Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Government spending and business cycles

Consider a real business cycle model in which the representative household has preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t\right) - \frac{N_t^2}{2} \right]$$

over consumption (C_t) and labor (N_t), and output is produced by the representative firm using labor, according to the production function

$$Y_t = z_t N_t^{\alpha}, \quad 0 < \alpha < 1,$$

where z_t is a productivity shock that follows a Markov process. Households own the firms and receive their profits as dividends each period. The only asset households can trade is a non-contingent bond.

Each period, a given amount G_t of output must be allocated to government spending, where the G_t shock is i.i.d. over time and is treated as given by all the agents in the economy. Government spending is financed with lump-sum taxes.

1. (18 points) Define a recursive competitive equilibrium.

Answer

Since net asset supply is always zero and there is no capital, the aggregate state vector consists of the shocks z and G. A household's individual state is its bond holdings, b. We have the value function

$$V(z, G, b) = \max_{c,n,b'} \ln(c) - \frac{n^2}{2} + \beta \mathbb{E} V(z', G', b') \tag{1}$$

subject to

$$c + q(z, G)b' = b + w(z, G)n + \Delta(z, G) - T(z, G) \tag{2}$$

where $q(z, G)$ is the price of the bond, $w(z, G)$ is the wage, $\Delta(z, G)$ is the dividends, and $T(z, G)$ is the lump-sum tax. The solution to this problem gives individual decision rules $c(z, G, b)$, $n(z, G, b)$, $b'(z, G, b)$.

A recursive competitive equilibrium consists of

- A value function $V(z, G, b)$ and individual decision rules $c(z, G, b)$, $n(z, G, b)$, $b'(z, G, b)$
- Aggregate decision rules $C(z, G)$, $N(z, G)$, $B'(z, G)$
- Prices $w(z, G)$, $\Delta(z, G)$, $q(z, G)$
- Taxes $T(z, G)$

such that

- The value function and individual decision rules maximize (1) subject to (2), taking the prices as given.
- The pricing functions $w(z, G)$, $\Delta(z, G)$ are consistent with firms' profit maximization:

$$w(z, G) = z\alpha N(z, G)^{\alpha-1} \tag{3}$$

$$\Delta(z, G) = zN(z, G)^{\alpha} - w(z, G)N(z, G) \tag{4}$$

- Aggregate and individual decision rules are consistent:

$$C(z, G) = c(z, G, 0), \quad N(z, G) = n(z, G, 0), \quad B'(z, G) = b'(z, G, 0) \tag{5}$$

- Government budget is balanced:

$$T(z, G) = G \tag{6}$$

- Markets clear:

$$C(z, G) = zN(z, G)^{\alpha} - G \tag{7}$$

$$B'(z, G) = 0 \tag{8}$$

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2. (8 points) Derive an optimality condition relating the equilibrium labor N_t to the exogenous parameters and aggregate state variables.

Answer

From the first-order conditions for c and n in (1), we get

$$w_t \frac{1}{C_t} = N_t \tag{9}$$

Substituting in the equilibrium conditions, this becomes

$$\frac{z_t \alpha N_t^{\alpha-1}}{z_t N_t^{\alpha} - G_t} = N_t \tag{10}$$

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3. (8 points) Determine how a positive shock to z_t affects N_t. What if $G_t = 0$? Provide some intuition.

Answer

We can rewrite the optimality condition as

$$\frac{\alpha N_t^{\alpha-1}}{N_t^{\alpha} - G_t/z_t} = N_t \tag{11}$$

When $G_t > 0$, it is easy to show that N_t is decreasing in z_t. When $G_t = 0$, N_t is independent of z_t. The intuition is that, when $G_t = 0$, the income and substitution effects exactly cancel each other out. When $G_t > 0$, the income effect dominates.

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4. (10 points) Determine how a positive shock to G_t affects N_t and C_t. Provide some intuition.

Answer

From the same optimality condition, it is clear that a positive shock to G_t increases N_t. Furthermore, since

$$\frac{z_t \alpha N_t^{\alpha-1}}{C_t} = N_t,$$

a shock to G_t necessarily moves N_t and C_t in opposite directions, so a positive shock to G_t reduces C_t. This is a pure income effect: a higher G_t makes the household poorer, inducing it to work more and consume less.

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5. (6 points) Derive an expression relating the equilibrium price of the bond to other endogenous variables.

Answer

The price of the bond is given by the Euler equation

$$q_t \frac{1}{C_t} = \beta \mathbb{E} \frac{1}{C_{t+1}}, \tag{12}$$

or

$$q_t = \beta \mathbb{E} \frac{C_t}{C_{t+1}} \tag{13}$$

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6. (8 points) Determine how a positive shock to G_t affects the price of the bond. Remember that G_t shocks are i.i.d. Provide some intuition.

Answer

We established earlier that a positive shock to G_t reduces C_t. Since G_t is i.i.d., a positive shock to G_t does not affect $\mathbb{E}\frac{1}{C_{t+1}}$, and therefore a positive shock to G_t unambiguously decreases q_t. Intuitively, since period-t consumption falls, the marginal utility of consumption in period t rises, lowering the desire to buy the bond, and thereby lowering its equilibrium price.

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7. (6 points) Is this a good model for explaining business cycle dynamics in the US? Would it be able to match business cycle co-movements observed in the data? What model elements are missing?

Answer

The model does not include capital. This is not only grossly counterfactual in itself, but also prevents the model from matching several key business cycle statistics. First, as shown above, the model would imply a zero or negative correlation between z_t and N_t, whereas the correlation between productivity and labor is positive in the data. Adding capital could resolve this problem, since productivity shocks would then increase labor supply through the intertemporal substitution channel. Second, the model lacks a propagation mechanism: a shock today has no effect on future variables (beyond any built-in persistence of the shock itself); this could also be resolved by adding capital to the model.

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2 OLG with Education

Demographics:

- In each period t, $N = 1$ young agents are born. Each lives for two periods.
- Households come in J types, indexed by j, of mass μ_j (with $\sum_j \mu_j = N$).

Preferences: $\mathcal{U}\left(c_t^y\right) + \beta\mathcal{U}\left(c_{t+1}^o\right)$ where $\mathcal{U}$ has nice properties.

Endowments: Each young of type j is endowed with two idiosyncratic parameters:

- $\varphi_j \in (0, 1)$ determines the age profile of labor efficiency (see below).
- $a_j > 0$ is a learning ability (see below).

Human capital technology:

- Young agents can invest goods e_j to increase their labor efficiency when old.
- Investing e_j when young yields labor efficiency $h_j\left(e_j\right) \equiv 1 - \varphi_j + a_j\mathcal{H}\left(e_j\right)$ when old.
- $\mathcal{H}\left(\right)$ is a human capital production function that is strictly increasing, strictly concave, and satisfies Inada conditions.

Technology:

- There is one good that can be used for consumption c or investment in education e.
- The good is produced from efficiency units of time only.
- A young agent produces φ_j units of output. An old agent produces $h_j\left(e_j\right)$ units of output.
- The resource constraint is given by

$$C_t + E_t = L_t \tag{14}$$

  where L_t denotes aggregate labor input in efficiency units, C_t denotes aggregate consumption, and E_t denotes aggregate education spending.

Markets: There are competitive markets for consumption. In addition, agents trade one period bonds with interest rate R that are in zero net supply.

Questions:

1. [16 points] Define a solution to the household problem. Interpret the first-order conditions.

   Answer ______________________________

   The easy way of doing this: first choose e to maximize lifetime earnings.[1] Then choose consumption to maximize utility. The complicated way (omitting time subscripts):

   $$\max_{e_j, b_j} \mathcal{U}\left(\varphi_j - e_j - b_j\right) + \beta \mathcal{U}\left(h_j\left(e_j\right) + R' b_j\right) \tag{15}$$

   with FOCs:

   $$\mathcal{U}'\left(\varphi_j - e_j - b_j\right) = \beta \mathcal{U}'\left(h_j\left(e_j\right) + R' b_j\right) R' \tag{16}$$

   $$a_j \mathcal{H}'\left(e_j\right) = R' \tag{17}$$

   A solution consists of $\left(e_j, b_j, c_j^y, c_j^o\right)$ that satisfy the 2 focs and the budget constraints.

   Interpretation:

   - (16) is a standard Euler equation. Giving up a unit of consumption and buying a bond...
   - (17) is a no-arbitrage condition. The household has access to two riskless assets. Their rates of return must be equal (as long as the household can borrow to finance education).

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2. [7 points] Explain why the education decision does not depend on preferences or the age profile of endowments (φ).

   Answer ______________________________

   The household maximizes lifetime earnings. Permanent income hypothesis applies without borrowing constraints.

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3. [13 points] Define a competitive equilibrium.

   Answer ______________________________

   For convenience define the aggregation function $\mathcal{A}\left(x_t\right) \equiv N \sum_j \mu_j x_{j,t}$.

   Objects:

   - household: $\left(e_j, b_j, c_j^y, c_j^o\right)$ for all t;
   - aggregates: C_t, E_t

[1] Based on a Penn State exam in 2016.

- prices: R_t

Equations:

- household: 4 conditions (see above)
- market clearing:
  - goods: RC
  - bonds: $\mathcal{A}(b) = 0$
- identities: $L_t \equiv \mathcal{A}(\alpha) + \mathcal{A}(h(e_{t-1}))$, $C = \mathcal{A}(c_t^y) + \mathcal{A}(c_t^o)$. $E_t = \mathcal{A}(e_t)$.

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4. [13 points] Now assume that borrowing is constrained by $b' \geq \bar{b}$. Characterize the solution for an agent with a binding borrowing constraint. Show that agents with higher early endowments (φ_j) invest more in education. Explain the intuition.

Answer

The borrowing constraint binds when $R' < a_j h'(e_j)$ at the optimum. The household would like to raise e more, but cannot borrow to finance this. Then the FOC becomes

$$\mathcal{U}'\left(\varphi_j - e_j - \bar{b}\right) = \beta \mathcal{U}'\left(h_j(e_j) + R'\bar{b}\right) a_j \mathcal{H}'(e_j) \tag{18}$$

This solves for e_j.

The RHS of (18) is decreasing in e_j, while the LHS is increasing. Hence, the solution for e_j is unique (no surprise). For φ_j it's the other way around. Apply the implicit function theorem (or totally differentiate) to see that higher φ_j leads to lower e_j. The intuition is the poor households would like to borrow to fund consumption and education, but they cannot. So they cut down on both. It is not optimal to just cut down on consumption when young because that would violate consumption smoothing (young marginal utility would be too high).

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5. [10 points] Assume that all agents are identical (they draw the same φ and a). There is no borrowing limit. Show that a higher β increases education investment.

Answer

Since everyone is identical, there is no borrowing or lending. The bond market may be removed from the model. The entire equilibrium now boils down to

$$\mathcal{U}'(\varphi_j - e_j) = \beta \mathcal{U}'(h_j(e_j))\, a_j \mathcal{H}'(e_j) \tag{19}$$

Patient households are willing to give up consumption when young to invest in human capital. This drives down the interest rate and raises e.

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3 Money as a Bubble

Demographics: Time is discrete and goes on forever. There is a unit measure of infinitely lived households.

Preferences:

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(c_t) \tag{20}$$

Endowments:

- In each period, each agent receives an idiosyncratic endowment $y > 0$ that is drawn i.i.d. across agents and over time from a compact set.
- At the beginning of time, agents are also endowment with one unit of "money," which is in fixed supply.

Technologies: Endowments can only be eaten today: $\int c_{i,t} di = \int y_{i,t} di$.

Government: The government taxes the money holdings of each agent at the end of the period at the constant rate τ. It returns the money collected to the households as lump-sum transfers z in the same period.

Markets: There are competitive markets for goods (price p) and money (numeraire).

We study the recursive competitive equilibrium.

Questions:

1. [17 points] Set up the household's dynamic program.

   Answer

   Let m denote real money holdings. Let $\mathcal{G}$ be the law of motion for the aggregate state.

   Aggregate state: density of endowments and money holdings at the beginning of the period: $S = \mathcal{F}(y, m)$.

   Individual state: (y_i, m_i).

   Budget constraint: $y + m + z = x + c$ where x denotes saving. Next period's money holdings are

$$M' = \frac{M'}{P'} \frac{P'}{P} P = m' \pi' P = x(1 - \tau) P \tag{21}$$

   where $\pi' = p'/p$. Therefore

$$m' = \frac{1 - \tau}{\pi'} (y + m + z - c) \tag{22}$$

   This makes sense. The rate of return on money is negative inflation net of the tax.

Bellman:

$$V\left(y,m;S\right)=\max_{c,m'}\mathcal{U}\left(c\right)+\beta\mathbb{E}V\left(m',y';\mathcal{G}\left(S\right)\right) \tag{23}$$

subject to (22). Note: Since y is i.i.d., we could also make "cash on hand" $y+m$ the individual state.

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2. [8 points] Derive and Euler equation and define a solution to the household problem in recursive language.

Answer

This problem yields a standard Euler equation of the form

$$\mathcal{U}'\left(c\right)\geq\beta\mathbb{E}\frac{1-\tau}{\pi'}\mathcal{U}'\left(c'\right) \tag{24}$$

with equality if $m'>0$. A solution consists of a value function and policy functions for consumption $\mathcal{C}\left(y,m;S\right)$ and money demand m', $\mathcal{M}\left(y,m;S\right)$, that solve the Bellman equation in the standard sense (max point by point and fixed point).

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3. [7 points] Write out the government budget constraint.

Answer

For convenience, define the aggregation function $\mathcal{A}\left(x\right)\equiv\int_{y\times m}x\left(y,m\right)\mathcal{F}\left(y,m\right)dydm$. Then:

- on each household, the government spends $\mathcal{Z}\left(S\right)$; this is also aggregate spending
- from each household, the government collects (in nominal terms) $\tau M'=\tau m'p'$; or in real terms: $\tau m'\pi'$
- aggregate revenue is therefore $\tau\mathcal{A}\left(\mathcal{M}\right)\pi\left(S\right)$ where $\mathcal{M}$ is the household's money demand function (see above)
- budget constraint: $\mathcal{Z}\left(S\right)=\tau\mathcal{A}\left(\mathcal{M}\right)\pi\left(S\right)$

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4. [15 points] Define a recursive competitive equilibrium.

Answer

Objects:

- Law of motion of the aggregate state $S'=\mathcal{G}\left(S\right)$.
- Household value function and policy functions.
- Transfer function of the government $\mathcal{Z}\left(S\right)$.

- Inflation $\pi(S)$

Equilibrium conditions:

- Household
- Government budget constraint
- Market clearing for goods: $\mathcal{A}(\mathcal{C}) = Y$ where Y is the (constant) total endowment.
- Market clearing for money: $\mathcal{M}' = \mathcal{A}(\mathcal{M}) = \mathcal{M}/\pi(S)$.
- Consistency: Policy functions induce $S' = \mathcal{G}(S)$.

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5. [10 points] Sketch an argument that a stationary equilibrium where money is valued does not exist when the tax rate τ is too high.

Answer

Consider $\tau \approx 1$. Note that stationarity requires $\pi = 1$. Then the Euler equation implies that no household holds money. The reason is that $\mathcal{U}'(c)$ is bounded from below by the consumption values with $m = 0$. Therefore, the RHS of the Euler equation approaches 0 as $\tau \to 1$.

End of exam.