Macroeconomics Qualifying Examination

August 2023

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 OLG with Money

Demographics: A unit mass of households is born in each period. Each lives for two periods.

Endowments: e_y when young and e_o when old. Endowments are in units of a single consumption good.

Preferences: $u(c_t) + \beta u(z_{t+1})$ where c is young consumption and z is old consumption. Assume log utility.

Technology: Endowments can only be eaten. This is (for now) a closed economy, so that $e_y + e_o = c_t + z_t$.

Markets: There a competitive markets for goods (numeraire) and one period bonds that are in zero net supply (interest rate $R = 1 + r$).

Questions

1. [20 points] Go as far as you can towards solving for the equilibrium allocation and interest rate. Hint: You should find (among other things) that

$$c = \frac{e_y + e_o/R'}{1+\beta} \tag{1}$$

Provide intuition for how the equilibrium interest rate varies with the values of endowments and β.

Answer

Euler equation: $u'(c) = \beta R' u'(z')$ which implies in this case $z'/c = \beta R'$.[1]

Budget constraint: $e_y + e_o/R' = c + z'/R'$. Present value of consumption equals present value of income.

Closed form solution for young consumption follows from $c + z' = c(1+\beta)$ and thus $e_y + e_o/R' = c(1+\beta)$ which gives (1). Old consumption is then given by

$$z' = \beta R' c = \frac{\beta}{1+\beta}\left(R' e_y + e_o\right) \tag{2}$$

Bond market clearing: $b = 0$ or $c = e_y$. Then from the decision rule $\beta e_y = e_o/R'$ or

$$R' = e_o / \left(\beta e_y\right) \tag{3}$$

An equilibrium then consists of (c, z, R) that solve the two decision rules and (3).

[1] Base on a qualifying exam question at UT Austin, June 2019.

Intuition: When $e_y = e_o$, we have constant consumption and therefore $\beta R = 1$. The incentives to postpone (R) and pull forward (β) consumption balance each other. Higher e_y/e_o increases young saving. To maintain zero saving, R must fall.

From here on assume that $\beta = 1$ and $e_y = e_o = e$. This implies (unsurprisingly) that $R = 1$ in equilibrium.

2. [15 points] Now assume that households can borrow and lend from abroad at a fixed interest rate R. Characterize the stationary equilibrium. Hint: only one equation changes relative to the closed economy. Which one?

Answer

Decision rules and budget constraints are unchanged. Only goods market clearing changes (and bond market clearing, but that's redundant by Walras' law). One way of writing this:

$$e_y + e_o = c_t + z_t + x_t \tag{4}$$

where x_t is foreign consumption. Foreigners consume $x_t = R_t b_t - b_{t+1}$ or, in stationary equilibrium, $x = rb$.

A second way of deriving this: For the young, $e_y + b' = c$. For the old, $z' = e_o - R'b'$. Assuming a stationary equilibrium (presumably obvious) and adding up those two budget constraints gives $e_y + e_o + b = c_t + z_t + Rb$ or $e_y + e_o = c_t + z_t + rb$. If the young borrow ($b > 0$), then the rest of the world is paid rb in each period.

From the young budget constraint, we have $b = c - e$, so that

$$2e = c + z + r(c - e) \tag{5}$$

$$(1 + R)e = c + Rc + rc \tag{6}$$

$$c = e\frac{1 + R}{2R} \tag{7}$$

where $2R = 1 + R + r$. Then

$$b = c - e \tag{8}$$

$$= -e\frac{r}{2R} \tag{9}$$

When $r = 0$ this (obviously) coincides with the closed economy ($b = 0$). When $r > 0$ we have $b < 0$ and households save (which makes sense).

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3. [10 points] Now assume that there are two countries that trade with each other. They are identical, except for the endowments. The home country has high endowments when young,

$e_{y,h} = e(1+\epsilon)$, and low endowments when old, $e_{o,h} = e(1-\epsilon)$. In the foreign country, it's the other way around: $e_{y,f} = e(1-\epsilon)$ and $e_{o,f} = e(1+\epsilon)$.

Define a stationary world equilibrium. Guess and verify the market clearing interest rate.

Answer

Decision rules and budget constraints are still the same. Goods market clearing changes to

$$4e = c_h + z_h + c_f + z_f \tag{10}$$

Bond market clearing becomes $b'_h - b'_f = c_h - e_{h,y} + c_f - e_{f,y} = 0$ or $c_h + c_f = 2e$. In words: since there is no intergenerational trade, total consumption of the young must equal their endowments.

Given the symmetry, guess $R = 1$. With the current assumptions, we have

$$c_h = \frac{e_{y,h}}{2}(1 + 1/R) \tag{11}$$

and the same for f. Since $e_{y,h} + e_{y,f} = 2e$, we have $c_h + c_f = e(1+1/R)$ and $z_h + z_f = e(1+R)$. Goods and bond markets clear when $R = 1$.

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4. [15 points] Next, we add fiat money in the home country only. The initial old are endowed with M units of money. The stock of money is constant over time. Define a *stationary* equilibrium with valued fiat money. Does it exist? Explain the intuition. Hint: There is no need to resolve the household problem.

Answer

Decision rules and budget constraints are unchanged (because money and bonds need to pay the same rate of return). Goods market clearing is also unchanged. The only change is to bond market clearing. Since $b'_h = e_{y,h} - c_h - m'$ and $b'_h + b'_f = 0$, we have $c_h + m - e_{h,y} + c_f - e_{f,y} = 0$ or

$$c_h + c_f + m = 2e \tag{12}$$

From the decision rules: $c_h + c_f = e(1 + 1/R)$ (unchanged), so that

$$m = 2e - e(1 + 1/R) \tag{13}$$

It follows that $m > 0 \iff R > 1$. But the return on m in a stationary equilibrium is zero (constant nominal and real m). No stationary equilibrium with valued fiat money exists.

Intuition: $R = 1$ is consistent with no saving. Then fiat money is not valued. $R > 1$ is needed for households to hold assets. But with constant m, the return on money is zero.

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2 State-dependent taxation

Redistributive government transfers are often raised in recessions, particularly in response to low employment. The following model will be used to study the effects of such a policy. Consider a real business cycle model, in which the representative household has preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln\left(c_t - \frac{n_t^{1+\theta}}{1+\theta}\right), \quad \theta > 0,$$

where c_t is the household's consumption and n_t the household's labor. Output is produced according to

$$Y_t = z_t K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is capital, N_t is labor, and z_t is an aggregate productivity shock that follows a Markov process. Capital accumulates through investment and depreciates at rate δ. Households own the capital stock and rent it out to firms in a competitive market. They also supply labor in a competitive market.

The government taxes labor income at a proportional rate τ_t and redistributes the tax revenue back in the same period, in the form of a lump-sum transfer. The tax rate τ_t follows the rule

$$\tau_t = 1 - \phi N_{t-1}^{\sigma}, \quad \phi > 0, \quad \sigma > 0,$$

and N_{t-1} is last period's **aggregate** employment level.

1. (15 points) Write down the household's problem in a recursive competitive equilibrium. Clearly state what the individual and aggregate state variables are. Does the behavior of taxes introduce any additional state variables, relative to the usual real business cycle model?

2. (5 points) Write down the government budget constraint in a recursive competitive equilibrium.

3. (20 points) Derive the condition that expresses equilibrium labor hours as a function of the aggregate state variables.

4. (5 points) Using the above condition, derive the elasticity $\partial \ln N_t / \partial \ln z_t$ (the elasticity of equilibrium labor hours with respect to the same period's z_t, keeping the values of other aggregate state variables fixed). Using the expression for this elasticity, explain how the economy's amplification mechanism depends on θ. Provide some intuition.

5. (5 points) Suppose (for this question only) that part of the tax revenues were used on a wasteful government activity, instead of being transferred back lump-sum. How would this change affect equilibrium hours worked? How does the answer depend on the assumptions of this problem?

6. (10 points) Suppose that the following are empirical facts:

(a) Labor hours are more persistent than average output per hour.

(b) Labor hours are positively correlated with average output per hour, but only weakly so (the correlation is much smaller than one).

Using the condition derived in part 3, explain how state-dependent taxation affects the model's ability to match the data on the above two dimensions. (5 points each).

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3 Government debt enters utility

Demographics: An infinitely lived representative household.

Preferences: $\mathbb{E}\sum_{t=0}^{\infty}\beta^t\mathcal{U}\left(c_t\times\mathcal{V}\left(b_t/c_t\right)\right)$ where c is consumption and b denotes bond holdings. The utility functions $\mathcal{U}$ and $\mathcal{V}$ are both strictly increasing and strictly concave.

Endowments: In each period, an aggregate shock s is realized. The household receives endowment $y\left(s\right)$. The shocks is drawn from a Markov chain.

Technology: Endowments can only be eaten: $c=y$.

Markets: There are competitive markets for

- goods (numeraire)
- risk-free, one period government bonds b' (price p)
- state contingent claims $z'\left(s'\right)$ with prices $q\left(s'|s\right)$.

Government: The government imposes lump-sum taxes $\tau\left(s\right)$. The budget constraint is $B=\tau+pB'$.

Questions:

1. [15 points] Set up the household's dynamic program.

   Answer

   The aggregate state is simply s.

   Hh Budget constraint: $y\left(s\right)+b+z\left(s\right)-\tau=c+pb'+\sum_{s'}q\left(s'|s\right)z'\left(s'\right)$.

   Household state is $b,z\left(s\right),s$. z is holding of Arrow securities.

   Bellman equation $V\left(b,z,s\right)=\max_{c,b',z'(s')}\mathcal{U}\left(c\times\mathcal{V}\left(b/c\right)\right)+\beta\mathbb{E}V\left(b',z',s'\right)+\lambda\times BC$

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2. [15 points] Derive the Lucas asset pricing / Euler equations

$$\frac{d\mathcal{U}}{dc}=\frac{1}{p}\beta\mathbb{E}\left[\frac{d\mathcal{U}\left(.'\right)}{db}+\frac{d\mathcal{U}\left(.'\right)}{dc}\right] \tag{14}$$

$$=\frac{1}{q\left(s'\right)}\beta\Pr\left(s'|s\right)\frac{d\mathcal{U}\left(.'\right)}{dc} \tag{15}$$

   Answer

   Notation: $\frac{d\mathcal{U}}{dc}=\mathcal{U}'\left(c\times\mathcal{V}\left(b/c\right)\right)\times\left[\mathcal{V}\left(b/c\right)-\mathcal{V}'\left(b/c\right)\left(b/c\right)\right]$ and $\frac{d\mathcal{U}}{db}=\mathcal{U}'\times\mathcal{V}'$.

   FOCs:

- c: $\frac{d\mathcal{U}}{dc} = \lambda$ where λ is the Lagrange multiplier for the budget constraint.
- b': $\beta \mathbb{E} V_b(.') = \lambda p$
- $z(s')$: $\beta \mathbb{E} V_{z(s')}(.') = \lambda q(s')$

Envelope

- $V_b = \frac{d\mathcal{U}}{db} + \lambda$
- $V_{z(s')} = \lambda \times \mathbb{I}_{s=s'}$

Combine to derive the Lucas asset pricing equations.

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3. [5 points] Explain the Lucas asset pricing equations in words. How does the preference for bond holdings affect the average risk-free rate, holding everything else equal?

Answer

- Give up p units of consumption to buy a bond. Tomorrow this yields the direct utility from holding the bond plus one unit of consumption.
- Or give up $1/q(s')$ units of consumption to buy a state contingent claim. Then get one unit of consumption in state s' tomorrow.
- Preference for bonds lower the risk free rate (since $\frac{d\mathcal{U}}{db} > 0$).

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4. [25 points] Consider the same environment, but with individual level shocks instead of aggregate shocks. Specifically, assume that there are N types of agents, indexed by j. Each type draws an endowment y_j that follows a Markov chain. Define a recursive competitive equilibrium.

Answer

Let Y denote the vector of all y_j. Let B denote the vector of all b_j. The aggregate state $\mathcal{A}$ then consists of Y, B and the vector Z which collects all $z_j(\mathcal{A})$. There are aggregate laws of motion $Z' = \mathcal{F}(\mathcal{A})$ and $B' = \mathcal{G}(\mathcal{A})$.

Households solve essentially the same problem as before:

$$V(b, z, \mathcal{A}) = \max_{c, b', z'(\mathcal{A}')} \mathcal{U}(c \times \mathcal{V}(b/c)) + \beta \mathbb{E} V(b', z', \mathcal{A}') + \lambda \times BC \tag{16}$$

where Z and B follow their (endogenous, deterministic) laws of motion and Y' is drawn randomly according to the transition matrices for the y_j.

An RCE consists of value function V, decision rules, such as $c = \mathcal{C}(b, z, \mathcal{A})$, and price functions, such as $p = \mathcal{P}(\mathcal{A})$ or $\mathcal{Q}(\mathcal{A}'|\mathcal{A})$. These satisfy:

(a) Household: as usual.

(b) Market clearing: $\sum_j \mathcal{B}\left(b_j\left(\mathcal{A}\right), z_j\left(\mathcal{A}\right), \mathcal{A}\right) = 0$ where b_j' is given by $\mathcal{B}$. Similar for state contingent claims. For goods: $\sum_j \mathcal{C}\left(b_j\left(\mathcal{A}\right), z_j\left(\mathcal{A}\right), \mathcal{A}\right) = \sum_j y_j\left(\mathcal{A}\right)$

(c) Consistency: $\mathcal{B}\left(b_j\left(\mathcal{A}\right), z_j\left(\mathcal{A}\right), \mathcal{A}\right) = b_j\left(\mathcal{A}'\right)$ etc.

End of exam.