Macroeconomics Qualifying Examination

August 2021

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- The total time is 3 hours. The total number of points is 180.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

Special instructions for remotely administered exams:

- This exam is open book. You may consult books and notes.
- To ask questions during the exam: call, text, or facetime Lutz Hendricks at 919-886-6885.

1 Habit Formation

Demographics: A representative household with unit mass.

Preferences:

$$\sum_{t=0}^{\infty} \beta^t \left\{ U\left(c_t\right) + G\left(l_t - L_{t-1}\right) \right\} \tag{1}$$

where c is consumption, l is individual leisure, and L is average leisure in the economy (which individuals take as given). The utility functions U and G are strictly increasing and strictly concave.

Technology:

$$z_t F\left(K_t, 1 - L_t\right) + \left(1 - \delta\right) K_t = c_t + K_{t+1} \tag{2}$$

where z is a Markov technology shock and K is aggregate capital. F has nice properties.

Questions:

1. [7 points] State the dynamic program of a social planner.

   Answer

   Need to define an auxiliary state $\bar{L}_t = L_{t-1}$. Then the Bellman equation is given by

$$V\left(K, \bar{L}, z\right) = \max U\left(zF\left(K, 1 - L\right) + \left(1 - \delta\right) K - K'\right) + G\left(L - \bar{L}\right) + \beta \mathbb{E} V\left(K', L, z'\right) \tag{3}$$

2. [13 points] Derive and interpret the first-order conditions.

   Answer

   First-order conditions:

$$U'\left(c\right) = \beta \mathbb{E} V_K\left(.'\right) \tag{4}$$

$$G'\left(L - \bar{L}\right) + \beta \mathbb{E} V_L\left(.'\right) = U'\left(c\right) z F_L \tag{5}$$

   Envelope conditions:

$$V_K = U'\left(c\right)\left(zF_K + 1 - \delta\right) \tag{6}$$

$$V_L = -G' \tag{7}$$

Standard Euler equation:

$$U'(c) = \beta \mathbb{E} U'(c') \left(z' F_K(.') + 1 - \delta\right) \tag{8}$$

"Static" condition:

$$G'\left(L - \bar{L}\right) = U'(c)\, z F_L + \beta \mathbb{E} G'(L' - L) \tag{9}$$

Interpretation: Standard for the Euler equation. For the static condition: Working another hour costs utility from leisure G'. Today, it produces zF_L which can be eaten. It also reduces the habit stock by 1, which gains G' tomorrow.

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3. [5 points] Define a solution in sequence language.

Answer

Solution: $\{K, C, L\}$ that solve the Euler equation, the static condition, the RC, and boundary conditions (initial condition and standard TVC).

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4. [10 points] Now consider the competitive equilibrium. Households take aggregate L as given. Define a competitive equilibrium. Hint: there is no need to resolve the household problem.

Answer

This is now a completely standard growth model, except that $\bar{L}$ appears in G (but everybody takes it as a parameter, so it does not affect the equlibrium conditions). Equilibrium is therefore the same as for the planner, except that the static condition changes to $G'\left(L - \bar{L}\right) = U'(C)\, z F_L$.

In more detail: Sequences $\{c_t, l_t, K_t, w_t, q_t\}$ that satisfy:

- household: Euler equation, static condition, budget constraint, TVC
- firm: 2 standard first-order conditions
- market clearing: resource constraint, $l = L$, and the rest implicit
- 6 equations / 5 objects

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5. [10 points] Now consider the deterministic steady state (where $z_t = \bar{z}$ for all t). Propose a tax-transfer policy that implements the first-best. Provide intuition for why it works.

Answer

Now we have $L = \bar{L}$ so that utility from leisure is always $G(0)$.

Planner: $G'(0) = U'(c)\, z F_L + \beta G'(0)$ or $G'(0) = \frac{U'(c) z F_L}{1-\beta}$.

Equilibrium: $G'(0) = U'(c)\, zF_L$.

Other than that, the equilibrium conditions are the same. Everything else equal, the planner chooses less leisure (more work) than the household would in equilibrium.

Intuitively, we don't want to distort the intertemporal allocation (the Euler equation is already at first-best), but we want households to internalize that taking more leisure today reduces utility tomorrow. So we want to tax leisure or subsidize wages.

Consider a wage subsidy at rate τ, so that the household faces the wage rate $w = (1+\tau)\, zF_L$ and the static condition becomes $G'(0) = U'(c)\,(1+\tau)\, zF_L$. If we set $1+\tau = 1/(1-\beta)$, we fix the static condition. A lump-sum tax is needed to balance the government budget without distorting the intertemporal allocation.

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2 Asset Pricing: Bonds Give Utility

Demographics: There is a single representative household who lives forever.

Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}U\left(c_{t}\times G\left(b_{t}/c_{t}\right)\right) \tag{10}$$

where c is consumption and b denotes bond holdings. The utility functions U and G are strictly increasing and strictly concave.

Endowments: At $t=0$, households are endowed with b_0 bonds (issued by the government) and $k_0 = 1$ shares of a Lucas fruit tree.

Technology: The tree yields d_t units of consumption as fruit. d_t follows a Markov chain. Fruit can only be eaten.

Government: The government issues one period bonds and balances the budget through lump-sum taxes.

Markets: There are competitive markets for consumption (numeraire), trees (price p), and bonds (price q).

Questions:

1. [7 points] State the household's dynamic program.

   Answer

   This is inspired by Krishnamurthy and Vissing Jorgensen (JPE, 2010). The Bellman equation is given by

$$V\left(k,b,d\right)=\max_{c,k',b'}U\left(cG\left(b/c\right)\right)+\beta\mathbb{E}V\left(k',b',d'\right) \tag{11}$$

$$+\lambda\left[\left(p+d\right)k+b+z-c-pk'-qb'\right] \tag{12}$$

   where z is the lump-sum transfer.

2. [18 points] Derive and interpret the Lucas asset pricing equations.

   Answer

   Define $\frac{dU}{dC}\equiv U'\left(cG\left(b/c\right)\right)\left[G\left(b/c\right)-G'\left(b/c\right)\frac{b}{c}\right]$ and $\frac{dU}{db}\equiv U'\left(cG\left(b/c\right)\right)G'\left(b/c\right)$. Then we have the first-order conditions

$$\frac{dU}{dC} = \lambda \tag{13}$$
$$\lambda p = \beta \mathbb{E} V_k (.') \tag{14}$$
$$\lambda q = \beta \mathbb{E} V_b (.') \tag{15}$$
$$V_k = \lambda (p + d) \tag{16}$$
$$V_b = \lambda + \frac{dU}{db} \tag{17}$$

These simplify to

$$\frac{dU}{dc} = \beta \mathbb{E} \left\{ \frac{dU(.')}{dc'} \frac{p' + d'}{p} \right\} \tag{18}$$
$$= \frac{1}{q} \beta \mathbb{E} \left\{ \frac{dU(.')}{dc'} + \frac{dU(.')}{db'} \right\} \tag{19}$$

As a quick sanity check: if $G' = 0$, we get the conditions for the standard Lucas model as a special case.

Intuition:

- Give up a unit of consumption and buy shares. The rate of return is the usual $\frac{p'+d'}{p}$, which is eaten tomorrow. The only difference relative to the standard Lucas model is that marginal utility has a different functional form.
- Give up a unit of consumption and buy $1/q$ bonds which pay 1 unit of consumption tomorrow. We then get the marginal utility of eating $1/q$ plus the utility from holding the additional bonds.

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3. [5 points] Define a competitive equilibrium.

Answer

In sequence language: $\{c_t, b_t, k_t, q_t, p_t\}$ that satisfy:

- household: 2 FOCs + budget constraint (and transversality)
- government budget constraint: $qb' = b + z$
- market clearing for goods ($c = d$), bonds (implicit), shares ($k = 1$).

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4. [15 points] Assume that $U(cG(b/c)) = \frac{[cG(b/c)]^{1-\sigma}}{1-\sigma}$. Further assume that the government manages to hold b/c constant over time at $\phi > 0$ (don't worry about the feasibility of

this assumption). How does the presence of utility from holding bonds ($G' > 0$) affect the expected returns for equities (shares) and bonds? Provide intuition.

Answer

The key point to realize is that only the term $U'(d \times G(\phi))$ is time-varying in the marginal utility $\frac{dU}{dC}$. With the CRRA utility function, the MRS is given by the standard $\beta\,(d'/d)^{-\sigma}$. Hence we get the standard equity pricing equation

$$1 = \mathbb{E}\left\{\beta\,(d'/d)^{-\sigma}\,\frac{p'+d'}{p}\right\} \tag{20}$$

It follows that the return process for equities is the same as in a model where bonds do not deliver utility. The intuition is that the government has neutralized the new feature of the model: bond holdings now longer affect the marginal utility of consumption.

However, for bond pricing we have

$$1 = \frac{1+G'(\phi)}{q}\,\mathbb{E}\left\{\beta\,(d'/d)^{-\sigma}\right\} \tag{21}$$

It follows that the bond price rises by factor $1+G'(\phi)$ relative to the case where bonds do not deliver utility. Hence, the equity premium rises (as one might expect).

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3 Public capital

Consider a real business cycle model in which the representative household has preferences

$$\mathbb{E}_0 \sum_{t=0} \beta^t u\left(C_t, N_t\right)$$

The production function is

$$Y_t = J_t^\gamma K_t^\alpha N_t^{1-\alpha}$$

where Y_t is output, K_t is capital, and N_t is labor. J_t is *public capital*, such as infrastructure (e.g. roads or bridges), public schools, or public hospitals. The numbers γ and α are parameters and satisfy $\gamma \geq 0$, $\alpha \in (0,1)$.

Capital K_t is owned by the households and evolves according to the law of motion

$$K_{t+1} = (1-\delta) K_t + I_t, \quad \delta \in (0,1]$$

Public capital J_t evolves according to the law of motion

$$J_{t+1} = (1-\lambda) J_t + G_t, \quad \lambda \in (0,1],$$

where G_t is period-t government spending on new public capital; in other words, new spending takes one period to become productive. The government spending G_t is a shock that follows an exogenous Markov process.

The economy-wide resource constraint is

$$C_t + I_t + G_t = Y_t$$

1. (15 points) The social planner takes the process for G_t as given. Write down the social planner's problem in recursive form. Derive its optimality conditions.

Answer

$$V(J,K,G) = \max_{C,N,K'} u(C,N) + \beta \mathbb{E} V(J',K',G')$$

subject to

$$C + K' + G = J^\gamma K^\alpha N^{1-\alpha} + (1-\delta) K$$

and

$$J' = (1-\lambda) J + G$$

The first-order and envelope conditions lead to the Euler equation

$$u_C = \beta \mathbb{E} \left(1 - \delta + \alpha \left(J'\right)^\gamma \left(K'\right)^{\alpha-1} \left(N'\right)^{1-\alpha}\right) u_C'$$

and intra-temporal optimality

$$(1-\alpha) J^\gamma K^\alpha N^{-\alpha} u_C = u_N$$

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2. (5 points) Describe the channels through which a shock to G affects both current and future output. Is the direction of the effect ambiguous? How is this different from a model with $\gamma = 0$?

Answer

An increase in G lowers C and K' and, if preferences exhibit an income effect on labor supply, increases N. If N increases, current output increases; otherwise, there is no effect on current output. The fall in K' would, all else equal, lead to a decrease in future output. However, if $\gamma > 0$, an increase in G also mechanically increases next period's J, making the effect on future output ambiguous.

Now, let's consider the competitive equilibrium of this economy. Assume that the government finances government spending G_t using proportional taxes on households' labor income.

3. (15 points) Define a recursive competitive equilibrium for this economy. Very clearly state what are the aggregate and individual state variables.

Answer

The individual state is k. The aggregate states are J, K, G. A RCE consists of

- A value function $V(k, J, K, G)$ together with individual decision rules

$$c(k, J, K, G), \quad n(k, J, K, G), \quad k'(k, J, K, G)$$

- Aggregate decision rules

$$C(J, K, G), N(J, K, G), K'(J, K, G)$$

- Pricing functions $w(J, K, G)$, $r(J, K, G)$
- A tax function $\tau(J, K, G)$

such that

- The value function together with individual decision rules solves the household's problem:

$$V(k, J, K, G) = \max_{c,n,k'} u(c, n) + \beta \mathbb{E} V(k', J', K', G')$$

subject to

$$c + k' = (1 - \tau(J, K, G)) w(J, K, G) n + (1 - \delta + r(J, K, G)) k$$

- The pricing functions are consistent with firm's profit maximization and therefore satisfy

$$w(J, K, G) = (1 - \alpha) J^{\gamma} K^{\alpha} N(J, K, G)^{-\alpha}$$

$$r(J, K, G) = \alpha J^{\gamma} K^{\alpha - 1} N(J, K, G)^{1 - \alpha}$$

- The government budget constraint is satisfied:

$$G = \tau(J,K,G)\,w(J,K,G)\,N(J,K,G)$$

- Aggregate consistency holds:

$$C(J,K,G) = c(K,J,K,G)$$

$$N(J,K,G) = n(K,J,K,G)$$

$$K'(J,K,G) = k'(K,J,K,G)$$

- Markets clear:

$$C(J,K,G) + K'(J,K,G) + G = J^{\gamma}K^{\alpha}N^{1-\alpha} + (1-\delta)K$$

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4. (5 points) Does the the recursive competitive equilibrium allocation coincide with the solution to the planner's problem? Provide a simple proof for your answer.

Answer

No: the household's intratemporal optimality condition (after substituting in the wage) looks like

$$(1-\tau)(1-\alpha)J^{\gamma}K^{\alpha}N^{-\alpha}u_C = u_N$$

which cannot coincide with the social planner's optimality condition for any value of the aggregate state.

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5. (5 points) Suppose that government spending is financed using a combination of government debt and proportional taxes on labor income. Explain how your definition of recursive competitive equilibrium would need to be modified. In particular, write down the new government budget constraint and explain how it differs from the previous one. You do not have to fully re-define the equilibrium - just carefully explain what would change.

Answer

In this case, government debt, denoted B, is another aggregate state variable in the definition of the RCE. The government budget constraint now reads

$$G + B = \tau(J,K,B,G)\,w(J,K,B,G)\,N(J,K,B,G) + q(J,K,B,G)\,B'(J,K,B,G)$$

where q denotes the price of government bonds.

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4 An Incomplete-markets Model With Labor Supply

An economy consists of a large measure of infinitely-lived households. Each household chooses consumption c_t and working hours n_t in each period, and its utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

Households receive **idiosyncratic shocks** to their productive ability y each period, which are drawn from a finite Markov chain π with representative element $\pi_{ij} = Prob(y' = y_j | y = y_i)$. If a household with ability y supplies n working hours, its **efficiency units of labor** are yn.

Output is produced using efficiency units of labor according to

$$Y_t = L_t^{\alpha}, \quad 0 < \alpha < 1,$$

where Y_t = output and L_t = efficiency units of labor. Firms hire efficiency units of labor in a competitive market.

Households own the firms: the firms' profits each period are distributed as dividends equally among all the households.

Households cannot trade state-contingent claims but can borrow and lend at an exogenous, fixed interest rate r, subject to a borrowing constraint $a_t \geq -B$, where we assume that $\beta(1+r) < 1$ and $B \geq 0$.

We will consider the recursive competitive equilibrium (not necessarily a stationary one).

1. (15 points) Write down the household's recursive problem. Clearly explain what you are doing.

Answer

The aggregate state variable here is the distribution of agents across asset levels and ability levels (see explanation below). The individual state variables are asset level a and ability y. Then we have the household's problem

$$V(a, y, \Phi) = \max_{c,n,a'} u(c, n) + \beta \mathbb{E} V(a', y', \Phi')$$

subject to the budget constraint

$$c + a' = w(\Phi) yn + (1+r) a + \Delta(\Phi)$$

and the borrowing constraint

$$a' \geq -B,$$

and subject to the law of motion for the aggregate state

$$\Phi' = \mathcal{H}(\Phi)$$

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2. (10 points) Derive the household's Euler equation, clearly taking into account when the borrowing constraint does and does not bind.

Answer

Let λ and μ be the Lagrange multipliers on the budget constraint and the borrowing constraint, respectively. The first-order conditions for c and a' are

$$u_c(c, n) = \lambda$$

$$-\lambda - \mu + \beta \mathbb{E} V_1(a', y', \Phi') = 0$$

and the envelope condition is

$$V_1(a, y, \Phi) = (1 + r)\lambda$$

Combining, we get

$$u_c(c, n) = \mu + \beta(1 + r)\mathbb{E}u_c(c', n')$$

So, we have

$$u_c(c, n) \geq \beta(1 + r)\mathbb{E}u_c(c', n'),$$

with equality if the borrowing constraint does not bind.

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3. (5 points) What is the aggregate state in the household's problem (if any)? Why do we need to keep track of this aggregate state (if any)?

Answer

The aggregate state is the joint distribution Φ of households across asset and ability levels. We have to keep track of this aggregate state (even though the interest rate r is fixed here), because we need it to compute wages and dividends.

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4. (10 points) Write down the equilibrium expressions for

   (a) The aggregate efficiency units of labor.

   (b) The wage per efficiency unit of labor.

   (c) The dividends received by each household.

Answer

$$L(\Phi) = \int_{A \times Y} yn(a, y, \Phi)\, \Phi(da, dy)$$

$$w(\Phi) = \alpha (L(\Phi))^{\alpha - 1},$$

$$\Delta(\Phi) = (L(\Phi))^{\alpha} - w(\Phi) L(\Phi) = (1-\alpha)(L(\Phi))^{\alpha}$$

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5. (5 points) How do your answers to questions 3 and 4 change if the production function is $Y_t = L_t$?

Answer

In this case, the wage equals 1 independently of the distribution, and profits equal zero independently of the distribution. This means we no longer need to have the distribution as an aggregate state.

End of exam.