Macroeconomics Qualifying Examination

June 2020

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- The total number of points is 200.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

Special instructions for remotely administered exams:

- This exam is open book. You may consult books and notes.
- To ask questions during the exam: call, text, or facetime Lutz Hendricks at 919-886-6885.

1 OLG Asset Pricing

Demographics: Time is discrete. In each period, a unit mass of households is born. Each lives for 2 periods.

Endowments: Each young household has one unit of work time. Old people do not work. The initial old are endowed with K_1 units of capital.

Preferences: $u\left(c_t^y\right) + \beta \mathbb{E} u\left(c_{t+1}^o\right)$. Assume log utility: $u(c) = \ln(c)$.

Technology: $Y_t = z_t K_t^\alpha L_t^{1-\alpha} = K_{t+1} + c_t^y + c_t^o$. z_t is a technology shock that takes on $j = 1, ..., N$ discrete values with i.i.d. probabilities π_j. Note that capital fully depreciates.

Markets: All markets are competitive. There are markets for consumption (numeraire), capital rental (price q), labor rental (wage w), bonds (gross interest rate R).

Questions:

1. [11 points] State the household problem and derive the first-order conditions.
2. [14 points] Define a competitive equilibrium in sequence language. (Strictly speaking, this is not proper, but we shall run with it.)
3. [9 points] Show that the saving rate is constant in equilibrium: $K' = \frac{\beta}{1+\beta} w$.
4. [9 points] Show that $(c^o)' = z' \alpha \left(K'\right)^{\alpha-1} \beta c^y$.
5. [11 points] Solve for the gross return of the bond for given K':

$$R' = \frac{\alpha \left(K'\right)^{\alpha-1}}{E\left(1/z'\right)} \tag{1}$$

   Provide intuition.

6. [10 points] Holding K' constant, does an increase in uncertainty raise or reduce the risk free rate? Explain the intuition. An increase in uncertainty is a mean-preserving spread in the distribution of z'. Think of this specifically as increasing the largest value that z can take on, while decreasing the smallest in such a way that the mean stays the same.
7. [8 points] Again, holding K' constant, does an increase in uncertainty increase or decrease the equity premium, defined as $\mathbb{E}\left\{q'\right\} - R'$? Explain the intuition.

1.1 Answer[1]

1. Household:

$$\max u\left(w-k'-b'\right)+\beta\mathbb{E}u\left(R'b'+q'k'\right) \tag{2}$$

Assuming an interior solution, we get the standard Lucas asset pricing equations:

$$\begin{aligned} u'\left(c_t^y\right) &= \beta\mathbb{E}u'\left(c_{t+1}^o\right)R_{t+1} & (3)\\ &= \beta\mathbb{E}u'\left(c_{t+1}^o\right)q_{t+1} & (4)\end{aligned}$$

2. Equilibrium: Stochastic sequences $\left\{c_t^y, c_{t+1}^o, k_t, b_t, K_t, L_t, q_t, w_t, R_t\right\}$ that satisfy:

   (a) household: 2 Euler equations and 2 budget constraints;

   (b) firm: 2 standard first-order conditions;

   (c) market clearing: $k=K$, $L=1$, goods (RC), $b=0$.

3. Constant saving rate: The Euler equation with $b=0$ and log utility is

$$\frac{1}{c^y}=\beta\mathbb{E}\frac{q'}{q'k'}=\frac{\beta}{k'} \tag{5}$$

Substituting this into the budget constraint of the young yields the constant saving rate.

4. Law of motion for consumption: The budget constraint implies

$$c^o=q'k'=z'\alpha\left(k'\right)^{\alpha-1}k' \tag{6}$$

Replace k' with βc^y to obtain the result.

5. Risk free return: Substituting the solution for c^o into the Euler equation yields

$$\frac{1}{c^y}=\beta R'\mathbb{E}\frac{1}{z'\alpha\left(k'\right)^{\alpha}}\underbrace{\frac{k'/\beta}{c^y}}_{1} \tag{7}$$

or

$$R'=\frac{\alpha\left(k'\right)^{\alpha-1}}{E\left(1/z'\right)} \tag{8}$$

Intuition: More capital next period reduces the return on capital and the interest rate. If z is constant, this reverts to the standard condition that equates the rates of return on capital and bonds. The discount factor does not matter; it affects the attractiveness of both assets in the same way and leaves their relative returns unchanged.

[1] Based on UCLA Spring 2019.

6. Increase in uncertainty: $1/z$ is a convex function. Therefore, a mean preserving spread increases its expected value. Simple example: Let z take on either value 2 with certainty or values 0 and 4 with probability 1/2 each. $\mathbb{E}z = 2$ in both cases, but $\mathbb{E}1/z = \infty$ under the mean preserving spread. The risk-free rate therefore falls. Intuition: flight to safety.

7. Equity premium: $\mathbb{E}q' = \mathbb{E}z'\alpha\left(k'\right)^{\alpha-1}$ is not affected by increasing uncertainty. Therefore, the equity premium rises (as R' falls). Intuition: holding capital gets riskier. Households prefer the safer asset.

2 Two Skill Model

Demographics: There are two types of households: skilled and unskilled ($j \in \{u, s\}$). Each type has unit mass. All live forever.

Preferences: $\mathbb{E}\sum_{t=1}^{\infty}\beta^t u\left(c_t\right)$ with $u\left(c\right) = c^{1-\sigma}/\left(1-\sigma\right)$.

Technology: $Y_t = \sum_j c_{j,t} + k_{t+1} = f\left(k_t, L_{u,t}, z_t L_{s,t}\right)$ where $L_{j,t}$ is labor demand, f is increasing in all arguments and has constant returns to scale, and z is a Markov productivity shock.

Endowments: Each agent is endowed with one unit of work time (of one skill) in each period. Agents start out with capital $k_{j,1}$.

Markets: There are competitive rental markets for skilled and unskilled labor (wages w_j) and for capital (rental price q). There is a market for goods (numeraire).

Questions:

1. [19 points] Write down the social planner's dynamic programming problem. Let ω_j be the welfare weight for a worker of skill j. Derive the planner's first order conditions and define a solution (you may use sequence language, even though the problem is stochastic).

   (a) Provide intuition for the solution and relate it to that of a one-household growth model.

2. [4 points] What can you say about the evolution of consumption for the two skills? Would your conclusion change if the utility function were different?

3. [22 points] Define a Recursive Competitive Equilibrium.

4. [9 points] What happens to the distribution of consumption in $t+1$ when there is a skill bias shock (in the sense that z_{t+1} is unexpectedly high)? Compare the planner's outcome with the equilibrium. Some equilibrium outcomes are ambiguous. Is it possible that skill biased technical change hurts skilled workers?

2.1 Answers

1. Social planner: $\max \mathbb{E} \sum_j \omega_j \sum_t \beta^t u(c_{j,t})$ subject to $\sum_j c_{j,t} + k_{t+1} = f(k_t, 1, z_t)$. This can be broken down into the static problem of allocating consumption across agents in each period and the dynamic problem of how much to save.
   (a) Static problem: the planner equates weighted marginal utilities across agents: $\omega_j u'(c_{j,t})$ is the same for all agents. Let's call $U(C) = \max \sum_j \omega_j u(c_{j,t})$ subject to $C = \sum_j c_{j,t}$.
   (b) Dynamic problem: $V(k; z) = \max U(f(k, 1, z) - k') + \beta \mathbb{E} V(k'; z')$. This is now simply a one-household growth model with utility function U.
   (c) FOC: $U'(C) = \beta \mathbb{E} V'(k'; z')$.
   (d) Envelope: $V'(k; z) = U'(C) f_k$.
   (e) Euler: $U'(C) = \beta \mathbb{E} \{U'(C') f_k(k', 1, z')\}$.
   (f) Solution: stochastic sequences $\{c_{j,t}, C_t, k_t\}$ that satisfy static optimality, Euler equation, resource constraint, standard TVC.

2. With CRRA utility, if the ratio of marginal utilities is constant over time, so is the ratio of consumption values. Without CRRA, this would not be true.

3. RCE:
   (a) The aggregate state consists of the shock z and the distribution of capital over households, which is just a vector: $S = (k_u, k_s)$. Let the law of motion for S be $S' = G(S, z)$.
   (b) Each household solves a perfectly standard problem:

$$V_j(k_j; S, z) = \max u\left(w_j(S, z) + q(S, z) k_j - k'_j\right) + \beta \mathbb{E} V_j\left(k'_j; G(S, z), z'\right) \quad (9)$$

   The solution is standard: a value function and a policy function $k'_j = \kappa_j(k_j; S)$ that solve the dynamic program in the usual sense.
   (c) Firm: This is a standard static problem with first order conditions (price functions):

$$w_j(S, z) = f_j(K(S, z), L_u(S, z), z(S, z) L_s(S, z)) \quad (10)$$

   and $q(S, z) = f_k(K(S, z), L_u(S, z), zL_s(S, z))$.
   (d) Objects:
      i. Value functions and policy functions for the two households.
      ii. Price functions $w_j(S, z)$ and $q(S, z)$.
      iii. Law of motion for the aggregate state.
   (e) Equilibrium conditions:
      i. Value functions and policy functions solve the households' problems in the usual sense.

ii. Firm first order conditions.

iii. Consistency: $S_j' = G_j(S,z) = \kappa_j(S_j; S, z)$ where S_j and G_j denote the jth elements of each object.

iv. Market clearing:

A. labor: $f_j(K(S,z), 1, 1) = w_j(S,z)$ where $K(S,z) = \sum_j S_j$;

B. capital: $f_k(K(S,z), 1, 1) = q(S,z)$.

4. Skill bias shock:

(a) For the planner, things are unambiguous. Consumption increases for all agents in proportion. We even know that consumption increases because aggregate consumption increases in TFP in a one-household model.

(b) In RCE, things are ambiguous. If both worker types hold the same k_j, the shock changes the relative incomes of the two types. It is not clear whether the skill premium w_s/w_u increases or decreases. For example, with perfect substitution across skill types, the skill premium increases. Then consumption of the skilled increases more than consumption of the unskilled. But the converse can happen if the types are strong complements. Then an increase in the effective supply of skilled labor can drive down the skilled labor income share. Since both types will have the same growth rate of expected consumption, this means that the relative consumption of the skilled will decline.

3 New Keynesian Model

Setup

- We consider a New Keynesian model with sticky prices.
- Suppose the economy is described by the following log-linearized system (all variables in deviations from the steady state).

$$X_t = \mathbb{E}_t X_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1}\right) + u_t \tag{11}$$

$$\pi_t = \kappa X_t + \beta \mathbb{E}_t \pi_{t+1} + e_t \tag{12}$$

where $\beta \in (0,1)$, X_t is output gap, i_t is the nominal interest rate, and π_t is inflation. We assume that the process for the cost-push shocks is

$$e_t = \rho_e e_{t-1} + \varepsilon_t, \tag{13}$$

where ε_t is a white noise process and $\rho_e \in (0,1)$. We assume that the central bank chooses i_{t+j}, π_{t+j}, and X_{t+j} under commitment to minimize the loss function

$$L_t = \mathbb{E}_t \left[\frac{1}{2} \sum_{j=0}^{\infty} \beta^j (\pi_{t+j}^2 + \lambda X_{t+j}^2) \right], \tag{14}$$

subject to (12), where in the initial period $E_t \pi_{t+1}$ can be treated as given.

Note that we don't include (11) as a constraint because the central bank can always satisfy the constraint with an appropriate choice of the nominal interest rate. We can therefore assume that the central bank chooses π_{t+j} and X_{t+j} to minimize (14) subject to (12).

Questions:

1. [6 points] Set up the central bank's problem.
2. [16 points] Derive the first order conditions with respect to
   (a) π_t $(j = 0)$
   (b) π_{t+j} $\forall j \geq 1$
   (c) X_{t+j} $\forall j \geq 0$
3. [11 points] Show that the first order conditions and (12) imply

$$\mathbb{E}_t \pi_{t+1} = \frac{\lambda}{\kappa}(X_t - \mathbb{E}_t X_{t+1}). \tag{15}$$

and

$$X_t = \mathbb{E}_t X_{t+1} \left(1 + \beta + \kappa^2/\lambda\right) + \frac{\kappa}{\lambda} \mathbb{E}_t e_{t+1} - \beta \mathbb{E}_t X_{t+2} \tag{16}$$

4. [10 points] We assume from now on that the central bank always ($\forall j \geq 0$) sets policy according to the first order conditions corresponding to 2b and 2c, and that it ignores the first order condition with respect to π_t from 2a. This implies that (15) also holds for π_t in period t such that

$$\pi_t = \frac{\lambda}{\kappa}(X_{t-1} - X_t). \tag{17}$$

Show that

$$X_t = \frac{1}{(1 + \beta + \frac{\kappa^2}{\lambda})}\left(\beta \mathbb{E}_t X_{t+1} + X_{t-1} - \frac{\kappa}{\lambda} e_t\right). \tag{18}$$

5. [17 points] Guess that the output gap is given by an equation of the following form

$$X_t = AX_{t-1} + Be_t, \tag{19}$$

where A and B are coefficients. Show that

$$B = -\frac{\kappa}{\kappa^2 + \lambda[1 + \beta(1 - A - \rho_e)])}. \tag{20}$$

You don't need to solve for A, but we assume $A \in (0, 1)$.

Hint: Recall that

$$e_t = \rho_e e_{t-1} + \varepsilon_t \tag{21}$$

and use this to derive an equation for $E_t X_{t+1}$ as a function of X_{t-1}.

6. [6 points] Use the above result and (15) to determine equilibrium inflation as a function of X_{t-1} and e_t.

7. [8 points] Assume $\rho_e = 0$. How does commitment help the central bank to achieve its objective in response to a cost shock? A precise verbal answer is sufficient.

3.1 Solution

1. Problem based on Walsh (2003) "Monetary Theory and Policy"

$$L_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \Bigg\{ (\pi_{t+j}^2 + \lambda X_{t+j}^2) + \psi_{t+j}[\pi_{t+j} - \kappa X_{t+j} - \beta \mathbb{E}_t \pi_{t+j+1} - e_{t+j}] \Bigg\}$$

2. FOCs:

$$\pi_t + \psi_t = 0 \tag{22}$$
$$\mathbb{E}_t(\pi_{t+j} + \psi_{t+j} - \psi_{t+j-1}) = 0 \tag{23}$$
$$\mathbb{E}_t(\lambda X_{t+j} - \kappa\psi_{t+j}) = 0 \tag{24}$$

3. Using (24) in (23)

$$\mathbb{E}_t\pi_{t+j} = \frac{\lambda}{\kappa}(\mathbb{E}_t X_{t+j-1} - \mathbb{E}_t X_{t+j}) \ \forall j \geq 1 \tag{25}$$

Expected inflation:

$$\mathbb{E}_t\pi_{t+1} = \psi_t - \mathbb{E}_t\psi_{t+1} = \frac{\lambda}{\kappa}\left[X_t - \mathbb{E}_t X_{t+1}\right] \tag{26}$$

Phillips Curve:

$$\mathbb{E}_t\pi_{t+1} = \mathbb{E}_t\kappa X_{t+1} + \beta\mathbb{E}_t\pi_{t+2} + \mathbb{E}_t e_{t+1} \tag{27}$$
$$= \kappa\mathbb{E}_t X_{t+1} + \mathbb{E}_t e_{t+1} + \beta\frac{\lambda}{\kappa}\mathbb{E}_t\left[X_{t+1} - X_{t+2}\right] \tag{28}$$

Applying (15) and collecting terms:

$$X_t = \mathbb{E}_t X_{t+1}\left(\kappa^2/\lambda + 1 + \beta\right) + \kappa/\lambda\mathbb{E}_t e_{t+1} - \beta\mathbb{E}_t X_{t+2} \tag{29}$$

4. Use (17) in (12)

$$\pi_t = (\kappa X_t + \beta\mathbb{E}_t\pi_{t+1} + e_t) \tag{30}$$
$$\frac{\lambda}{\kappa}(X_{t-1} - X_t) = (\kappa X_t + \beta\frac{\lambda}{\kappa}\mathbb{E}_t(X_t - \mathbb{E}_t X_{t+1}) + e_t) \tag{31}$$
$$X_t(1 + \frac{\kappa^2}{\lambda} + \beta) = \beta\mathbb{E}_t X_{t+1} + X_{t-1} - \frac{\kappa}{\lambda}e_t \tag{32}$$
$$X_t = \frac{1}{(1 + \frac{\kappa^2}{\lambda} + \beta)}\left(\beta\mathbb{E}_t X_{t+1} + X_{t-1} - \frac{\kappa}{\lambda}e_t\right) \tag{33}$$

5. Solution for B:

$$X_t = AX_{t-1} + Be_t \tag{34}$$
$$\mathbb{E}_t X_{t+1} = AX_t + B\rho_e e_t \tag{35}$$
$$= A^2 X_{t-1} + (A + \rho_e)Be_t. \tag{36}$$

Use this in (18).

$$X_t(1 + \frac{\kappa^2}{\lambda} + \beta) = \beta \mathbb{E}_t X_{t+1} + X_{t-1} - \frac{\kappa}{\lambda} e_t \quad (37)$$

$$X_t = \frac{(\beta A^2 + 1)}{(1 + \frac{\kappa^2}{\lambda} + \beta)} X_{t-1} + \frac{(\beta B(A + \rho_e) - \frac{\kappa}{\lambda})}{(1 + \frac{\kappa^2}{\lambda} + \beta)} e_t \quad (38)$$

Therefore,

$$B = \frac{(\beta B(A + \rho_e) - \frac{\kappa}{\lambda})}{(1 + \frac{\kappa^2}{\lambda} + \beta)} \quad (39)$$

$$= -\frac{\kappa}{\kappa^2 + \lambda[1 + \beta(1 - A - \rho_e)])} \quad (40)$$

6. From (15) we have:

$$\pi_t = \frac{\lambda}{\kappa}((1 - A)X_{t-1} - Be_t) \quad (41)$$

7. The central bank can commit to responding to the lagged output gap (equation 41). In response to a positive cost shock, the central bank needs to lower output by less because expected inflation for the next period is reduced by the output gap, and this reduces inflation today (equation 12).

End of exam.