Final Exam. Econ720. Fall 2012

Professor Lutz Hendricks

- Answer all questions.
- Write legibly! Write legibly! Write legibly!
- Write on only one side of each sheet.
- Clearly number your answers.
- The total time is 2 hours.
- A good answer should explain what you are doing. For example: "To find the consumption function, I take first order conditions, then use the budget constraint to solve for c." Then comes the math...

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1 Money in the Utility Function

[55 points] Demographics: Time is continuous. There is a single representative household who lives forever.

Preferences:

$$\int_{t=0}^{\infty} e^{-\rho t} u(c_t, m_t) dt \tag{1}$$

where c is consumption and m denotes real money balances.

Endowments: Households work 1 unit of time at each instant, earning a wage w_t. Households are initially endowed with k_0 units of capital and m_0 units of real money.

Technology:

$$f(k_t) - \delta k_t = c_t + \dot{k}_t \tag{2}$$

Money: nominal money grows at exogenous rate $g(M)$. New money is handed to households as a lump-sum transfer: $\dot{M}_t = p_t x_t$.

Questions:

1. The household's budget constraint is given by

$$\dot{k}_t + \dot{m}_t = w + r_t k_t + x_t - c_t - \pi_t m_t - g(\dot{m}_t) \tag{3}$$

where $g(\dot{m}_t)$ is the cost of adjusting the money stock. $g'(0) = 0$ and $g''(\dot{m}_t) > 0$. State the Hamiltonian. If you cannot figure this out, assume $g(\dot{m}) = 0$ and proceed (for less than full credit).

2. State the first-order conditions.
3. Define a competitive equilibrium.
4. Characterize the steady state to the extent possible. What is the effect of a permanent change in $g(M)$?
5. What is the optimal rate of inflation? Explain.

2 McCall Model With On-the-job Search

[45 points] Consider a McCall model where employed workers receive job offers. Time is continuous. The worker lives forever and maximizes the expected present value of income, discounted at rate r.

When unmployed, the arrival rate of job offers is α_0. When employed it is α_1. Offers are drawn from the cdf $F(w)$. Employed workers lose their jobs with probability λ and move into unemployment next period.

Questions:

1. State the Bellman equation for an unemployed worker. Hint: rU = [current payoff] + [expected "capital gain"].
2. State the Bellman equation for an employed worker. Explain it. Hint: With Possion events, the probability of being hit by 2 events at the same time is 0. Therefore, the probability of getting an offer is α_1 and the probability of being layed off is λ.
3. Clearly, unemployed workers set a reservation wage such that $U = W(w_R)$, while employed workers accept any job with $w' > w$. Derive
$$w_R - b = (\alpha_0 - \alpha_1) \int_{w_R}^{\infty} [W(w') - W(w_R)] dF(w') \tag{4}$$
4. Explain (4) in words.
5. Why might a worker accept a job with $w < b$?
6. How would a higher b affect the reservation wage? What is the intuition?
7. Now add the possibility of quits to the model. The worker enters the period with w. Then he observes all the shocks (wage offer, loss of job). Then he decides whether or not to quit and move into unemployment next period. Write down and explain the Bellman equation.
