Macroeconomics Qualifying Examination

June 2023

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 An OLG Model with Shocks

Demographics: In each period, mass 1 of identical young households are born. Each lives for 2 periods.

Preferences: $\mathcal{U}(c_t^y) + \beta \mathbb{E} \mathcal{U}\left(c_{t+1}^o\right)$. $\mathcal{U}$ is well behaved.

Endowments: w goods when young. The initial old hold M_0 units of fiat money.

Technology: Investing one unit of the good yields r' units tomorrow. r' is stochastic with realizations r_k with probabilities γ_k. The realization of r' is an aggregate shock. The resource constraint is

$$c_t^y + c_t^o + s_{t+1} = s_t r_t + w \tag{1}$$

Government:

- The government issues fiat money M, which pays nominal interest i.
- It imposes lump sum tax τ on the old. This is paid in the form of money (not goods).
- The government budget constraint in nominal terms is given by

$$M_{t+1} = (1+i) M_t - p_t \tau_t \tag{2}$$

- p is the price level.
- i is fixed and τ_t is an exogenous sequence.

Markets: There are competitive markets for goods and money.

Timing:

- At the start of period t, the old hold M_t.
- They receive interest and pay the lump-sum tax.
- They now hold M_{t+1}, which they sell to the young.

Questions:

1. [15 points] Set up the household problem, derive the first-order conditions, and define a solution.

   Answer

   - BC when young: $w_t = c_t^y + s_{t+1} + q_{t+1}$ where q_{t+1} is real saving in money.[1]

[1] Based on a Penn State 2017 qualifying exam.

- BC when old: $c^o = sr + qR - \tau$ where $R \equiv (1+i)/(1+\pi)$ is the real return on money, where $1+\pi_{t+1} \equiv p_{t+1}/p$.
- Std Euler: $\mathcal{U}'(c^y) = \beta \mathbb{E} \mathcal{U}'\left(c^{o'}\right) R' = \beta \mathbb{E} \mathcal{U}'\left(c^{o'}\right) r'$
- Note that the real return on money is generically state dependent. The young cannot choose m_{t+1}.

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2. [20 points] Define a Competitive Equilibrium in sequence language (a list of N objects and N equations for each period). Your definition should only contain real variables (and the inflation rate). You may ignore the fact that old consumption is stochastic and write your equilibrium as if consisted of deterministic sequences.

Answer

CE objects: $c_t^y, c_t^o, s_t, q_t, m_t, \pi_t, R_t$ (7 equations)

CE conditions:

- household: 2 bc, 2 Euler (4)
- government budget constraint (1)

$$\frac{M_{t+1}}{p_{t+1}}(1+\pi_{t+1}) = (1+i)\frac{M_t}{p_t} - \tau_t \tag{3}$$

or

$$m_{t+1}(1+\pi_{t+1}) = (1+i)\, m_t - \tau_t \tag{4}$$

or

$$m_{t+1} = R_{t+1} m_t - \frac{\tau_t}{1+\pi_{t+1}} \tag{5}$$

- goods market: RC (1)
- Money market (1): The young save $p_t q_{t+1}$. They hold M_{t+1}. Therefore,

$$q_{t+1} = \frac{M_{t+1}}{p_t} = m_{t+1}(1+\pi_{t+1}) \tag{6}$$

- Note that $q_{t+1} R_{t+1} = m_{t+1}(1+\pi_{t+1}) \frac{1+i}{1+\pi_{t+1}} = m_{t+1}(1+i)$
- Definition $R_t = (1+i)/(1+\pi)$ (1)

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3. [10 points] Verify that Walras' law holds.

Answer

Goods market clearing: $c_t^y + c_t^o + s_{t+1} = w + s_t r_t$

Using the budget constraints:

$$(w - s_{t+1} - m_{t+1} (1 + \pi_{t+1})) + (s_t r_t + m_t (1 + \pi_t) R_t - \tau_t) + s_{t+1} = w + s_t r_t \tag{7}$$

Cancel terms to find

$$m_{t+1} (1 + \pi_{t+1}) = m_t (1 + i) - \tau_t \tag{8}$$

which is the government budget constraint.

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4. [15 points] Define an Arrow-Debreu (period zero trading) equilibrium for this economy. This should be a list of N equations in N unknowns. The definition requires you to derive the household's first-order conditions. Be sure to write out the household budget constraint appropriate for Arrow-Debreu trading. Let a shock history be $H_t = (r_0, \ldots, r_t)$. For simplicity, assume there is no government.

Answer

The price of the good is $p(H_t)$.

Household budget constraint when born in H_t:

$$(w - c^y(H_t) - s(H_t))\, p(H_t) = \sum_k [c^o(r_k, H_t) - s(H_t)\, r_k]\, p(r_k, H_t) \tag{9}$$

where (r_k, H_t) is the history with realization r_k following H_t.

Household problem: We can substitute the budget constraint into the problem to substitute out consumption (somewhat unusually):

$$\max_s \mathcal{U}(w - s) + \beta \sum_k \gamma_k \mathcal{U}(s r_k) \tag{10}$$

with standard Euler equation.

Market clearing: same as RC above:

$$w + s(H_{t-1})\, r_k = c^y(r_k, H_{t-1}) + c^o(r_k, H_{t-1}) + s(r_k, H_{t-1}) \tag{11}$$

Equilibrium: c^y, c^o, s, p; all functions of H_t; that satisfy

- household: Euler and 2 budget constraints
- goods market clearing

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2 Business cycles

This problem analyzes a real business cycle model in which the economy is hit by two shocks - neutral and investment-specific. The representative household has utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(C_t) + v(1 - N_t)], \quad 0 < \beta < 1,$$

where C_t is consumption, N_t is labor hours, and the functions u and v are both differentiable, strictly increasing, and strictly concave. Output is produced according to

$$Y_t = z_t K_t^{\alpha} N_t^{1-\alpha}, \quad 0 < \alpha < 1$$

Output is split every period between consumption and investment,

$$C_t + I_t = Y_t,$$

and capital accumulates according to

$$K_{t+1} = (1 - \delta) K_t + \psi_t I_t$$

There are **two** aggregate shocks: a shock to production, z_t, and a shock to the capital accumulation process, ψ_t, which follow independent Markov chains.

1. (10 points) Write down the social planner's problem in recursive form.

Answer

$$V(z, \psi, K) = \max_{C,N,K'} u(C) + v(1 - N) + \beta \mathbb{E} V(z', \psi', K') \tag{12}$$

subject to

$$C + \frac{K' - (1 - \delta)K}{\psi} = z K^{\alpha} N^{1-\alpha} \tag{13}$$

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2. (15 points) Derive the social planner's inter-temporal and intra-temporal optimality conditions.

Answer

The first-order and envelope conditions yield

$$\frac{u'(C_t)}{\psi_t} = \beta \mathbb{E}(1 - \delta + \alpha \psi_{t+1} z_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha}) \frac{u'(C_{t+1})}{\psi_{t+1}} \tag{14}$$

and

$$v'(1 - N_t) = (1 - \alpha) z_t K_t^{\alpha} N_t^{-\alpha} u'(C_t) \tag{15}$$

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3. (5 points) Can a shock to ψ_t (all else equal) simultaneously raise employment and output per worker? Prove your answer formally. Explain in words how the result would differ when considering shocks to z_t.

Answer

No. This is trivial from the production function:

$$\frac{Y_t}{N_t} = z_t K_t^{\alpha} N_t^{-\alpha}$$

is decreasing in N_t for given z_t and K_t. So, any shock that leaves z_t and K_t unaffected must move Y_t/N_t and N_t in opposite directions. Of course an increase in z_t could increase both since it directly affects Y_t.

Notes and common mistakes: The answer follows directly from the expression for the production function. It did not need the use of (14) or (15). Also, figuring out how N_t actually depends on ψ_t is both impossible analytically from the information given (e.g. we do not know the process for ψ_t) and unnecessary, since the question did not actually ask how ψ_t affects N_t.

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4. (15 points) Can a shock to ψ_t (all else equal) simultaneously raise consumption and investment? Prove your answer formally. Explain in words how the result would differ when considering shocks to z_t.

Answer

No. We prove this in two steps. First, $C_t + I_t = z_t K_t^{\alpha} N_t^{1-\alpha}$ means that, if C_t and I_t both increase, then N_t must increase as well (since z_t and K_t are assumed given). Second, C_t and N_t cannot both increase, because of (15), which shows that C_t and N_t must be negatively related for a given z_t, K_t. Again, a shock to z_t would enable this positive co-movement.

Notes and common mistakes: The answer follows from the resource constraint $C_t + I_t = Y_t$, together with (15). It did not need the use of (14). Also, figuring out how C_t and I_t actually depends on ψ_t is both impossible analytically from the information given (e.g. we do not know the process for ψ_t) and unnecessary, since the question did not actually ask how ψ_t affects C_t and I_t.

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5. (5 points) Would the model match the US business cycle data better than with either shock alone? Explain your answer.

Answer

Yes. If the only shocks are to ψ_t, the previous two questions already show that this would induce a negative co-movement between Y_t and Y_t/N_t, and a negative co-movement between C_t and I_t, both of which are counterfactual. Shocks to z_t enable positive co-movement among these variables. On the other hand, the correlation between N_t and Y_t/N_t, while positive, is significantly less than one. Adding shocks to ψ_t weakens this correlation.

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6. (10 points) Consider the model's steady state for a fixed $z_t = \bar{z}$, $\psi_t = \bar{\psi}$. Derive an equation that characterizes the steady-state capital-output ratio, K/Y. How does an increase in $\bar{\psi}$ affect the steady-state capital-output ratio? How does the effect differ from an increase in $\bar{z}$? Give some intuition.

Answer

The Euler equation (14) is steady state becomes

$$1 = \beta(1 - \delta + \alpha\bar{\psi}\bar{z}K^{\alpha-1}N^{1-\alpha}), \tag{16}$$

which we rewrite

$$1 = \beta(1 - \delta + \alpha\bar{\psi}\frac{Y}{K}) \tag{17}$$

It then follows that the capital-output ratio is

$$\frac{K}{Y} = \frac{\alpha\bar{\psi}}{\frac{1}{\beta} - 1 + \delta}, \tag{18}$$

which is increasing in $\bar{\psi}$ and independent of $\bar{z}$. An increase in $\bar{\psi}$ raises the efficacy of capital accumulation without directly affecting the production technology. On the other hand, an increase in $\bar{z}$ affects the production function directly, and so raises both K and Y in the same proportion, thereby keeping the ratio unchanged.

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3 Unemployment insurance

An economy consists of a continuum of ex-ante identical people, each of whom, at each point, can be either employed or unemployed. Each agent lives forever and has utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1,$$

where u is differentiable, strictly increasing, and strictly concave. If a person is employed in period t, they become unemployed with exogenous probability $\delta \in (0,1)$ the following period, and remain employed with probability $1-\delta$. If a person is unemployed in period t, they become employed with exogenous probability $p \in (0,1)$ the following period, and remain unemployed with probability $1-p$. A person produces an exogenous amount y when employed and an exogenous amount $h < y$ when unemployed. Output cannot be stored across periods.

The government collects an exogenous tax T each period from each employed person, and provides unemployment benefits b to each unemployed person. The government budget must be balanced in every period. Assume $h + b < y - T$.

People can trade a non-contingent bond, which is in zero net supply, subject to some exogenous borrowing limit $\bar{A}$.

We will be considering a recursive competitive equilibrium (not necessarily a stationary one).

1. (15 points) Write down the recursive problem for a person of each employment status. Briefly explain if the problem needs an aggregate state and, if so, what it is.

   Answer

   The aggregate state is the joint distribution Φ of assets and employment status. This is needed for two reasons: it affects the market-clearing interest rate r, and also affects the endogenous unemployment benefit b through the government budget constraint.

   An individual's states are their assets a and their current employment status $i \in \{e, u\}$. The! value function for an employed individual ($i = e$) is

   $$V(e, a, \Phi) = \max_{c,a'} u(c) + \beta(1-\delta)V(e, a', \Phi') + \beta\delta V(u, a', \Phi') \tag{19}$$

   subject to the budget constraint

   $$c + a' = y - T + (1 + r(\Phi))a, \tag{20}$$

   the borrowing constraint

   $$a' \geq -\bar{A}, \tag{21}$$

   and the law of motion for Φ,

   $$\Phi' = \mathcal{H}(\Phi) \tag{22}$$

The value function for an unemployed individual ($i = u$) is

$$V(u, a, \Phi) = \max_{c,a'} u(c) + \beta p V(e, a', \Phi') + \beta(1-p)(u, a', \Phi') \tag{23}$$

subject to the budget constraint

$$c + a' = h + b(\Phi) + (1 + r(\Phi))a, \tag{24}$$

the borrowing constraint

$$a' \geq -\bar{A}, \tag{25}$$

and the law of motion for Φ,

$$\Phi' = \mathcal{H}(\Phi) \tag{26}$$

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2. Write down the following objects in a **stationary** competitive equilibrium:

   (a) (3 points) The expression for the unemployment rate.
   (b) (3 points) The government budget constraint.
   (c) (4 points) The asset market clearing condition.

Answer

(a) The unemployment rate evolves according to

$$n_{u,t+1} = (1-p)n_{u,t} + \delta(1 - n_{u,t}),$$

and so in stationary equilibrium we get

$$n_u = \frac{\delta}{p + \delta} \tag{27}$$

(b) This is $T(1 - n_u) = bn_u$, which can also be written as

$$pT = \delta b \tag{28}$$

(c)

$$\int_{\mathcal{A}\times\{e,u\}} a'(i, a)\Phi(da, di) = 0 \tag{29}$$

where $\mathcal{A}$ is the asset space, Φ is the stationary distribution of assets and employment status, and $a'(i, a)$ is the individual decision rule given the stationary distribution.

For the remainder of the problem, consider a stationary equilibrium and assume that $\bar{A} = 0$: agents can save but cannot borrow.

3. (20 points) Denote by r^* the highest interest rate consistent with equilibrium. Derive the expression for r^*. Show that $\beta(1+r^*) < 1$. Describe the wealth distribution.

Answer

$\bar{A} = 0$ means that no borrowing is allowed. Market clearing implies that in equilibrium there must be no lending. This immediately tells us that the wealth distribution is degenerate: the employed always consume $c_e = y - T$, and the unemployed always consume $c_u = h + b$, where by assumption we have $c_u < c_e$. Next, the Euler equations are

$$u'(c_e) \geq \beta(1+r)[(1-\delta)u'(c_e) + \delta u'(c_u)] \tag{30}$$

and

$$u'(c_u) \geq \beta(1+r)[pu'(c_e) + (1-p)u'(c_u)] \tag{31}$$

We can show that, if (30) holds, then (31) must hold with strict inequality; in other words, the borrowing constraint for an unemployed person must bind. Since u is concave, we know that $u'(c_u) > u'(c_e)$, and therefore (30) implies $u'(c_e) > \beta(1+r)u'(c_e)$, so we must have $\beta(1+r) < 1$. Then (31) implies

$$u'(c_u) > pu'(c_e) + (1-p)u'(c_u) > \beta(1+r)[pu'(c_e) + (1-p)u'(c_u)].$$

So, (31) holds with strict inequality, and (30) implies

$$\beta(1+r) \leq \frac{u'(c_e)}{(1-\delta)u'(c_e) + \delta u'(c_u)} \tag{32}$$

The highest interest rate consistent with equilibrium therefore occurs when (30) holds with equality; in other words, r^* is given by

$$\beta(1+r^*) \leq \frac{u'(c_e)}{(1-\delta)u'(c_e) + \delta u'(c_u)} < 1 \tag{33}$$

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4. (10 points) Suppose that there is an increase in T, with a corresponding change in b. Determine how this policy change would affect r^*, and provide some intuition.

Answer

An increase in T, accompanied by an increase in b, raises c_u and lowers c_e. This means that

$$\frac{u'(c_e)}{(1-\delta)u'(c_e) + \delta u'(c_u)} = \frac{1}{1-\delta+\delta\frac{u'(c_u)}{u'(c_e)}}$$

increases, by concavity of u. So, r^* increases. Intuitively, r^* is the interest rate such that the employed are just indifferent between saving and borrowing. When income rises for the unemployed and falls for the employed, this lowers an employed person's desire to save, thus raising the equilibrium interest rate.

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5. (5 points) Does the above policy change affect aggregate unemployment and output? If it does, explain how (formal proof not necessary). If it does not affect unemployment and output, discuss how you could extend/modify the model so that it does.

Answer

As shown above, the stationary unemployment rate is $n_u = \frac{\delta}{p+\delta}$ and aggregate output is $(1 - n_u)y + n_u h$. Both are exogenous, because y and h are exogenous, and so are p and δ. Thus they are unaffected by policy. This would be changed in a model where p and/or δ are endogenous; for example, if agents choose how much effort to exert to search for a job, or if jobs differ and agents choose which jobs to accept.

End of exam.