertemporal allocation is not affected.