Macroeconomics Qualifying Examination

August 2024

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 The Cake Eating Problem

Consider the following cake eating problem.

- At time zero, the agent is endowed with a cake of size $x(0) = 1$.
- The agent maximizes
$$\max \int_0^\infty e^{-\rho t} \ln(y(t))\, dt \tag{1}$$
by choosing a path of consumption $y(t)$.
- Each slice of cake eaten reduces the size of the pie: $\dot{x}(t) = -y(t)$.
- The agent cannot eat more than the entire cake: $x(t) \geq 0 \ \forall t$.

Questions:

1. [15 points] Derive the first order condition for $y(t)$ and the law of motion for the co-state. Provide intuition.

Answer

The current value Hamiltonian is $\mathcal{H} = u(y) - \mu y$ with first-order condition $u'(y) = \mu$ and $g(\mu) = \rho$.

Intuition:

- The costate μ is the marginal value of more cake. Since cake can be eaten (one-for-one), that marginal value must equal the marginal utility of consumption.
- The law of motion says that marginal utility grows at the discount rate. In general, there is a trade-off between earning interest (postpone consumption) and discounting (consume early). Here, the interest rate is zero and discounting induces the agent to consume early.

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2. [20 points] Solve for the time path of consumption. Hint: Recall that $\int_0^\infty e^{-\alpha t} dt = 1/\alpha$.

Answer

The first order condition implies that μ grows at the constant rate ρ:

$$\mu(t) = u'(y(t)) = \mu(0)\, e^{\rho t} \tag{2}$$

Hence
$$y(t) = u'^{-1}\left[\mu(0)\, e^{\rho t}\right] \tag{3}$$

With log utility, $u'(y) = 1/y$ so that

$$y(t) = \frac{1}{\mu(0)\, e^{\rho t}} \tag{4}$$

and therefore $y(0) = 1/\mu(0)$.

The TVC implies that the agent eats the entire cake:

$$y(0) \int_0^\infty e^{-\rho t} dt = \frac{y(0)}{\rho} = x(0) \tag{5}$$

Then

$$y(t) = \rho x(0) e^{-\rho t} \tag{6}$$

For completeness (but that's just algebra), we can also compute the path of the remaining stock:

$$x(t) = x(0) - \int_0^t y(\tau) d\tau = x(0) - x(0) \left[1 - e^{-\rho\tau}\right] \tag{7}$$

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3. [10 points] How does your solution change if the time horizon is finite (T) and the agent is paid $p \times x(T)$ utils for whatever cake is left over at the end?

Answer

Only the transversality condition changes to $\mu_T = u'(y(T)) = e^{-\rho T} p$. Then

$$y(t) = e^{-\rho t} e^{\rho T} / p \tag{8}$$

This implies $y(T) = 1/p$ or $u'(y(T)) = p$. That makes sense. At the end of time, the agent should be indifferent between eating a bit more and "selling" the remaining cake.

Relative to the previous problem, the entire path of $y(t)$ simply shifts down.

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2 Money with Heterogeneity

Consider the following endowment economy in discrete time.

Demographics: There is a unit mass of infinitely lived households.

Preferences: $\mathbb{E} \sum_{t=1}^{\infty} \beta^t \mathcal{U}(c_t)$ where $\mathcal{U}$ is well behaved.

Endowments:

- At the beginning, households are endowed with heterogenous money holdings that add up to M (in nominal terms).
- Each period, every household draws $y \in \{y_L, y_H\}$ with i.i.d. probabilities π_L or $\pi_H = 1 - \pi_L$.

Technology: Endowments can only be eaten.

Government: The government taxes money holdings at the start of the period at rate τ. It returns the revenues via lump-sum transfers. Each household gets the same nominal transfer $P_t q_t$. Hence, the stock of money is constant over time.

Markets: There are spot markets for money (numeraire) and consumption (price P_t). There is no borrowing.

Consider the stationary recursive competitive equilibrium.

Questions:

1. [20 points] State the household's dynamic program.

   Answer

   Individual state: $s = (y, m)$ where m denotes real money holdings (before tax).

   Since we are looking for a stationary equilibrium, we don't have to worry about aggregate states yet. But we could keep track of it here. It is the joint distribution of the individual states with law of motion $D' = \mathcal{H}(D)$.

   This is a standard consumption-saving problem with one asset and a borrowing constraint. The asset return is constant (stationary equilibrium) at R. Decision rules are $\mathcal{C}(s)$ for consumption and $\mathcal{M}(s)$ for real money holdings.

   Budget constraint: $Pc + m'P' = Py + (1-\tau)Pm + Pq$ or

   $$c + m'(1+\pi') = y + (1-\tau)m + q \tag{9}$$

   where q is the lump-sum transfer (real).

   Bellman equation:

   $$V(y, m) = \max_{m'} \mathcal{U}(y + [1-\tau]m - m'(1+\pi')) + \beta \mathbb{E} V(y', m') \tag{10}$$

   subject to $m' \geq 0$.

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2. [10 points] Derive the household's Euler equation.

   Answer

   Interior FOC:

   $$\mathcal{U}'(c)(1+\pi') = \beta \mathbb{E} V_m(y', m') = \beta \mathbb{E} \mathcal{U}'(c')(1-\tau) \tag{11}$$

   Standard Euler equation for rate of return $R' = (1-\tau)/(1+\pi')$.

   Taking into account the borrowing constraint: It binds when RHS < LHS.

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3. [20 points] Define a stationary recursive competitive equilibrium. You need not write out the equation for the law of motion of the aggregate state, but you should state it in words.

   Answer

Define aggregation function $\mathcal{A}(\mathcal{F}) \equiv \int_s \mathcal{F}(s) D(s) ds$ where D is the density over states.

Government budget constraint: $q = \tau \mathcal{A}(\bar{\mathcal{M}})$ where $\bar{\mathcal{M}}(s) = m$. I.e., $\mathcal{A}(\bar{\mathcal{M}})$ is today's real money stock.

RCE:

- Aggregate state: just D with law of motion $\mathcal{H}$.
- Household: V and decision rules. They solve the Bellman equation in the usual sense.
- Government: q that satisfies the budget constraint
- Price functions: $R = \mathcal{R}(D)$. But with stationarity, $\pi = 0$ (because the nominal money stock is constant over time) so that $R = 1 - \tau$.
- Market clearing:
  - Resource constraint for goods: $Y = \mathcal{A}(\mathcal{C})$ where $Y = \pi_L y_L + \pi_H y_H$
  - Money: really none; the price level adjusts to whatever households demand.
- Stationarity: $D = \mathcal{H}(D)$ with a consistency condition that says in words: If D implies that the mass of agents in some set of states is x, then the decision rules imply that the same mass is in that state next period.

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3 Business Cycles

This problem is about a real business cycle model in which people care about the consumption of others; for example, because of preferences for social status. The economy has a large number of identical infinitely-lived households. Each household has preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t - \gamma C_t) - \frac{n_t^{1+\theta}}{1+\theta} \right], \qquad 0 < \beta < 1, \quad \theta > 0, \quad 0 < \gamma < 1,$$

where c_t is that individual household's consumption in period t, n_t is that household's labor in period t, and C_t is **aggregate** consumption in period t.

Output is produced using capital K_t and labor N_t according to the production function

$$Y_t = z_t K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where z_t is an aggregate productivity shock that follows a Markov process. Output each period is split between consumption and investment in new capital. Existing capital depreciates at rate δ.

Households own the capital and rent it out to firms in a competitive market, as well as supplying labor in a competitive market.

1. (15 points) Write down the household's recursive problem in a recursive competitive equilibrium (you do *not* need to write the entire definition of a recursive competitive equilibrium). What are the aggregate and individual states? Is aggregate consumption an aggregate state in the household's problem? Why or why not?

2. (15 points) Derive the equilibrium conditions, and explain which ones differ from an economy with $\gamma = 0$.

3. (25 points) Consider the steady state of this economy when z_t is fixed at $\bar{z} = 1$. How does the economy's steady-state *capital-labor ratio* K/N depend on γ? How do the steady-state *levels* of N and K depend on γ? Prove your answers and explain.

Now, let's examine the efficiency properties of this economy. (This part refers to the stochastic economy, not the steady state.)

4. (10 points) Write down the social planner's problem in recursive form.

5. (20 points) Prove (by comparing the appropriate optimality conditions) that the competitive equilibrium is inefficient. Provide some intuition for the inefficiency.

End of exam.