Final Exam. Econ720. Fall 2024

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- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- The total time is 120 minutes.
- The total number of points is 120.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

Name:	PID:

1 Habit Formation

We add habit formation to the standard growth model in continuous time. We study the planner's problem.

Demographics: There is a single, infinitely lived representative household.

Preferences: The household maximizes $\int_0^\infty e^{-\rho t} u(c_t, z_t) dt$ where c is consumption and

$$z_t = \int_{-\infty}^{t} e^{-\gamma(t-\tau)} c_\tau d\tau \tag{1}$$

is a weighted average of lagged consumption. Assume $\rho > 0$ and $\gamma > 0$.

Endowments: At date 0 the household is endowed with k_0 units of the good and consumption "habit" z_0.

Technology: The household can produce new goods using only goods. The resource constraint is $\dot{k} = f(k) - \delta k - c$.

Note that Leibniz's rule

$$\frac{d}{dt} \int_a^t f(x_j, t) dj = f(x_t) + \int_a^t [\partial f(x_j, t)/\partial t] dj \tag{2}$$

implies that $\dot{z} = c - \gamma z$.

Questions:

1. [10 points] Write down the current value Hamiltonian. Be clear about what the state and control variables are.

   Answer

   State variables are k and z. The control is just c. The laws of motion are given.

   Hamiltonian:

   $$H = u(c, z) + \lambda (f(k) - \delta k - c) + \mu (c - \gamma z) \tag{3}$$

2. [17 points] Derive the first-order conditions. Define a solution to the planner's problem.

   Answer

First order conditions:

$$c: \; u_c - \lambda + \mu = 0 \tag{4}$$
$$k: \; -\dot{\lambda} + \rho\lambda = \lambda\left(f'(k) - \delta\right) \tag{5}$$
$$z: \; -\dot{\mu} + \rho\mu = u_z - \mu\gamma \tag{6}$$

Solution: Functions of time (c, z, k, λ, μ) that solve: 3 first-order conditions, 2 laws of motion, boundary conditions (k_0, z_0 given and 2 TVCs).

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3. [12 points] Characterize the steady state. You should obtain 3 equations in 3 unknowns.

Answer

Determined recursively: $g(u_c) = 0$ solves for k_{ss}: $f'(k_{ss}) - \delta = \rho$. $\dot{k} = 0$ solves for consumption. $\dot{z} = 0$ solves for $z = c/\gamma$.

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4. [6 points] How does the presence of habit formation affect steady state consumption and capital? Explain the intuition.

Answer

Habit formation has no effect. Steady state values of k, c, z are independent of $u()$. Intuition: With constant z, marginal utility is shifted by a constant. The intertemporal allocation is not affected.

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2 Stochastic Sidrauski Model

Demographics:

- There is a representative household of unit mass who lives forever.

Preferences:

- The household values consumption c, not working (where work time is l), and real money holdings m according to
$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\mathcal{U}\left(c_t\right)-\mathcal{V}\left(l_t\right)+\mathcal{X}\left(m_t\right)\right] \tag{7}$$
where $\mathcal{U},-\mathcal{V},\mathcal{X}$ are strictly increasing and strictly concave.
- $m_t = M_t/P_t$ denotes real money held at the start of period t.

Endowments:

- At the beginning of time, households are endowed with M_0 units of money.

Technology:

- Each household produces output from labor only.
- The resource constraint is $c_t + g_t = \mathcal{F}\left(l_t\right) = l_t^{\alpha}$ where $0 < \alpha < 1$ and g is government consumption.

Government:

- Government spending g is stochastic: $g_t = \left(1-f_t\right)\mathcal{F}\left(l_t\right)$ where $0 < f < 1$ is drawn from a Markov process. Households take g_t as given.
- The government issues money to pay for g_t.
- The budget constraint is $P_t g_t = M_{t+1} - M_t$.

Markets:

- The household operates the technology. There are no firms.
- There are competitive markets for money (numeraire) and goods (price P_t).

Note:

- The household can sell m_t during period t and still enjoy $\mathcal{X}\left(m_t\right)$. Hence, the budget constraint in nominal terms is given by
$$P_t\mathcal{F}\left(l_t\right) + P_t m_t = P_t c_t + m_{t+1}P_{t+1} \tag{8}$$
- The household does not know P_{t+1} when making decisions in period t.

Questions:

1. [15 points] State the household's dynamic program in real terms. Clearly state what the aggregate and individual state variables are. Also state what the controls are. Hint: can the household control m'?

   Answer ____

   Aggregate state $S = (f, \bar{m})$ where $\bar{m}$ is the amount of real money that "everyone else" holds.

   Since inflation is not known when M' is chosen, the control cannot be real money holdings m'. It has to be $M'/P \equiv \tilde{m} = m'(1+\pi')$. If inflation turns out higher than expected, the household ends up with "too little" money next period.

   The Bellman equation is given by

   $$V(m,S) = \max_{c,l,\tilde{m}} \mathcal{U}(c) - \mathcal{V}(l) + \mathcal{X}(m) + \beta \mathbb{E} V\left(\frac{\tilde{m}}{1+\pi'}, S'\right) \tag{9}$$

   subject to the budget constraint in real terms

   $$\mathcal{F}(l) + m = c + \tilde{m} \tag{10}$$

   ____

2. [15 points] Derive the household's Euler equation

   $$\mathcal{U}'(c) = \beta \mathbb{E}\left\{\frac{\mathcal{U}'(c') + \mathcal{X}'(m')}{1+\pi'}\right\} \tag{11}$$

   and static optimality condition $\mathcal{U}'(c)\mathcal{F}'(l) = \mathcal{V}'(l)$. Explain what they say in words.

   Answer ____

   FOCs

   - $\mathcal{U}'(c) = \lambda$
   - $\mathcal{V}'(l) = \lambda \mathcal{F}'(l)$
   - and for money demand:

   $$\beta \mathbb{E} \frac{V_m(\tilde{m}, S')}{1+\pi'} = \lambda \tag{12}$$

   Envelope: $V_m(m;S) = \mathcal{X}'(m) + \lambda$

   Simplify to obtain Euler and static condition.

   In words:

   (a) Euler: Money is the only asset with rate of return $\frac{1}{1+\pi'}$. Giving up a unit of consumption today buys $\frac{1}{1+\pi'}$ units of real money tomorrow. These yield direct marginal utility $\mathcal{X}'(m')$ plus the usual utility derived from selling the money and buying consumption.

(b) The static condition is standard: MRS = MRT.

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3. [15 points] Express **equilibrium** hours worked and consumption as functions of f_t. Assume $\mathcal{U}(c) = \ln(c)$ and $\mathcal{V}(l) = l^{\gamma}/\gamma$ with $\gamma > 1$.

Answer

Resource constraint: $\mathcal{F}(l) = c + (1-f)\mathcal{F}(l)$ or $c = f\mathcal{F}(l) = fl^{\alpha}$.

Static condition:

$$1/c = \frac{l^{\gamma-1}}{\alpha l^{\alpha-1}} = \frac{l^{\gamma-\alpha}}{\alpha} \tag{13}$$

Combine the two conditions for c to obtain

$$l = (\alpha/f)^{1/\gamma} \tag{14}$$

Therefore, l is decreasing in f. Also

$$c = \alpha^{\alpha/\gamma} \times f^{(\gamma-\alpha)/\gamma} \tag{15}$$

which is increasing in f (because $\gamma > \alpha$).

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4. [10 points] In the previous question, you found that today's consumption and leisure depend only (positively) on today's shock f_t. Explain the intuition. Why is the effect of f_t positive? Why don't expectations of future f_{t+j} appear in the solutions for consumption and labor supply?

Answer

Higher f is basically a positive income effect (the government eats less today). This causes consumption and leisure to rise.

At first, it would seem that there is also a complicated intertemporal effect. Expected future government spending changes (in an unknown direction, depending on the process governing f). The inflation rate changes. That affects incentives to save.

But in this model, saving is always zero (endowment economy). Households think they can save by buying money. But in reality that's not feasible. All output must be eaten today. That's why the static condition plus goods market clearing together solve for c and l regardless of m or expectations. In equilibrium, the interest rate has to ensure that households do not save.

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3 Monopoly pricing

Consider a firm that faces a downward sloping demand curve $\mathcal{Q}(p)$ for its product and price adjustment costs.

Output is produced from labor only. Hence, the firm must satisfy $\mathcal{Q}(p) = z\mathcal{F}(n)$ where z is an i.i.d. productivity shock and $\mathcal{F}$ is the strictly concave production function.

The firm maximizes the expected discounted value of profits $\mathbb{E}\sum_{t=0}^{\infty}\beta^t\pi(p_t, z_t)$

The timing is as follows:

1. The firm enters the period with last period's price p_{-1} and draws productivity shock z.
2. The firm decides whether to keep the price ($p = p_{-1}$) or change it and pay cost C.
3. Once the price has been fixed, the firm produces $\mathcal{Q}(p)$ and earns profit $\pi(p, z) = p\mathcal{Q}(p) - wn$ where w is the wage. Employment must satisfy $\mathcal{Q}(p) = z\mathcal{F}(n)$.
4. The firm draws a new z' and moves into the next period with $p'_{-1} = p$.

[20 points] State the firm's dynamic program. Hint: Think of the firm as making two sequential decisions. Each decision gets its own value function.

Answer

This is a two stage decision problem. At the beginning of the period, the firm decides whether or not to change the price:

$$V(p_{-1}, z) = \max\{\pi(p_{-1}, z) + \beta\mathbb{E}V(p_{-1}, z'), W(z)\} \tag{16}$$

At this stage, p_{-1} is a state. The firm also knows the current shock z. If the firm does not adjust the price, there is no more decision to be made.

If the firm decides to change the price, it reaches the second stage of the decision problem. p_{-1} is now irrelevant and the problem becomes

$$W(z) = \max_p\{\pi(p, z) - C + \beta\mathbb{E}V(p, z')\} \tag{17}$$

End of exam.