Macroeconomics Qualifying Examination

June 2024

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 OLG Planning Problem

Consider an OLG production economy with standard assumptions, except that households live for T periods.

Demographics: In period t, $N_t = (1+n)^t$ identical households are born. Each lives for T periods.

Endowments: Each household provides ℓ_a units of labor at age a. At the beginning of time, households are endowed with k_a units of capital that sum to K_1.

Preferences: Households value consumption according to $\sum_{a=1}^{T} \beta^a \mathcal{U}(c_{\tau,a})$ where $c_{\tau,a}$ denotes consumption of persons born in period τ when aged a.

Technology: Goods are produced from capital and labor according to

$$K_{t+1} + C_t = F(K_t, L_t) + (1-\delta) K_t \tag{1}$$

where K is aggregate capital and L is aggregate labor input. C is aggregate consumption.

Questions:

1. [20 points] State the social planner's problem. Let $\omega_\tau N_\tau$ denote the weight that the planner assigns to cohort τ.

   Answer

   Let $V_\tau \equiv \sum_{a=1}^{T} \beta^a \mathcal{U}(c_{\tau,a})$. The the planner chooses $\{K_t, c_{\tau,a}\}$ to maximize $\sum_{\tau=1}^{\infty} \omega_\tau N_\tau V_\tau$ subject to resource constraints. Here, it is understood that the planner can only choose consumption from $t=1$ onwards (i.e., the planner cannot alter young consumption for those already born at $t=1$). Alternatively, we could use the weights ω_τ, but that's just different notation.

   Let's define an aggregation function $\mathcal{A}(x,t) \equiv \sum_{a=1}^{T} x_{\tau(a,t),a} N_{\tau(a,t)}$ where $\tau(a,t) = t-a+1$ is the birth year that goes with age a and period t. The resource constraints are then given by (1), where $C_t = \mathcal{A}(c,t)$ and $L_t = \mathcal{A}(\ell)$.

2. [20 points] Derive the planner's first order conditions. For now, do not substitute out the Lagrange multipliers. Explain these equations in words.

   Answer

   FOCs are

$$\omega_\tau N_\tau \beta^a \mathcal{U}'(c_{\tau,a}) = \lambda_{t(\tau,a)} N_\tau \tag{2}$$

   and

$$\lambda_t = \lambda_{t+1} \left[F_K(K_{t+1}, L) + 1 - \delta \right] \tag{3}$$

   In words:

- λ_t is the value of a unit of the good at date t. It can be eaten by any cohort τ at its marginal utility. All cohorts should have the same marginal utility.
- The second first-order condition is a standard Euler equation.

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3. [15 points] Substitute out the Lagrange multipliers and derive an Euler equation and static optimality condition. Explain the static condition in words.

Answer

Euler:

$$\omega_t \mathcal{U}'(c_{t,1}) = \omega_{t+1}\mathcal{U}'(c_{t+1,1})\left[F_K(K_{t+1}, L) + 1 - \delta\right] \tag{4}$$

The same would hold for any age a.

Static condition:

$$\lambda_t = \omega_{\tau(t,a)}\beta^a \mathcal{U}'\left(c_{\tau(t,a),a}\right) \tag{5}$$

for all ages a that are alive at date t. Here, $\tau(t,a)$ is the birth date that goes with year t and age a.

In words: The planner hands out consumption such that the marginal utility of a cohort is always proportional to the planner's weight ω. Perhaps easier to interpret: The ratio of marginal utilities of two cohorts is always the same (equal to the ratio of their ωs).

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4. [5 points] With log utility, solve for the relative consumption of two cohorts that live at the same time t. How does that consumption ratio depend on age? Explain the intuition.

Answer

Consider cohorts τ and $\hat{\tau}$ with age gap $a - \hat{a} = \hat{\tau} - \tau$. The consumption ratio is given by

$$\frac{c_{\tau,a}}{c_{\hat{\tau},\hat{a}}} = \frac{\omega_\tau}{\omega_{\hat{\tau}}}\beta^{\hat{\tau}-\tau} \tag{6}$$

is independent of age. The planner wants to keep MRS = MRT = 1.

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2 Productive government spending and business cycles

Consider a real business cycle model in which the representative household has preferences over consumption C_t and labor N_t given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) + v(1 - N_t)], \quad 0 < \beta < 1$$

where u and v are twice continuously differentiable, strictly increasing, and strictly concave. The production function is

$$Y_t = G_t^{\phi} K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1, \quad 0 \leq \phi < 1 - \alpha$$

where Y_t is output, K_t is capital, and N_t is labor. G_t is a government spending shock that follows an exogenous Markov process. Capital K_t evolves according to the law of motion

$$K_{t+1} = (1 - \delta) K_t + I_t, \quad \delta \in (0, 1]$$

The economy-wide resource constraint is

$$C_t + I_t + G_t = Y_t$$

Questions:

1. (10 points) Consider the problem of a social planner who takes the production function, the process for G_t, and resource constraint as given, and seeks to maximize the expected utility of the representative household. Write this problem recursively. Clearly state what the state variables and choice variables are.

   Answer ______________________________

   The states are G and K. The choice variables are C, N, and next period's capital K'. The social planner's value function is

   $$V(K, G) = \max_{C, K', N} u(C) + v(1 - N) + \beta \mathbb{E}_{G'|G} V(K', G') \tag{7}$$

   subject to the resource constraint

   $$C + K' + G = G^{\phi} K^{\alpha} N^{1-\alpha} + (1 - \delta) K \tag{8}$$

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2. (20 points) Derive the first-order conditions and envelope conditions for the planner's problem and use them to derive the Euler equation and the intra-temporal optimality condition.

   Answer ______________________________

The first-order conditions to the above maximization problem are

$$u'(C) = \lambda \tag{9}$$

$$v'(1-N) = \lambda(1-\alpha)G^{\phi}K^{\alpha}N^{-\alpha} \tag{10}$$

and

$$\lambda = \beta\mathbb{E}_{G'|G}V_K(K', G') \tag{11}$$

The envelope condition is

$$V_K(K, G) = (1 - \delta + \alpha G^{\phi}K^{\alpha-1}N^{1-\alpha})\lambda \tag{12}$$

Combining (9) and (10) yields the intra-temporal optimality condition

$$v'(1-N_t) = (1-\alpha)G_t^{\phi}K_t^{\alpha}N_t^{-\alpha}u'(C_t) \tag{13}$$

Combining (9) and (11) with (12) gives us the Euler equation

$$u'(C_t) = \beta\mathbb{E}_t[1 - \delta + \alpha G_{t+1}^{\phi}K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}]u'(C_{t+1}) \tag{14}$$

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3. (20 points) Suppose that $\phi = 0$. Is it possible that a positive shock to G_t, all else equal, would increase period-t consumption and investment? Prove your answer formally. Briefly explain how the answer may be different if $\phi > 0$.

Answer

If $\phi = 0$, this is impossible. This can be seen by inspecting (13) together with the resource constraint

$$C_t + K_{t+1} + G_t = (1-\delta)K_t + G_t^{\phi}K_t^{\alpha}N_t^{1-\alpha} \tag{15}$$

If $\phi = 0$, (15) implies that a simultaneous increase in C_t and K_{t+1} requires an increase in N_t. However, (13) implies that a shock to G_t (which does not directly enter (13) when $\phi = 0$) moves C_t and N_t in opposite directions. This is a contradiction.

If $\phi > 0$, an increase in G_t might theoretically increase C_t and K_{t+1} simultaneously, because (from (15)) an increase in G_t now raises output for a given N_t, and also because an increase in G_t now raises the marginal product of labor, which may (from (13)) be consistent with a simultaneous increase in C_t and N_t.

For the remainder of this problem, consider the competitive equilibrium of this economy. Households own the capital and rent it out to firms. The government finances its expenditure process G_t using some combination of lump-sum taxes and proportional taxes on labor income. The government does not issue debt.

4. (5 points) Write down the government budget constraint in a recursive competitive equilibrium. If any objects are functions of the aggregate state, you should make this explicit.

Answer

$$G = T(K, G) + \tau(K, G)w(K, G)N(K, G)$$

where T denotes the lump-sum tax, τ denotes the proportional tax rate on labor income, and w is the equilibrium wage.

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5. (15 points) If the only taxes are proportional taxes on labor income, does the competitive equilibrium employment level coincide with the one chosen by the social planner? Prove your answer and provide some economic intuition.

Answer

Equilibrium yields the following condition, where τ_t is the tax rate in period t (verify this):

$$v'(1 - N_t) = (1 - \tau_t)w_t u'(C_t), \tag{16}$$

which, after substituting for the wage, gives

$$v'(1 - N_t) = (1 - \tau_t)(1 - \alpha)G_t^{\phi} K_t^{\alpha} N_t^{-\alpha} u'(C_t) \tag{17}$$

It is easy to verify that (17) is inconsistent with (13) since we must have $\tau_t > 0$; so the equilibrium allocation is necessarily inefficient. The proportional tax on labor acts as a wedge that disincentivizes working.

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3 Wealth redistribution and labor supply

An economy consists of a continuum of households with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln \left(c_t - \frac{1}{2} n_t^2 \right)$$

where c_t denotes consumption and n_t denotes labor effort.

Households receive **idiosyncratic** shocks to their productive ability y each period, which are drawn i.i.d. from a distribution $\pi(y)$ with finite support. Output is produced using labor with a linear technology: if a household with productive ability y supplies n labor hours, its labor income is simply yn.

The only asset traded in this economy is a non-contingent bond in zero net supply, which agents can trade subject to some exogenous borrowing limit $\bar{A}$.

We will consider a recursive competitive equilibrium (not necessarily a stationary one).

Questions:

1. (10 points) Write down the household's problem in recursive form.

   Answer

$$V(a, y) = \max_{c,n,a'} \ln \left(c - \frac{1}{2} n^2 \right) + \beta \sum_{y'} \pi(y') V(a', y') \tag{18}$$

   subject to

$$c + a' = (1 + r)a + yn \tag{19}$$

   and

$$a' \geq -\bar{A} \tag{20}$$

2. (20 points) Derive a household's intra-temporal and inter-temporal optimality conditions. Briefly discuss whether the the inter-temporal optimality condition holds as an equality or a strict inequality.

   Answer

   Let λ and μ be the multipliers on the budget constraint (19) and the borrowing constraint (20), respectively. The first-order and envelope conditions are

$$\frac{1}{c - \frac{1}{2} n^2} = \lambda \tag{21}$$

$$\frac{1}{c - \frac{1}{2}n^2} n = \lambda y \tag{22}$$

$$\lambda = \mu + \beta \mathbb{E}_{y'} V_a(a', y') \tag{23}$$

and

$$V_a(a, y) = \lambda(1 + r) \tag{24}$$

Combining (21) with (22) gives the intra-temporal condition

$$n = y, \tag{25}$$

which implies that a household's labor supply is independent of its bond holdings. This is due to the functional form of the utility function (GHH preferences). Combining (21), (23), and (24) gives

$$\frac{1}{c - \frac{1}{2}n^2} \geq \beta(1 + r)\mathbb{E}_{y'} \frac{1}{c' - \frac{1}{2}(n')^2}, \tag{26}$$

with equality if the borrowing constraint does not bind, and strict inequality if it does.

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3. (5 points) Write down the expression for aggregate labor supply.

Answer

Above, we found that $n = y$. This means (since y is i.i.d.) that the aggregate efficiency units of labor are equal to

$$\sum_y \pi(y) y n = \sum_y \pi(y) y^2 \tag{27}$$

We are able to write the aggregate labor independently of the distribution of wealth in this case.

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4. Imagine that the economy is in a stationary equilibrium. The government implements a one-time redistribution of wealth in which each household receives a transfer $T(a)$ which is a function of the household's bond holdings a (the transfer may be positive or negative).

   (a) (5 points) Write down the government's budget constraint that $T(a)$ must satisfy, since the total amount of transfers must be zero.

   Answer

   $$\int_{A \times Y} T(a) \Phi(da, dy) = 0,$$

   where A is the asset space, Y is the set of possible y, and Φ is the stationary joint distribution of assets and productivity.

(b) (10 points) How would this redistribution of wealth affect aggregate labor supply? Prove your answer. Briefly explain.

Answer

As we explained above, an individual's labor is independent of their assets, and so aggregate labor is independent of the asset distribution. This is due to the functional form of the utility function (GHH preferences). Redistribution of wealth does not affect aggregate labor supply, which is still $\sum_y \pi(y)y^2$.

End of exam.