Macroeconomics Qualifying Examination

August 2020

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total number of points is 200.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

Special instructions for remotely administered exams:

- This exam is open book. You may consult books and notes.
- To ask questions during the exam: call, text, or facetime Lutz Hendricks at 919-886-6885.

1 Open Economy with Capital Adjustment Costs

Setup

- Demographics: There is a representative infinitely lived household.
- Endowments: The household has an initial capital stock k_0 and debt d_{-1}. It supplies l units of labor inelastically.
- Preferences: The per period utility function $u(c_t)$ over consumption is strictly concave and increasing. The household discounts future utility at rate β.
- Technology: The production function is:

$$y_t = A_t k_t^{\alpha} l^{1-\alpha} = A_t F(k_t) \tag{1}$$

  where y_t is output, A_t is a deterministic productivity factor, $k_t > 0$ is physical capital, l is the constant labor supply, and $\alpha \in (0,1)$.

  The law of motion of capital is:

$$k_{t+1} = (1-\delta)k_t + i_t \tag{2}$$

  where $\delta \in [0,1]$ is the depreciation rate.

  There is also a capital adjustment cost

$$\Psi(i_t, k_t) = \frac{\phi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 \tag{3}$$

  with $\phi > 0$. The capital adjustment cost enters as a resource cost such that the household's sequential budget constraint is

$$y_t + d_t = c_t + i_t + \Psi(i_t, k_t) + (1+r)d_{t-1}. \tag{4}$$

- Markets: The representative household of the small open economy operates the firm. The household can trade on international goods and bond markets. The household faces a constant interest rate r and a No-Ponzi game constraint

$$\lim_{j\to\infty} \frac{d_{t+j}}{(1+r)^j} \leq 0.$$

Questions

1. [9 points] Set up the household's dynamic program.

2. [11 points] Derive and interpret the Euler equations

$$u'(c) = \beta u'(c') \left[(1-\delta) \left(1 + \frac{\partial \Psi(i', k')}{\partial i} \right) + A' F'(k') - \frac{\partial \Psi(i', k')}{\partial k'} \right] / \left[1 + \frac{\partial \Psi(i, k)}{\partial i} \right] \quad (5)$$

$$= \beta u'(c') [1 + r] \quad (6)$$

3. [9 points] From now on, assume that

$$\beta(1 + r) = 1 \quad (7)$$

and

$$\delta = 0. \quad (8)$$

Assume that we are in a steady state with $A_t = \bar{A}\ \forall t \geq 0$. Describe how k depends on $\bar{A}$ and r (maintaining $\beta(1+r) = 1$) and explain your answer.

4. [10 points] The present value budget constraint requires that consumption and interest payments on outstanding debt equal permanent non-financial income:

$$c_t + r d_{t-1} = y_t^p \equiv \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{A_{t+j} k_{t+j}^{\alpha} l_{t+j}^{1-\alpha} - (k_{t+j+1} - k_{t+j}) - \Psi(i_{t+j}, k_{t+j})}{(1+r)^j} \quad (9)$$

Take (9) as given and assume productivity is constant at $\bar{A}$. Suppose the initial inherited debt level is d_{-1}. Find the debt level for all $t \geq 0$.

5. [13 points] Now suppose the small open economy is in the steady state and agents assume productivity will be at $\bar{A}$ forever. Then, in period 0, the economy is hit by a temporary positive productivity shock, such that

$$A_t = \begin{cases} \bar{A} & \text{for } t \leq -1 \\ A' > \bar{A} & \text{for } t = 0 \\ \bar{A} & \text{for } t > 0 \end{cases} .$$

(a) Describe how investment changes in response to the productivity shock. Explain your answer.

(b) We now define the trade balance as

$$tb_t \equiv y_t - c_t - i_t - \Psi(i_t, k_t). \quad (10)$$

and the current account as

$$ca_t = tb_t - r d_{t-1} \quad (11)$$

Describe how consumption, the trade balance, the current account and debt change in response to the productivity shock. You should explain what happens in the initial period and in later periods. Explain your answer.

6. [7 points] Describe in words how your answer to Question 5a would change if the productivity shock had some persistence, e.g.,

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_t \tag{12}$$

with $\rho \in (0,1)$. ε_t is white noise and the shock in period $t = 0$ is $\varepsilon_0 > 0$.

How does ϕ affect the volatility of investment in this case?

1.1 Answers

1. Household dynamic program:

$$V\left(k, \hat{d}; A\right) = \max u(c) + \beta V\left(k', \hat{d}'; A'\right) \tag{13}$$

where $\hat{d} = d$ and

$$c = (1-\delta) k + AF(k) + \hat{d}' - k' - \Psi(k' - [1-\delta] k, k) - (1+r) \hat{d} \tag{14}$$

2. Euler equations:

$$u'(c)[1 + \Psi_1] = \beta V_1\left(k', \hat{d}'; A'\right) \tag{15}$$

$$u'(c) = -\beta V_2\left(k', \hat{d}'; A'\right) \tag{16}$$

$$V_1(.) = u'(c)\left[1 - \delta + AF'(k) + (1-\delta)\frac{\partial \Psi(i,k)}{\partial i} - \frac{\partial \Psi(i,k)}{\partial k}\right] \tag{17}$$

$$V_2(.) = -\beta V_2(.') (1+r) \tag{18}$$

Simplify to arrive at the standard Euler equations. Interpretations are standard. Both assets must have the same marginal rate of return.

3. Steady state:

- With $\delta = 0$, steady state investment is 0.
- Assumption (7) and the equilibrium conditions imply that the Euler equation becomes

$$\frac{1}{\beta} = A_{t+1} F'(k_{t+1}) + (1-\delta)$$

$$r = A_{t+1} F'(k_{t+1}) \tag{19}$$

Hence, $F'(k) = r/A$ and

$$k = \kappa\left(\frac{A}{r}\right), \kappa'(\cdot) > 0 \tag{20}$$

4. Debt level:

   With strictly concave utility, (7) implies that consumption is constant

$$c_{t+1} = c_t \tag{21}$$

   From (20), (9), and (21), we obtain for all $t \geq 0$

$$\begin{aligned} c_t &= \bar{c} = -rd_{t-1} + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\bar{A}\bar{k}^{\alpha} l^{1-\alpha}}{(1+r)^j} \\ &= -rd_{t-1} + \bar{A}\bar{k}^{\alpha} l^{1-\alpha} \\ d_t &= \bar{d} = d_{-1} \ \forall t \geq 0 \end{aligned}$$

5. Productivity shock:

   (a) Investment:

   From (19) we know that $k_t = \bar{k}$ $\forall t > 0$ and therefore $i_t = 0$ $\forall t \geq 0$ and therefore $\Psi(i_t, k_t) = 0$ $\forall t \geq 0$.

   Investment is zero because the productivity increase is temporary and there is no reason to increase investments in t to have a higher capital stock in $t+1$. Since investment is 0, the strength of the investment adjustment cost does not matter.

   (b) Trade balance etc:

   Since investment and the capital stock never change, y^p rises in the initial period and then goes back to its initial level. Call the change in output in the inital period Δy. Then we know from (9) that $\Delta c = \frac{r}{1+r}\Delta y$ in the inital period and thus in all periods.

   It follows that households save the remaining income, so that in the initial period $\Delta tb = \frac{1}{1+r}\Delta y > 0$. This happens simply because the marginal propensity to consume is less than one. Household smooth consumption over time. With initial debt given, $\Delta ca = \Delta tb$ in the initial period.

   From period 1 onwards, consumption remains higher by Δc and interest payments are lower by $r\Delta tb = \Delta c$. Hence, debt stays constant over time. The current account returns to the initial level because $\Delta ca = \Delta tb - r\Delta d = 0$.

   In other words, the economy is again in steady state, but with lower debt. In effect, agents save some of the additional output in period 0 (trade surplus) and the eat the savings in later periods (trade deficit).

6. Persistent productivity shock:

   Now investment would increase since (19) implies that $k_1 > \bar{k}$. A higher ϕ increases the capital adjustment costs, leading to a small increase in i and a lower volatility in investments.

2 CIA Model

Demographics: There is a single representative household who lives forever.

Preferences: The household values consumption c and dislikes work time h according to

$$\sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - \phi h_t\right] \tag{22}$$

with $\phi > 0$.

Endowments: In each period, the household is endowed with enough time so that h_t is not constrained. We also assume that $h_t \geq 0$ never binds. At the start of period 0, the household also has K_0 units of capital, M_0 units of money, and $B_0 = 0$ one period bonds.

Technology:

$$K_{t+1} = F\left(K_t, A_t h_t\right) + (1-\delta) K_t - c_t \tag{23}$$

where $F(K, Ah) = K^{\alpha} (Ah)^{1-\alpha}$ and A_t is an exogenously given sequence.

Money:

- The government costlessly prints money and hands it to the household as lump-sum gifts, so that $M_t = M_{t-1} + P_t \tau_t$. τ_t is an exogenous sequence.
- Money is required to purchase consumption: $M_t \geq P_t c_t$.

Questions:

1. [10 points] State the household's Bellman equation.
2. [14 points] Derive and interpret the first-order conditions (which still include the Lagrange multipliers on the budget constraint and CIA constraint, λ and μ).
3. [6 points] Show
$$\lambda = \lambda' \beta R' = (1/\pi') \beta (\lambda' + \mu') = (1/\pi') \beta u'(c') \tag{24}$$
and explain these expressions in words.
4. [8 points] Define a solution to the household problem in sequence language.
5. [6 points] Show that c_t is a function of only w_t when the CIA constraint does not bind. What is the intuition?
6. [10 points] Define a competitive equilibrium in sequence language.
7. [14 points] Consider the case of constant $A_t = \bar{A}$. Assume that the CIA constraint binds. Assume that the inflation rate is constant and positive: $\pi > 0$.
   (a) Show that a change in the inflation rate does not change the steady state real interest rate.
   (b) Show that steady state values of k, c, h all decrease with π. Provide intuition.

2.1 Answers

1. Houshold:

    (a) Define $m = M/P$. Then the nominal budget constraint is given by

    $$M' - P'\tau' = P\left[w + R\left(b + k\right) + M - c - k' - b'\right] \tag{25}$$

    which gives in real terms

    $$w + R\left(b + k\right) + m = c + k' + b' + \left(m' - \tau'\right)\pi' \tag{26}$$

    (b) Bellman equation: $V\left(k, m, b\right) = \max u\left(c\right) - \phi h + \beta V\left(k', m', b'\right)$ subject to CIA constraint $c \leq m$ and budget constraint.

2. FOCs:

    (a) $u'\left(c\right) = \lambda + \mu$: giving up a unit of consumption relaxes both constraints. Or: it takes a unit of money to buy a unit of consumption.

    (b) $\beta V_k\left(.'\right) = \lambda$: one unit of income buys one unit of capital.

    (c) $\beta V_b\left(.'\right) = \lambda$: same for bonds.

    (d) $\beta V_m\left(.'\right) = \lambda\pi'$: similar for money, but one unit of income buys $1/\pi'$ units of money tomorrow.

    (e) $\phi = \lambda w$: marginal value of working another hour is 0.

    (f) Envelope:

        i. $V_k = V_b = \lambda R$: one unit of capital or bonds pays R.

        ii. $V_m = \lambda + \mu$: one unit of money is a unit of income and a relaxation of the CIA constraint.

3. Simplify the FOCs to get (24).

4. Solution: (in sequence language) $\{c_t, h_t, k_t, m_t, b_t, \lambda_t, \mu_t\}$ that satisfy the 3 FOCs (24) and budget constraint and CIA constraint (or $\mu = 0$). There is indeterminacy of the portfolio composition (only $k + b$ is determined). Standard transversality.

5. When the CIA constraint does not bind: $\mu = 0$ and $c = u'^{-1}\left(\phi/w\right)$. Consumption only depends on the current wage (not anything in the future) because the marginal utility of leisure is a constant.

6. CE: Sequences of 7 household variables plus aggregates $\{K_t, H_t, M_t\}$ and prices $\{p_t, w_t, R_t\}$ that satisfy:

    (a) 6 household conditions

    (b) 2 standard firm FOCs

(c) government budget constraint

(d) market clearing: goods (RC), labor $h = H$, capital $k = K$, bonds $b = 0$, money $m = M/p$

7. Steady state: A steady state consists of $k, h, m, c, h, \lambda, \mu$ and R, π that satisfy:

   (a) $\beta R = 1$

   (b) $g = \pi$ where g is the constant money growth rate

   (c) $R = F_k + 1 - \delta$ with $F_k = \alpha (k/Ah)^{\alpha-1}$ and $w = (1-\alpha)(k/Ah)^{\alpha}$.

   (d) $F(k, Ah) = c + \delta k$. Or $F(k/Ah, 1) = c/Ah + \delta k/Ah$.

   (e) if CIA does not bind: $\mu = 0$ and $R\pi = 1$ and therefore $R = 1/g$.

   (f) If CIA binds: $c = m$. $R = 1/\beta$ and therefore inflation does not affect k/Ah. Therefore, it does not affect wages and λ. From $\lambda = (1/\pi')\, \beta u'(c')$, we have that higher π requires lower c. From the resource constraint, c/h is unchanged and therefore h must drop.

3 RBC with Comparison Utility

Demographics: There is a single representative agent who lives forever.

Preferences: $\mathbb{E} \sum_{t=0}^{\infty} \beta^t [u(c_t - \eta \bar{c}_t) - v(l_t)]$ where c is consumption and $\bar{c}$ is aggregate consumption (taken as given by each individual). l is hours worked.

Technology: $c_t + k_{t+1} = A_t k_t^{\alpha} l_t^{1-\alpha} + (1-\delta) k_t$ where A_t is governed by a finite Markov chain.

Endowments: At the beginning of time, households are endowed with capital k_0. In each period, households have a time endowment that is large enough so we don't have to worry about the upper bound on hours worked.

Markets: There are competitive markets for goods (numeraire), labor rental (w_t), shares in the firm (p_t), and bonds (gross return R_t; in zero net supply). Firms own the capital and households own firms.

Questions:

1. [11 points] Set up the household problem as a dynamic program.

2. [8 points] Derive the first-order conditions.

3. [4 points] Define a solution in recursive language.

4. [14 points] Set up the firm problem as a dynamic program. The firm discounts future profits at the stochastic rate R', which is the rate of return on stocks that the household takes as given.

5. [10 points] Derive the first order conditions and show

$$\mathbb{E}\frac{F_k\left(k';A'l'\right)+(1-\delta)}{R'\left(S'\right)}=1 \tag{27}$$

6. [4 points] Define a solution to the firm problem.

7. [8 points] Define a competitive equilibrium (in sequence language).

8. [9 points] Define a solution to the social planner's problem.

9. [5 points] Do the allocations differ? Assume CRRA preferences u. What is the intuition?

3.1 Solution[1]

1. Household problem:

   (a) Budget constraint: $px' = (p+d)\,x + wl - c$.

   (b) Then the household problem becomes:

   $$V\left(x,b;S\right)=\max_{c,l} u\left((p+d)\,x-px'+wl-c-\eta\bar{c}\left(S\right)\right)-v\left(l\right)+\beta\mathbb{E}V\left(x',b';S'\right) \tag{28}$$

   where the aggregate state is $S=(k,A)$.

2. FOCs are standard. We get the Lucas asset pricing equation

   $$u'\left(c-\eta\bar{c}\right)=\beta\mathbb{E}R'u'\left(c'-\eta\bar{c}'\right) \tag{29}$$

   where $R'=\left(p'+d'\right)/p$ and the static condition $wu'\left(c-\eta\bar{c}\right)=v'\left(l\right)$.

3. A solution is a value function and policy functions $x'=\xi\left(x;S\right)$ and $l=\ell\left(x;S\right)$ that solve the Bellman equation in the usual sense.

4. Firm: The firm maximizes the present value of "dividends"

   $$\max\mathbb{E}\sum_t D_t\left[F\left(k_t,A_tl_t\right)+(1-\delta)\,k_t-w_tl_t-k_{t+1}\right] \tag{30}$$

   where $D_t=\left[R_1\times\cdots\times R_t\right]^{-1}$ discounts date t payoffs to date 0.

   (a) Bellman equation

   $$V\left(k;S\right)=\max_{k',l} F\left(k,Al\right)+(1-\delta)\,k-wl-k'+\mathbb{E}V'\left(k';S'\right)/R'\left(S'\right) \tag{31}$$

[1] Based on Albany qualifying exam 2019

5. FOC for labor $w = AF_2$. For investment: $\mathbb{E}V'(k';S')/R'(S') = 1$ with $V'(k;S) = F_k + 1 - \delta$. Therefore (27).

6. Solution: Value function and policy functions that solve the Bellman equation in the usual way.

7. Equilibrium:

   (a) Optimality implies the standard Euler equation

   $$u'(c - \eta\bar{c}) = \beta\mathbb{E}\{u'(c' - \eta\bar{c}')[F_k(k';A'l') + (1-\delta)]\} \tag{32}$$

   and the static condition $u'(c - \eta\bar{c})AF_2 = v'(l)$.

   (b) Equilibrium is then (in sequence language): $\{c_t, l_t, k_t\}$ that satisfy the Euler equation, static optimality, and resource constraint. Plus TVC. With $\bar{c} = c$.

8. Planner: The planner's problem is totally standard, except that utility is $u([1-\eta]c)$. That leaves the Euler equation unchanged (the $1-\eta$ terms cancel), but it changes the static condition.

9. The allocations differ because the consumption externality reduces the marginal utility of consumption. The planner takes this into account when trading off consumption and leisure.

End of exam.