Final Exam. Econ720. Fall 2015

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- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- Clearly number your answers.
- The total time is 2 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Economy With Land and Heterogeneity

Demographics: There is a unit mass of farmers. Each lives forever.

Preferences: $\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{\ln\left(c_{it}\right)-Ah_{it}\right\}$ with $A>0$. h denotes hours worked.

Technologies:

- Each farmer produces output according to

$$y_{it}=f\left(L_{it},h_{it};z_t\right)=z_tL_{it}^{\theta}h_{it}^{1-\theta} \tag{1}$$

  L_{it} is farmer i's land.

- z_t is an iid **aggregate** productivity shock that takes on values $\mu+\sigma$ and $\mu-\sigma$ with equal probability.

- The aggregate resource constraint is $\int c_{it}di=\int y_{it}di$.

Endowments: At $t=0$, each farmer is endowed with land L_{i0}. Land is in fixed supply: $\int L_{it}di=L$. In each period, a farmer has a time endowment that is sufficiently large so that we don't have to worry about h_{it} hitting corners.

Markets: There are competitive markets for goods (numeraire), land (price $p\left(z\right)$), and state contingent claims (price $q\left(z',z\right)$).

Timing: The shock z_t is realized at the beginning of period t before markets open.

Questions:

1. State and solve the social planner's problem.

2. Consider a Recursive Competitive Equilibrium. What is farmer i's individual state? What is the aggregate state of the economy?

3. State the farmer's budget constraint. Assume that the household can buy and sell land (observing today's z) before producing.

4. Set up the household's Bellman equation and derive the first-order conditions.

5. Derive the Lucas asset pricing equations for land and contingent claims.

6. Define a farmer's solution in sequence language.

7. Define an equilibrium in sequence language.

8. Does the equilibrium allocation differ from the planner's? Why or why not? What can you say about the time path of consumption inequality?

2 MP Model With Skills

There is a unit mass of infinitely lived workers. At any time, a worker can be employed or unemployed. He can be skilled ($s = H$) or unskilled ($s = L$).

Workers value consumption, discounted at rate r. The unemployed eat b. The employed eat the bargained wage w_s; $s \in \{L, H\}$.

Matches are formed by a matching function $m(u_s, v_s)$.

Transitions:

1. Unskilled unemployed (mass u_L):
   (a) find a job with probability $\alpha_L = m(u_L, v_L) / u_L$
2. Skilled unemployed (mass u_H):
   (a) find a job with probability $\alpha_H = m(u_H, v_H) / u_H$
   (b) become unskilled with probability δ
3. Unskilled employed (mass e_L):
   (a) lose your job with probablity λ
   (b) become skilled with probability η
4. Skilled employed (mass e_H):
   (a) lose your job with probablity λ

The masses satisfy: $u_L + u_H + e_L + e_H = 1$.

For simplicity, assume that wages are renegotiated when workers change from low to high skill (so that all high skilled workers are paid the same wage).

Vacancies: Firms post vacancies that target a specific skill. v_s is the number of vacancies that target skill s. Posting a vacancy costs k.

Questions:

1. Write down the flow equations that determine the steady state values of u_s, e_s. By this, I mean equations of the form $\dot{u}_L = inflows - outflows = 0$.
2. Write down the Bellman equations for workers in all possible states.
3. Write down the Bellman equation for the vacancy posting firms.

4. Define a steady state. Assume free entry by firms creating vacancies and Nash bargaining for the wage.

End of exam.

3 Answers

3.1 Economy With Land[1]

1. Planner: Because of constant returns to scale and linear disutility of working, the size of each farm is indeterminate. It seems obvious that the planner assigns each farmer the same L/h and c. Hence, $L_i = L$. h is chosen to max

$$\ln\left(zL^{\theta}h^{1-\theta}\right) - Ah \tag{2}$$

   or $\max\,(1-\theta)\ln h - Ah$. FOC: $h = (1-\theta)\,A$.

2. RCE: The aggregate state is z and the joint distribution of individual states. Call that s. Farmer i's individual state is land L and a vector of state contingent claims a.

3. Household budget constraint:

$$f\left(L', h; z\right) + a\left(z\right) - p\left(L' - L\right) - \sum_{z'} q\left(z', z\right) a'\left(z'\right) = c \tag{3}$$

   The reason why L' shows up here: agents trade L before they produce and after observing z.

4. Household Bellman:

$$V\left(L, a; s\right) = \max \ln\left(f\left(L', h; z\right) + a\left(z\right) - p\left(L' - L\right) - \sum_{z'} q\left(z', z\right) a'\left(z'\right)\right) - Ah + \mathbb{E}\beta V\left(L', a'; s'\right) \tag{4}$$

   First order conditions:

$$p/c - f_L/c = \beta\mathbb{E}V_L\left(.'\right) \tag{5}$$
$$f_h/c = A \tag{6}$$
$$q\left(z', z\right)/c = \beta\mathbb{E}V_{a'(z')}\left(.'\right) \tag{7}$$

   Envelope:

$$V_L = p/c \tag{8}$$
$$V_{a(z)} = 1/c \tag{9}$$

5. Simplify:

$$\frac{p - f_L}{c} = \beta\mathbb{E}\frac{p'}{c'} \tag{10}$$
$$q\left(z', z\right)/c = \beta\mathbb{E}1/c' \tag{11}$$

   Note that these are just Lucas asset pricing equations.

[1] Inspired by the UCLA qualifying exam 1999.

6. Household solution: $c_{it}, h_{it}, L_{it}, a_{i,t+1}(z_{t+1})$ that satisfy:

   (a) 3 foc (one of which is specific to z_{t+1})

   (b) budget constraint

   (c) initial conditions, TVC

7. CE: Objects: $c_{it}, h_{it}, L_{it}, a_{i,t+1}(z_{t+1}), p_t, q_t(z_{t+1})$.

   Equilibrium conditions:

   - households (above: 4)
   - market clearing: $\int L_{it} di = L$, resource constraint (goods market), $\int a_{it}(z_{t+1}) di = 0$.

8. Implications for heterogeneity: Everyone has the same consumption growth rate. Since farmers initially have different land endowments, their initial consumption must differ. Hence, consumption levels differ permanently (by a constant factor). Since $f_h = (1-\theta) z (h/L)^{-\theta} = Ac$, farmers choose different h/L. Poor farmers choose higher h/L than rich farmers. Hence, the allocations differ from the planner's solution (unless the initial endowments L_{i0} happen to be all the same).

3.2 Answer: MP Model With Skills

1. Transition equations

$$\dot{e}_H = \alpha_H u_H + \eta e_L - \lambda e_H \tag{12}$$
$$\dot{e}_L = \alpha_L u_L - \eta e_L - \lambda e_L \tag{13}$$
$$\dot{u}_H = \lambda e_H - \delta u_H - \alpha_H u_H \tag{14}$$
$$\dot{u}_L = \lambda e_L - \delta u_H - \alpha_L u_L \tag{15}$$

   In steady state, all of these are constant.

2. Bellman:

$$rU_L = b + \alpha_L (W_L - U_L) \tag{16}$$
$$rU_H = b + \alpha_H (W_H - U_H) + \delta (U_H - U_L) \tag{17}$$
$$rW_L = w_L + \eta (W_H - W_L) - \lambda (W_L - U_L) \tag{18}$$
$$rW_H = w_H + \lambda (U_H - W_H) \tag{19}$$

3. Firms:

$$rV_s = -k + \alpha_s J_s \tag{20}$$
$$rJ_H = y_H - w_H + \lambda (V_H - J_H) \tag{21}$$
$$rJ_L = y_L - w_L + \lambda (V_L - J_L) + \eta (J_H - J_L) \tag{22}$$

4. Steady state: 4 labor quantities, 2 vacancy quantities, 8 values, 2 wages that satisfy

   (a) 8 Bellman equations

   (b) 4 flow equations

   (c) Nash bargaining: $W_s - U_s = J_s - V_s$ (or you can have more general bargaining weights) (2)

   (d) free entry: $V_s = 0$ (2)
