Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- The total number of points is 200.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Finitely Lived Capital

Demographics: A representative agent who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$ with $0 < \beta < 1$ and u well-behaved.

Technology:

- Output is produced according to

$$Y_t = F(K_t, L_t) = c_t + I_t \tag{1}$$

  where I_t denotes investment.

- Capital used consists of new capital k_t and one year old capital x_t: $K_t = k_t + x_t$.
- Capital that is older than one year cannot be used in production.
- Each unit of I produces one unit of **new** capital next period: $k_{t+1} = I_t$.

Endowments:

- At $t = 0$, households are endowed with k_0 units of new capital and with x_0 units of capital that is one year old.
- In each period, households are endowed with one unit of work time.

Markets: There are competitive markets for goods (numeraire), capital (rental price q_t), and labor (wage w_t).

Questions:

1. [9 points] Write down the household's dynamic program.
2. [13 points] Derive the Euler equation and explain what it means in words.
3. [12 points] Define a competitive equilibrium in sequence language.

1.1 Solution

1. Budget constraint: $c + k' = w + qk + qx$ with law of motion $x' = k$.

$$V(k, x) = \max_{k'} u(w + qk + qx - k') + \beta V(k', k) \tag{2}$$

2. First-order conditions:

$$u'(c) = \beta V_1(k', k) \tag{3}$$

with envelope condition

$$V_1(k, x) = u'(c)\, q + \beta V_2(k', k) \tag{4}$$
$$V_2(k, x) = u'(c)\, q \tag{5}$$

Euler equation:

$$u'(c) = \beta u'(c')\, q' + \beta u'(c'')\, q'' \tag{6}$$

Intuition: This is like a truncated version of a standard Euler equation without depreciation.

3. CE: Sequences of $\{c_t, K_{t,} L_{t,} k_{t,} x_t, q_t, w_t\}$ that satisfy:

   (a) Household: Euler equation, budget constraint, law of motion $x' = k$ and transversality.

   (b) Firm: 2 standard first-order conditions.

   (c) Market clearing: goods (RC), capital: $K = k + x$, labor $L = 1$.

2 Investment Adjustment Costs

Setup

- Demographics: There is a representative infinitely lived household.
- Endowments: The household has an initial bond position B_0. The household also owns the capital and the initial capital stock is K_0. The household supplies labor each period.
- Preferences: Household per-period utility is given by

$$U(C_t, N_t) = \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} \tag{7}$$

  where $\chi \geq 0$, C_t denotes consumption, and N_t denotes labor. The household discounts future utility at rate $\beta \in (0, 1)$.

- Technology: There is a representative firm with the production function

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{8}$$

  where A_t is productivity with the stochastic process

$$\ln A_t = \rho \ln A_{t-1} + \varepsilon_{a,t}. \tag{9}$$

  $\varepsilon_{a,t}$ has mean zero and $0 < \rho < 1$.

  The capital accumulation equation is subject to an investment adjustment cost:

$$K_{t+1} = \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1-\delta)K_t \tag{10}$$

  where I_t denotes investment, δ depreciation rate, and $\phi \geq 0$.

- Markets: The goods market is perfectly competitive and the goods price is the numeraire. There is a bond market (one-period bond in zero net supply); the interest rate of a bond B_t that pays out in period t is r_{t-1} (interest rate predetermined from period $t-1$). Only the household participates in the bond market.

  The firm rents capital and labor from the household; the wage rate is w_t and the rental rate of capital is R_t.

  The firm is owned by the household and pays the profits to the household in the form of dividends Π_t.

Questions

1. [22 points] Specify the household problem and set up the Lagrangian. Derive the first order conditions (let λ_t denote the multiplier on the per-period budget constraint, and μ_t the constraint on the capital accumulation equation).

2. [11 points] Define the price of capital as $q_t \equiv \frac{\mu_t}{\lambda_t}$. Use the first order conditions to determine the price of capital in the steady state.

3. [9 points] Suppose the economy is in the steady state and hit by a positive productivity shock. We assume $\phi > 0$. How do you expect q_t to respond on impact? Explain your answer.

2.1 Solution

1. The household maximizes lifetime utility by choosing $C_t, N_t, I_t, K_t, B_{t+1}$ subject to the per-period budget constraints and capital accumulation. The current value Lagrangian is

$$
\begin{aligned}
\mathcal{L} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \ln C_t - \theta \frac{N_t^{1+\chi}}{1+\chi} \\
&+ \lambda_t \big[w_t N_t + R_t K_t + \Pi_t + (1+r_{t-1}) B_t - C_t - I_t - B_{t+1} \big] \\
&+ \mu_t \left[\left(1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t + (1-\delta) K_t - K_{t+1} \right] \Bigg\}
\end{aligned}
$$

The first order conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= 0 \leftrightarrow \frac{1}{C_t} = \lambda_t \\
\frac{\partial \mathcal{L}}{\partial N_t} &= 0 \leftrightarrow \theta N_t^{\chi} = \lambda_t w_t \\
\frac{\partial \mathcal{L}}{\partial I_t} &= 0 \leftrightarrow \lambda_t = \mu_t \left(1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) \\
&+ \beta \mathbb{E}_t \mu_{t+1} \phi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= 0 \leftrightarrow \mu_t = \beta \mathbb{E}_t \Big[\lambda_{t+1} R_{t+1} + (1-\delta) \mu_{t+1} \Big] \\
\frac{\partial \mathcal{L}}{\partial B_{t+1}} &= 0 \leftrightarrow \lambda_t = \beta \mathbb{E}_t \lambda_{t+1} (1 + r_t)
\end{aligned}
$$

2. The first order condition wrt investment implies that in steady state with $I_t = I_{t-1} = I^*$ we have $\lambda_t = \mu_t$ and thus $q_t = 1$.

3. q_t jumps up because the MPK increases but the household is subject to the investment adjustment cost.

3 Recursive Equilibrium with Money

Demographics: There is a unit measure of infinitely lived households who are ex ante identical.

Preferences: $\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t)$ with $0<\beta<1$ and u well-behaved.

Endowments:

- In each period, each agent draws an endowment $y_t \in \{y^1, \ldots, y^N\}$. The draws are iid across households and over time. The transition probabilities are $\phi^{ij} = \Pr(y_{t+1} = y^j | y_t = y^i)$.
- By a law of large numbers, the aggregate endowment Y is constant over time.
- Goods can only be eaten so that the resource constraint is $Y = C$ where C is aggregate consumption.
- The stationary distribution of endowments is such that $\eta_i = \Pr(y_t = y^i)$.
- In $t = 0$, each agent is endowed with M units of fiat money. Money can be stored. Its supply is fixed over time.

We are characterizing the recursive competitive equilibrium.

Questions:

1. [18 points] Write down the household's Bellman equation. To keep things simple, assume that the distribution of endowments is stationary (we are starting out with η_i). Note that you may write the joint distribution of endowments and money holdings as $F(m|y^i)\,\eta_i$, where m denotes real money holdings. F may vary over time.
2. [18 points] Define a **stationary** recursive competitive equilibrium.
3. [12 points] Intuitively, why can a stationary equilibrium exist when there is uncertainty, but not when there is no uncertainty (all agents' endowments are constant over time)?

3.1 Solution[1]

1. The aggregate state is the joint distribution of households over money holdings and endowments. With the distribution of endowments stationary, we only need to keep track of $F(m|y^i)$. The individual states are (y, m). The Bellman equation is:

$$V(y, m, F) = \max_{c, m'} u(c) + \beta \mathbb{E} V(y', m', F') \tag{11}$$

[1]Based on UCLA comprehensive exam 2015.

subject to the budget constraint

$$c(y,m) + m'(y,m) P(F') / P(F) = y + m \tag{12}$$

where P is the price level. Implicit in the model is the assumption that agents cannot borrow, so that $m'(y,m) \geq 0$. The household also needs to know the law of motion for the aggregate state: $F' = G(F)$.

2. Stationary RCE:
   (a) Household value function and policy functions maximize the Bellman equation in the usual sense.
   (b) Price level P.
   (c) Joint distribution of agents over states F.
   (d) Market clearing:
   $$M/P = \sum_i \eta_i \int m \times dF\left(m|y^i\right) \tag{13}$$
   and
   $$Y = \sum_i \eta_i y^i = \sum_i \eta_i \int c\left(y^i, m\right) dF\left(m|y^i\right) \tag{14}$$
   (e) Stationarity condition: to be written.

3. Without uncertainty: The real return on money ($R = 1$) implies that $\beta R < 1$. Without uncertainty, agents would want to borrow against future endowments (think about the Euler equation). The money market could not clear.

   With uncertainty, the Euler equation becomes $u'(c) = \beta \mathbb{E} u'(c') > \beta u'(\mathbb{E} c')$. Intuitively, agents now want to hold money even if the interest rate is less than the discount rate. This is precautionary saving in order to insure against income shocks.

4 Optimal Monetary Policy in a New Keynesian Model

Setup

- We consider a New Keynesian model with sticky prices.
- Suppose the economy is described by the following log-linearized system (all variables in deviations from the steady state).

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) + E_t(z_{t+1} - z_t) + u_t \tag{15}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t \tag{16}$$

where x_t is the output gap, i_t is the nominal interest rate, π_t is inflation, $\kappa > 0$ is a constant, u_t is a demand shock, z_t is a productivity shock, and e_t is a cost shock. We assume that

$$u_t = \rho_u u_{t-1} + \xi_t \tag{17}$$

$$z_t = \rho_z z_{t-1} + \psi_t \tag{18}$$

$$e_t = \rho_e e_{t-1} + \varepsilon_t, \tag{19}$$

where $\xi_t, \psi_t, \varepsilon_t$ are white noise processes and $\rho_u, \rho_z, \rho_e \in (0, 1)$. The central bank sets policy each period (discretion, no commitment) to minimize

$$\frac{1}{2} E_t \left[\sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda x_{t+i}^2) \right]. \tag{20}$$

The central banker's problem here reduces to a sequence of single period problems of the form

$$\min \frac{1}{2}(\pi_t^2 + \lambda x_t^2) \tag{21}$$

subject to (16), where we can treat x_t as if it were the policy instrument and $E_t \pi_{t+1}$ can be treated as given.

Questions:

1. [7 points] Derive the first order condition of the central bank's problem under discretion.
2. [19 points] Derive the optimal policy of a central bank with discretion, i.e., find the policy functions for x_t and π_t where e_t is the state variable by guessing linear policy functions for π_t and x_t.
3. [17 points] Derive the interest rate rule implied by the optimal discretionary policy, i.e. express the nominal interest rate as a function of the shocks. If you did not derive the policy functions above, then use $\pi_t = \omega_1 e_t$ and $x_t = \omega_2 e_t$.

4. [14 points] How will x_t and π_t move on impact in response to a positive
   (a) cost shock?
   (b) productivity shock?
   (c) demand shock?

   Explain your answers.

4.1 Solution[2]

1. The first order condition is

$$\kappa\pi_t + \lambda x_t = 0$$

2. Using FOC in (16),

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\kappa^2}{\lambda}\pi_t + e_t$$

   Guess a solution of the form $\pi = Ae_t$. Then it must hold that

$$\begin{aligned} Ae_t &= \beta A\rho_e e_t - \frac{\kappa^2}{\lambda} Ae_t + e_t \\ A &= \beta A\rho_e - \frac{\kappa^2}{\lambda} A + 1 \\ &= \frac{\lambda}{\lambda(1-\beta\rho_e)+\kappa^2} \end{aligned}$$

   With inflation equal to

$$\pi_t = \left[\frac{\lambda}{\lambda(1-\beta\rho_e)+\kappa^2}\right] e_t$$

   the output gap then is

$$x_t = -\left(\frac{\kappa}{\lambda}\right)\pi_t = -\left[\frac{\kappa}{\lambda(1-\beta\rho_e)+\kappa^2}\right] e_t$$

3. From the Euler equation (15):

$$i_t = \sigma(E_t x_{t+1} - x_t) + E_t\pi_{t+1} + \sigma E_t(z_{t+1} - z_t) + \sigma u_t \tag{22}$$

[2]Based on Walsh (2017)

Using the above policy functions yields

$$
\begin{aligned}
i_t = \sigma \left((1-\rho_e) \left[\frac{\kappa}{\lambda(1-\beta\rho_e)+\kappa^2} \right] e_t \right) \\
+ \left[\frac{\lambda}{\lambda(1-\beta\rho_e)+\kappa^2} \right] \rho_e e_t \\
+ \sigma E_t(z_{t+1} - z_t) + \sigma u_t \\
= \left(\left[\frac{\sigma\kappa(1-\rho_e)+\lambda\rho_e}{\lambda(1-\beta\rho_e)+\kappa^2} \right] e_t \right) + \sigma(\rho_z - 1) z_t + \sigma u_t
\end{aligned}
$$

4. (a) cost shock: Inflation will increase and the output gap will decrease in response to the cost shock.
   (b) productivity shock: no effect
   (c) demand shock: no effect

   With the appropriate choice of the nominal interest rate, the CB can always satisfy equation (16) in response to productivity or demand shocks, while maintaining zero output gap and price stabilization. Hence, these shocks don't affect inflation and output gap.

   For example, the CB can simultaneously maintain a zero output gap and price stability in response to a negative demand shock if it lowers the nominal interest rate.

   In contrast, the cost shock in the Phillips curve yields a trade-off between inflation and the output gap.

End of exam.