Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists o**f 4** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Stochastic Discount Factors

Demographics: A unit mass of ex-ante identical, infinitely lived households.

Endowments:

- At the beginning of time, each household draws a discount factor $\beta \in \{\beta_L, \beta_H\}$.
- In each subsequent period, each household draws a new discount factor from a Markov process. π_{jk} is the probability of drawing β_k conditional on having β_j. The individual draws are independent of each other.
- In each period, each household receives a constant endowment of y consumption goods.

Preferences: $\mathbb{E} \sum_{t=0}^{\infty} \beta_t^t \mathcal{U}(c_t)$. The notation β_t^t takes the discount factor at date t (either β_L or β_H) to the power t.

Technology: Endowments can only be eaten.

Markets: There are competitive markets for goods (numeraire) and for one-period bonds (in zero net supply; price p).

We consider a **recursive competitive equilibrium**.

1. [5 points] What is the aggregate state of the economy?

   Answer

   The aggregate state S is the joint distribution of (β, b). Call its law of motion $\mathcal{H}$.

2. [10 points] Write down the household's Bellman equation.

   Answer

   $V(b, j, S) = \max_{b'} \mathcal{U}(y + b - p(S)\, b') + \beta_j \sum_k \pi_{jk} V(b', k, \mathcal{H}(S))$.

3. [20 points] Define a recursive competitive equilibrium. Hint: Write the aggregate state as the fraction of persons with discount factor β_j and, for each j, the distribution of $b|j$. That's also useful for writing out aggregate quantities.

   Answer

   Let μ_j be the (exogenous) mass of households with discount factor β_j. Let Γ_j denote the distribution of bond holdings for households with β_j. In stationary equilibrium, we can think of the aggregate state as $S = (\Gamma_L, \Gamma_H)$ with law of motion $\Gamma_j' = \mathcal{H}_j(\Gamma_L, \Gamma_H)$.

   (a) Household: value function V and decision rule $b' = \mathcal{B}(b, j, S)$. These solve the household problem in the usual sense.

   (b) Price function: $p(S)$.

(c) Consistency:

$$\mu_j \Gamma'_j\left(\bar{b}\right) = \sum_k \mu_k \pi_{kj} \int \mathbb{I}\left\{\mathcal{B}\left(b, k, S\right) \leq \bar{b}\right\} d\Gamma_k\left(b\right) \tag{1}$$

where, of course, Γ'_j obeys the law of motion $\mathcal{H}_j$.

The LHS is the mass of persons who are in state j tomorrow with bonds below $\bar{b}$. The RHS gives the mass of all persons who transition to state j and who choose $b' \leq \bar{b}$. First, we sum over discount factors today. μ_k persons are in state k today. Fraction $\pi_{k,j}$ transitions to state j tomorrow. The integral is the fraction of those households who choose $b' \leq \bar{b}$.

(d) Market clearing:

i. Goods: $y = C\left(S\right)$ where $C\left(S\right) = \sum_j \mu_j C_j\left(S\right)$ and $C_j\left(S\right) = \int \left[y - \mathcal{B}\left(b, j, S\right)\right] d\Gamma_j\left(b\right)$

ii. Bonds: $0 = \sum_j \mu_j B_j\left(S\right)$ where $B_j\left(S\right) = \int b \times d\Gamma_j\left(b\right)$.

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2 Housing

Demographics: A representative household who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t \mathcal{U}(c_t, h_t)$ where c is non-durable consumption and h is housing consumption.

Technologies:

- Goods: $\mathcal{F}(K_{c,t}, L_{c,t}) = C_t + I_{K,t}$ and $K_{t+1} = (1-\delta) K_t + I_{K,t}$.
- Housing: $\mathcal{G}(K_{h,t}, L_{h,t}) = I_{h,t}$ and $H_{t+1} = (1-\delta_h) H_t + I_{H,t}$. Housing consumption is equal to the housing stock: $h_t = H_t$.
- Both production functions have constant returns to scale.
- $K_t = K_{c,t} + K_{h,t}$.

Endowments:

- At the beginning of time: H_0 and K_0.
- In each period: $L_{c,t} + L_{h,t} = 1$.

Markets:

- There are competitive spot markets for consumption goods (numeraire) and housing investment (price p_t).
- Households own K and rent it to the firms (rental price q).
- There is a competitive labor market with wage rate w.

Questions:

1. [15 points] State the household problem as a Dynamic Program. Be careful when writing out the budget constraint.

   Answer

   Budget constraint: It's best to write this out step-by-step. The household's income is $y = w + qk$. Spending is $I_k + pI_h + c$. Note the p in front of I_h. The household buys I_h from the $\mathcal{G}$ firm and uses it to accumulate h.

   So the budget constraint is
$$w + qk = I_k + pI_h + c \tag{2}$$

   In addition, $h' = (1-\delta_h) h + I_h$ and $k' = (1-\delta) k + I_k$. The easiest setup substitutes all three equations directly into the Bellman equation:

$$\begin{aligned} V(k,h) = \max_{k',h'} & \mathcal{U}(qk + w - I_k - pI_h, h) && (3) \\ & + \beta V((1-\delta) k + I_k, (1-\delta_h) h + I_h) && (4) \end{aligned}$$

2. [10 points] Derive and interpret the Euler equations.

Answer

FOCs:

$$\mathcal{U}_c(c,h) = \beta V_k(k',h') \tag{5}$$
$$= \beta V_h(k',h')/p \tag{6}$$

Envelope:

$$V_k = q_K \mathcal{U}_c + \beta(1-\delta) V_k(.') \tag{7}$$
$$V_h = \mathcal{U}_h + \beta(1-\delta_h) V_h(.') \tag{8}$$

Euler:

$$\mathcal{U}_c(c,h) = \beta(q_K' + 1 - \delta)\mathcal{U}_c(c',h') \tag{9}$$

For consumption, we get the standard Euler equation. For housing:

$$p\mathcal{U}_c(c,h) = \beta\left[\mathcal{U}_h(c',h') + (1-\delta_h)p'\mathcal{U}_c(c',h')\right] \tag{10}$$

In words: Giving up p units of c today buys one unit of I_h. Tomorrow, that yields consumption $\mathcal{U}_h(c',h')$. In addition, we can reduce I_h' by $(1-\delta_h)$ to keep h'' unchanged. That frees up $(1-\delta_h)p'$ units of income for consumption.

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3. [15 points] Define a competitive equilibrium in sequence language.

Answer

The firms solve standard problems with first-order conditions:

- $\mathcal{F}_k = q_k$ and $\mathcal{F}_l = w$
- $p\mathcal{G}_k = q_k$ and $p\mathcal{G}_l = w$

Objects:

(a) Household: c, I_k, I_h, k, h

(b) Firms: K_j, L_j for $j \in \{k,h\}$

(c) Prices: q, w, p

Equations:

(a) Household: 2 Euler equations, 3 constraints, TVC, initial conditions.

(b) Firms: 4 FOCs.

(c) Market clearing: goods (2, given), labor (given)

Identities: $K_k + K_h = k$.

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4. [15 points] Now suppose that the functional forms for $\mathcal{F}$ and $\mathcal{G}$ are the same, except for labor augmenting productivities. That is, $\mathcal{G}(K, L) = \mathcal{F}(K, AL)$. Rising A means that producing goods becomes relatively more efficient compared with producing housing (something that has happened in the data).
Also assume that $\mathcal{U}(c, h) = \ln(c) + \ln(h)$.
Compare two steady states that differ in A. What can you say about the values of the interest rate, p, c/h, and the expenditure ratio ph/c?

Answer

Interest rate: pinned down by the first Euler equation at $\beta(1 + q - \delta) = 1$.

Price: With constant returns to scale, $K_k/L_k = K_h/L_h$ and therefore $p = A$.

Consumption ratio: The second Euler equation implies that $\mathcal{U}_h/\mathcal{U}_c \propto p$. Hence, $c/h \propto p$. Expenditure shares are fixed.

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3 Investment taxation and labor supply (35 points)

Consider a representative-agent economy in which the representative household lives for two periods $(t = 0, 1)$, consumes in both periods, but produces only in period 0. The household's utility is

$$U(n, c_0, c_1) = \frac{c_0^{1-\sigma}}{1-\sigma} - \gamma\frac{n^{1+\phi}}{1+\phi} + \beta\frac{c_1^{1-\sigma}}{1-\sigma},$$

where n is labor hours in period 0, c_0 is consumption in period 0, and c_1 is consumption in period 1. We assume that the parameters of the utility function satisfy

$$\sigma \geq 0, \sigma \neq 1, \gamma > 0, \phi > 0, 0 < \beta < 1.$$

The household has a linear production technology that converts one unit of labor into one unit of the consumption good in period 0. The household also has a storage technology that converts one unit of the consumption good in period 0 into $A > 0$ units of the consumption good in period 1. In other words, if the household works n hours and stores k units in period 0, then period-0 output is n and period-1 output is Ak.

The government taxes the investment income (in other words, the output from storage) in period 1 at a proportional rate τ. Below, we will consider two different scenarios. In the first, all the tax revenue is thrown away (spent on some useless government expenditures). In the second, all the tax revenue is rebated back as a period-1 lump-sum transfer. In both cases, the household knows the period-1 policy in period 0. There are no taxes in period 0.

1. (20 points) Suppose that all the tax revenue is thrown away (spent on some useless government expenditures). How does this investment tax affect labor supply? Are there any parameter combinations $\sigma, \gamma, \phi, \beta, A$ (subject to the above assumptions) such that an *increase* in the tax rate would *increase* the equilibrium labor hours? Is it possible that an increase in the tax rate would increase *both* labor hours and period-0 consumption? Prove your answer and briefly explain the intuition.

2. (15 points) Now, suppose that, instead of being thrown away, all the tax revenue is rebated back as a lump-sum transfer to the household in period 1. Re-do the previous question. Explain how the answer is different and why.

4 Assets as a medium of exchange (55 points)

An economy has a measure 1 of infinitely-lived agents, all with identical preferences $\sum_{t=0}^{\infty} \beta^t u(c_t)$, over consumption c_t, where $\beta \in (0,1)$ and utility satisfies $u'(c) > 0$, $u''(c) < 0$. There are *two* types of agents, who differ in their endowments of the consumption good. One-half of the agents are Odds, and the other one-half are Evens. Odds have low endowments in even periods and high endowments in odd periods,

$$y_t^O = \begin{cases} y^\ell, & \text{if } t \text{ is even } (t = 0, 2, 4, ...), \\ y^h, & \text{if } t \text{ is odd } (t = 1, 3, 5, ...) \end{cases}$$

where $y^h > y^\ell > 0$. Evens have high endowments in even periods and low endowments in odd periods,

$$y_t^E = \begin{cases} y^h, & \text{if } t \text{ is even } (t = 0, 2, 4, ...), \\ y^\ell, & \text{if } t \text{ is odd } (t = 1, 3, 5, ...) \end{cases}$$

In addition, there is an asset called land. Land is in fixed supply equal to 1, and each unit of land provides $z \geq 0$ units of the consumption good (yes, the same consumption good) every period. Land does not depreciate. The consumption good itself **cannot** be stored over time. These assumptions imply that the total amount of the good available in each period is $0.5y^h + 0.5y^\ell + z$.

In period 0, the Odds own all the land. In other words, the agents with the low initial endowment are the ones who initially own all the land.

We will focus in all the questions on an equilibrium in which the price of a unit of land is *constant* over time. Call this price p.

1. (15 points) Suppose that the agents can lend and borrow among themselves, subject only to a natural borrowing limit (you *do not* need to say what is the natural borrowing limit). They can also buy and sell land, subject to the usual constraint that land holdings must be non-negative. Derive the equilibrium price of land.

Now, suppose that agents cannot lend or borrow at all. They can still buy and sell land, subject to the usual constraint that land holdings must be non-negative.

2. (15 points) Give conditions on parameters (y^h, y^ℓ, z, β) such that the equilibrium price of land is *the same* as in part 1.
3. (20 points) Suppose that the condition you derived in part 2 is not satisfied. Derive an equation that relates the equilibrium price of land p to the economy's parameters (y^h, y^ℓ, z, β). You do not need to explicitly solve for p, but your equation should not contain endogenous variables other than p. Is p higher or lower than in part 1? Also, is it possible to have $p > 0$ if $z = 0$?
4. (5 points) Discuss what is going on in this problem.