Macroeconomics Qualifying Examination

January 2021

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- The total number of points is 200.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

Special instructions for remotely administered exams:

- This exam is open book. You may consult books and notes.
- To ask questions during the exam: call, text, or facetime Lutz Hendricks at 919-886-6885.

1 OLG where only the old consume

Demographics: In each period, $N_t = (1+n)^t$ young are born. Each lives for two periods.

Preferences: Households only like to eat when old. Their utility is $u\left(c^o_{t+t}\right)$ where u has standard properties.

Endowments: The young work one unit of time each period. The initial old are endowed with K_0 units of capital.

Technology: $F(K_t, L_t) + (1-\delta) K_t = N_{t-1} c^o_t + K_{t+1}$ where F has constant returns to scale.

Government: The government levies lump-sum taxes τ_t on each young and ϕ_t on each old. It runs a balanced budget (so one tax is negative).

Markets: There are competitive markets for goods (numeraire), labor (wage w_t), and capital rental (q_t).

Questions

1. [15 points] Define the solution to the planner's problem. Assume that the planner discounts the utility of future generations at rate $\omega \in (0,1)$. Derive and interpret the Euler equation

$$u'(c^o_t) = \omega u'\left(c^o_{t+1}\right) \frac{f'(k_{t+1}) + 1 - \delta}{1+n} \tag{1}$$

   where f is the intensive form of F and $k \equiv K/L$.

2. [5 points] Characterize the Pareto optimal steady state.

3. [15 points] Define a competitive equilibrium. Take the sequence of τ_t as given.

4. [5 points] Derive the steady state capital stock for constant tax rates. Find the labor income tax rate that implements the planner's steady state.

5. [5 points] In words, what problem does the tax system solve in this economy?

1.1 Answers[1]

1. The resource constraint in per capita terms is

$$f(k_t) + 1 - \delta = c^o_t / (1+n) + k_{t+1}(1+n) \tag{2}$$

   where $k_t = K_t / L_t$ and $L_t = N_t$. The planner solves

$$\max \sum_{t=0}^{\infty} \omega^t u\left([1+n]\left[f(k_t) + 1 - \delta - k_{t+1}(1+n)\right]\right) \tag{3}$$

[1] Based on a question by Steve Williamson.

(a) A solution is a sequence $\{c_t^o, k_{t+1}\}$ that solve the Euler equation and resource constraint.

(b) Interpretation: Give up a unit of consumption to gain $(1+n)^{-1}$ units of k_{t+1}. Eat the output that the additional capital produces.

2. This is the standard modified Golden Rule formula:

$$f'(k_{MGR}) + 1 - \delta = \frac{1+n}{\omega} \tag{4}$$

3. The young save all their income:

$$k_{t+1}(1+n) = w_t - \tau_t \tag{5}$$

Wages equal marginal products:

$$w_t = f(k_t) - f'(k_t)\,k_t \tag{6}$$

Old consumption is given by

$$c_t^o = f'(k_t) + (1-\delta)\,k_t - \phi_t \tag{7}$$

The government budget constraint requires $(1+n)\,\tau_t + \phi_t = 0$. A CE is a sequence $\{c_t^o, k_{t+1}, \phi_t\}$ that satisfy

(a) the law of motion for k given by (5) and (6);

(b) the budget constraint (7)

(c) the government budget constraint

(d) with k_0 and τ_t given.

4. In steady state

$$k_{ss}(1+n) = f(k_{ss}) - f'(k_{ss})\,k_{ss} - \tau \tag{8}$$

Solve this while setting $k_{ss} = k_{MGR}$.

5. This is the standard social security avoids overaccumulation story.

2 RBC Model

Demographics: There is a single representative agent who lives forever.

Preferences: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \theta_t \ln c_t$ where $\beta \in (0,1)$ and $\theta_t \in (0,1)$ fluctuates between θ_E in even periods and θ_O in odd periods. Assume that $\theta_O > \theta_E$.

Technology: $z_t k_t = c_t + k_{t+1}$ where z_t is an i.i.d. random variable.

Endowments: Households own k_0 at the beginning of time.

We are solving the planner's problem.

Questions:

1. [10 points] Write down the planner's dynamic program. It is easiest to write this with separate value functions for even and odd periods.

2. [10 points] Derive an Euler equation of the form $u'(c) = \beta \mathbb{E} u'(c') z'$.

3. [10 points] Sketch how you could show that optimal consumption is a constant fraction of output: $c = \gamma_O z k$ in odd periods and $c = \gamma_E z k$ in even periods, where $\gamma_O > \gamma_E$. You should derive two equations that could be solved for γ_E and γ_O (but you don't need to solve them).

   Explain what this means for the time series behavior of consumption and capital.

4. [5 points] Derive an expression for the expected output growth rate $\mathbb{E}\{\ln y_{t+1} - \ln y_t\}$. Explain what the growth rate depends on and why.

2.1 Answers[2]

1. Bellman:

$$V_E(k,z) = \max_{k'} \theta_E \ln(zk - k') + \beta \mathbb{E} V_O(k', z') \tag{9}$$

$$V_O(k,z) = \max_{k'} \theta_O \ln(zk - k') + \beta \mathbb{E} V_E(k', z') \tag{10}$$

2. FOCs:

$$\frac{\theta_E}{zk - k'} = \beta \mathbb{E} V_O'(k', z') \tag{11}$$

   and analogously for odd periods. Here, I am abusing notation by writing $V_O' \equiv \partial V_O / \partial k$. The envelope conditions are

$$V_O'(k,z) = \frac{\theta_O z}{zk - k'} \tag{12}$$

[2] Based on a question by Steve Williamson.

Euler equation:

$$\frac{\theta_E}{zk-k'} = \beta \mathbb{E} \frac{\theta_O z'}{z'k'-k''} \tag{13}$$

where $c = zk - k'$. So this is a standard Euler equation that equates marginal utility today with expected marginal utility tomorrow where the interest rate is z'.

3. The solutions turn out to be

$$\gamma_O = \left(1-\beta^2\right) \frac{\theta_O}{\theta_O + \beta\theta_E} \tag{14}$$

and

$$\gamma_E = \left(1-\beta^2\right) \frac{\theta_E}{\theta_E + \beta\theta_O} \tag{15}$$

The easiest way of deriving this is to use the two Euler equations. Start from

$$\frac{\theta_E}{\gamma_E zk} = \beta \mathbb{E} \frac{\theta_O z'}{\gamma_O z'k'} = \beta \frac{\theta_O}{\gamma_O \left(1-\gamma_E\right) zk} \tag{16}$$

Write out the same for even periods. Now you have two equations that solve for γ_E and γ_O.

It follows that consumption (relative to output) is high in odd periods (when marginal utility is high). Conversely, investment is low in those periods. This is simply because households understand that they will like consuming less tomorrow compared with today when they are in odd periods.

4. Expected output growth in odd periods is given by

$$\mathbb{E} \ln z' + \ln \beta + \ln \left(\frac{\gamma_E}{\gamma_O}\right) \tag{17}$$

In even periods, the analogous expression holds. Expected output growth is independent of z because of log utility and full depreciation (income and substitution effects cancel). The third term reflects the fact that the effective discount rate is high in odd periods; therefore the saving rate is low.

3 Asset pricing

Demographics: There is a single representative agent who lives forever.

Preferences: $\mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$ with $\beta \in (0,1)$.

Technology: There are N trees. Tree i produces dividend $d_{i,t} = \theta_{i,t} y$ where $y > 0$ is a constant and $\theta_{i,t}$ are i.i.d. such that $\sum_i \theta_{i,t} = 1$. The resource constraint is $c_t = \sum_i d_{i,t} = y$.

Endowments: Households are endowed with one share of each tree at the beginning of time.

Markets: There are competitive markets for goods (numeraire), trees ($p_{i,t}$) and one period discount bonds (q_t) that are in zero net supply.

Questions:

1. [15 points] Derive the Lucas asset pricing equations for the trees and the bond.
2. [10 points] Solve for the prices of the trees and the bond.
3. [10 points] Derive the equity premium and explain its value.

3.1 Answers[3]

1. This is standard:
$$\mathbb{E}\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{\theta_{i,t+1}y+p_{i,t+1}}{p_{i,t}}\right\}=1 \tag{18}$$
and for the bond
$$\mathbb{E}\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\right\}\frac{1}{q_t}=1 \tag{19}$$
2. Because consumption is constant, we have $q_t=\beta$ and
$$p_{i,t}=\beta\mathbb{E}\left\{\theta_{i,t+1}y+p_{i,t+1}\right\} \tag{20}$$
Assuming no bubbles, the prices are constant (and equal the expected present values of the dividend streams). Hence,
$$p_i=\beta\frac{\mathbb{E}\theta_i' y}{1-\beta} \tag{21}$$
3. The equity premium must be zero because the trees are effectively risk free. The expected returns are given by
$$\mathbb{E}R_i'=\mathbb{E}\frac{d_{i_{t+1}}+p_{i,t+1}}{p_{i,t}} \tag{22}$$
$$=\frac{\mathbb{E}\theta_i y+\frac{\beta}{1-\beta}\mathbb{E}\theta_i y}{\frac{\beta}{1-\beta}\mathbb{E}\theta_i y}=\frac{1-\beta}{\beta}+1=\frac{1}{\beta} \tag{23}$$
This is also the expected return on the bond.

4 CIA model with hours choice

Demographics: There is a single representative household who lives forever.

Preferences: $\sum_{t=0}^{\infty}\beta^t\left\{\ln c_t-\phi h_t\right\}$ with $\phi>0$ and $\beta\in(0,1)$.

[3]Based on a question due to Steve Williamson.

Technology:

$$A_t K_t^{\theta} H_t^{1-\theta} + (1-\delta) K_t = C_t + K_{t+1} \tag{24}$$

with $\theta \in (0,1)$ and $A_t = \bar{A} > 0$.

Endowments: At the beginning of time, households are endowed with M_0 units of money and K_0 units of capital.

Money: The government issues money using lump-sum transfers according to $M_{t+1} - M_t = (1+\gamma)^t = T_t$ with $\gamma > 0$. Money is needed to buy consumption: $P_t c_t \le M_t + T_t$.

Markets: There are competitive markets for money (numeraire), goods (price P_t), labor (w_t), capital rental (q_t), and one period discount bonds that are in zero net supply (price d_t).

Questions:

1. [15 points] State the household's dynamic program.
2. [15 points] Derive the first-order conditions and define a solution. Derive and interpret

$$q' + 1 - \delta = R \equiv 1/d \tag{25}$$

$$\lambda = \lambda' \beta R' \tag{26}$$

$$\lambda (1+\pi') = \phi / w (1+\pi') = \beta u'(c') \tag{27}$$

   where λ is the Lagrange multiplier on the budget constraint.

3. [12 points] From now on, assume that the economy is in steady state. Derive a set of equations that can be solved for prices (q and w) and quantities (c, h, k).
4. [8 points] Show that steady state c, h, k are decreasing in inflation. Explain the intuition.

4.1 Answers

1. Household problem:

$$V(k, m, b; A) = \max u(c) - \phi h + \beta V(k', m', b'; A') \tag{28}$$

   subject to

$$c \le m + \tau \tag{29}$$

   where $\tau = T/P$ and

$$wh + b + k(q + 1 - \delta) + m + \tau = c + k' + db' + m'(1+\pi') \tag{30}$$

2. First-order conditions:

$$u'(c) = \lambda + \mu \tag{31}$$
$$\phi = \lambda w \tag{32}$$
$$\lambda d = \beta V_b(.') \tag{33}$$
$$\lambda = \beta V_k(.') \tag{34}$$
$$\lambda(1+\pi') = \beta V_m(.') \tag{35}$$

Envelope conditions:

$$V_b(.) = \lambda \tag{36}$$
$$V_k(.) = \lambda[q + 1 - \delta] \tag{37}$$
$$V_m = \lambda + \mu \tag{38}$$

Because there is no risk, we can easily derive that bonds and capital have the same return:

$$q' + 1 - \delta = R \equiv 1/d \tag{39}$$

We also have an Euler equation of the form

$$\lambda = \lambda' \beta R' \tag{40}$$

Solution: $\{c, h, k, m, b, \lambda, \mu\} \, \forall t$ that satisfy

(a) 5 FOCs (substitute out derivatives of V)

(b) budget constraint

(c) CIA or $\mu = 0$

(d) boundary conditions

3. Steady state:

(a) From constant λ we have $R = q + 1 - \delta = 1/\beta$.

(b) This implies a unique k/h that solves $q = A\theta(k/h)^{\theta-1}$ and thus a wage of $w = (1-\theta)A(k/h)^{\theta}$.

(c) Then c/h follows from the resource constraint:

$$c/h = A(k/h)^{\theta} - \delta k/h \tag{41}$$

(d) Finally, the level of everything is determined by

$$\lambda(1+\pi') = \phi/w(1+\pi') = \beta u'(c') \tag{42}$$

(e) With constant money growth and constant real money supply, the inflation rate simply equals the money growth rate.

4. It follows directly that higher inflation reduces c and therefore also h and k. Intuition: Leisure is the only good that is tax exempt. The constant taxes do not distort the intertemporal allocation, but they reduce hours worked.

End of exam.