Macroeconomics Qualifying Examination

January 2017

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Growth model with idiosyncratic productivity

Demographics: Unit mass population of agents who live forever.

Preferences: $E_0 \left[\sum_{t=0}^{\infty} \beta^t \log c_t^i\right]$ over a single consumption good.

Technology: At each period, each agent meets high-productivity investment project (H-project) with i.i.d. probability p and low-productivity ones (L-project) with complementary probability $1-p$. The investment technology is:

$$y_{t+1}^i = a_t^i z_t^i$$

where $z_t^i \geq 0$ is investment (in units of the consumption good) of agent i in t, and y_{t+1}^i is the output (in units of the consumption good) in subsequent period. $a_t^i = a^H$ if agent has H-project, and a^L otherwise, where $a^H > a^L \geq 0$. At beginning of each t, agents know whether they have H- or L-projects.

Credit market: Agents can borrow and lend to each other using risk-free one-period loan contracts. Borrowers face a credit constraint:

$$r_t b_t^i \leq \theta y_{t+1}^i, \tag{1}$$

where r_t and b_t^i are the gross interest rate and the amount of borrowing at t. This inequality states that the promised repayment amount in $t+1$ by each agent i cannot exceed an exogenous fraction $\theta \in [0,1]$ of the output produced by the agent.

Optimization problem: At each t, *after* learning a_t^i and taking as given the interest rate and outstanding debt obligation $r_{t-1} b_{t-1}^i$, each agent i solves

$$\max_{\{c_{t+j}^i, b_{t+j}^i, z_{t+j}^i\}_{j \geq 0}} E_t\left[\sum_{j=0}^{\infty} \beta^j \log c_{t+j}^i\right]$$

subject to:

$$c_{t+j}^i + z_{t+j}^i = y_{t+j}^i - r_{t+j-1} b_{t+j-1}^i + b_{t+j}^i \tag{2}$$
$$z_{t+j}^i \geq 0,$$

and credit constraint (1), for all $j \geq 0$.

Market clearing: In equilibrium, the interest rate must be such that the credit market clears: $p b_t^H + (1-p) b_t^L = 0$, where b_t^i denotes the individual net debt of an agent with i-project, where $i \in \{H, L\}$.

Questions:

1. Derive the first order conditions with respect to c_t^j, z_t^j and b_t^j, and the complementary slackness conditions.

2. Show that in equilibrium the interest rate must satisfy $a^L \leq r_t \leq a^H$ for all t. [Note that a solution to this question requires more thinking than algebra.]
   Assume $a^L < r_t < a^H$ for the remaining questions.

3. What is the optimal investment z_t^i of an agent with an L-project in t?
4. Show that the optimal investment z_t^i of an agent with an H-project in t is:

$$z_t^i = \frac{\beta e_t^i}{1 - \frac{\theta a^H}{r_t}},$$

where $e_t^i \equiv y_t^i - r_{t-1} b_{t-1}^i$ is agent i's net worth at t. [Hint: use the fact that $c_t^i = (1 - \beta) e_t^i$.]

5. Explain the economic intuition as to how a change in θ or r_t affects capital investment.
6. Show that the aggregate output at $t+1$ evolves according to:

$$y_{t+1} = \frac{\beta p y_t}{1 - \frac{\theta a^H}{r_t}}.$$

7. Explain the economic intuition as to how a change in θ or r_t affects output growth.

2 Overborrowing

Demographics: Unit mass population of households who live forever.

Endowments: In each period t, households receive an endowment of tradable goods y_t^T and an endowment of nontradable goods y_t^N.

Preferences: $E_0 \left\{\sum_{t=0}^{\infty} \beta^t \log(c_t)\right\}$, where the consumption basket c_t as an aggregation of tradable consumption c_t^T and nontradable consumption c_t^N:

$$c_t = \left[\omega \cdot (c_t^T)^{-\eta} + (1-\omega) \cdot (c_t^N)^{-\eta}\right]^{-\frac{1}{\eta}}$$

where $\eta > -1$ and $\omega \in (0,1)$ are exogenous parameters.

Technologies: Nontradable goods can only be eaten: $y_t^N = c_t^N$. Tradable goods can be eaten or traded with the rest of the world.

Asset market: The menu of foreign assets available is restricted to a one period, non–state contingent bond denominated in units of tradables that pays a fixed interest rate r, determined exogenously in the world market. Normalizing the price of tradables to 1 and denoting the price of nontradable goods by p^N the budget constraint is:

$$b_{t+1} + c_t^T + p_t^N c_t^N = (1+r)b_t + y_t^T + p_t^N y_t^N,$$

where b_{t+1} denotes bond holdings that households choose at the beginning of time t. Positive values of b denote net asset while negative values denote net debt. Households face an exogenous credit constraint:

$$b_{t+1} \geq -\left(\theta^N p_t^N y_t^N + \theta^T y_t^T\right)$$

which states that the amount of debt does not exceed a fraction θ^T of tradable income and a fraction θ^N of nontradable income.

1. Write the Lagrangian and derive first order conditions and complementary slackness.

2. Define a competitive equilibrium.

3. Consider an exogenous decrease in assets, $b_t\,(1+r)$. Assume that it decreases tradable consumption c_t^T. How does this affect the equilibrium price p_t^N and thus the credit constraint for everybody in the economy. Would the competitive equilibrium be Pareto efficient?

3 Firms Accumulate Capital

Demographics: A representative household who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$.

Endowments: k_0 at $t = 0$. One unit of work time in each period.

Technologies: $k_{t+1} = f(k_t, l_t) + (1 - \delta) k_t - c_t$. f has constant returns to scale.

Markets: goods (numeraire), labor rental (w), one period bonds (rate of return R).

- Only households trade bonds. Bonds are in zero net supply.
- Households own the firm and receive dividend income.
- There is a representative firm who lives forever. It owns the capital stock and maximizes the present discounted value of dividends. Dividends are given by $f(k, l) + (1 - \delta) k - wl - k'$.

Questions:

1. We consider a Recursive Competitive Equilibrium. What is the aggregate state variable?
2. Write down the household problem and define a solution in recursive language. You don't need to derive first-order conditions. They are standard.
3. Write down the firm's problem in recursive language.
4. Derive the firm's first order conditions and eliminate value function derivatives. Define a solution in recursive language.
5. Define a Recursive Competitive Equilibrium.

4 Matching With Random Productivities

Consider a standard MP model where workers draw random productivities after meeting firms.

Demographics: A unit mass infinitely lived workers.

Preferences: $\mathbb{E} \int_0^\infty e^{-rt} c_t dt$.

At $t = 0$ the worker is unemployed. While unemployed, he receives $c = b$ and he meets a vacant job with flow probability α_w.

While employed, the worker receives $c = w(y)$. Jobs separate with flow probability λ.

Firms post vacancies at flow cost k. Vacancies are filled with probability α_e. Firms maximize the discounted present value of profits $y - w(y) - k$.

After meeting a job, the worker draws a productivity $y \in [0, B]$ from cdf F. If matched, he produces y until the match separates.

Wages are determined by Nash bargaining after y has been observed. Workers get fraction θ of the surplus.

Job finding rates α_w and vacancy filling rates α_e are functions of labor market tightness (ratio of vacancies to unemployment). Though this does not matter for the questions below.

Questions:

1. Set up the Nash bargaining problem for a worker/firm pair with productivity y.

2. Write down the value functions of unemployed workers, employed workers, filled vacancies, unfilled vacancies. Assume that workers follow a reservation productivity strategy (they accept all matches with $y \geq y_R$).

3. Let $S_y = (V_w(y) - U) + (J(y) - V_V)$ be the surplus of a match with productivity y. Use the fact that $V_w(y) - U = \theta S_y$ and $J(y) - V_V = (1 - \theta) S_y$ to derive

$$rU = b + \frac{\alpha_w \theta k}{\alpha_e (1 - \theta)} \tag{3}$$

End of exam.

5 Answers

5.1 Growth model with idiosyncratic productivity[1]

1. Let the Lagrange multiplier for the budget constraint be λ_t^i, for the non-negative constraint be μ_t^i and for the credit constraint be ζ_t^i:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\log c_t^i + \lambda_t^i(a_{t-1}^i z_{t-1}^i - r_{t-1} b_{t-1}^i + b_t^i - z_t^i - c_t^i) + \mu_t^i z_t^i + \zeta_t^i(\theta a_t^i z_t^i - r_t b_t^i)\right]$$

   FOCs with respect to c_t^i, z_t^i and b_t^i:

$$\begin{aligned} \log'(c_t^i) &= \lambda_t^i \\ \lambda_t^i &= \beta a_t^i \mathbb{E}\lambda_{t+1}^i + \mu_t^i + \theta a_t^i \zeta_t^i \\ \lambda_t^i &= \beta r_t \mathbb{E}\lambda_{t+1}^i + r_t \zeta_t^i. \end{aligned}$$

   Complementary slackness conditions:

$$\begin{aligned} \mu_t^i z_t^i &= 0 \\ \zeta_t^i(\theta a_t^i z_t^i - r_t b_t^i) &= 0. \end{aligned}$$

2. Proof by contradiction: If $r_t < a^L$, then nobody lends (as even agents with the L-project prefer to invest in capital than lending) and the credit market cannot clear. If $r_t > a^H$ then nobody wants to borrow (as even agents with the H-project prefer not to borrow to invest in capital) and the credit market again cannot clear.

3. Given $r_t > a^L$, it is optimal for agents with an L-project to *not* invest, i.e., the non-negativity constraint $z_t^i \geq 0$ is binding. In other words, $z_t^i = 0$.

4. Given $r_t < a^H$, it follows that the credit constraint (1) binds. Substituting binding (1) and $c_t^i = (1-\beta)e_t^i$ into budget constraint (2) will yield the desired expression.

5. The expression shows that an increase in θ (representing a relaxation of the credit constraint) or a decrease in the interest rate r_t relaxes the credit constraint and increases the leverage of agents with H-projects.

6. The aggregate output is given by $y_{t+1} = pa^H z_t^H + (1-p)a^L z_t^L$. Substituting the credit market clearing conditions and the expressions for z_t^i from the two previous questions will yield the desired expression.

7. Similar to above, the expression shows that an increase in θ (representing a relaxation of the credit constraint) or a decrease in the interest rate r_t allows the economy to allocate more resources from agents with L-projects towards agents with H-projects, leading to a higher growth rate of output y_{t+1}/y_t.

[1] Based on Hirano and Yanagawa, Review of Economic Studies, 2016

5.2 Overborrowing[2]

1. FOCs:

$$\lambda_t = u_T(t)$$
$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1}$$
$$\lambda_t = \beta(1+r)E_t\lambda_{t+1} + \mu_t$$

   and complementary slackness condition:

$$\mu_t \cdot \left[b_{t+1} + \theta^N p_t^N y_t^N + \theta^T y_t^T\right] = 0$$

   where λ is the nonnegative multiplier associated with the budget constraint and μ is the nonnegative multiplier associated with the credit constraint.

2. Definitions are standard. Market clearing conditions for the nontradable and tradable markets:

$$c_t^N = y_t^N$$
$$c_t^T = y_t^T + (1+r)b_t - b_{t+1}.$$

3. If c_t^T declines, the first-order condition implies a lower p_t^N, which reduces the collateral value $\theta^N p_t^N y_t^N + \theta^T y_t^T$. Individual agents do not internalize the effect of their individual choices on the price p_t^N, leading to an externality. Hence competitive equilibrium will be Pareto inefficient.

5.3 Answer: Firms Accumulate Capital

1. The state is aggregate capital K. Agents know the law of motion $K' = G(K)$.

2. Household: Budget constraint:

$$a' = R(K')\,a + w(K) + d(K) - c \tag{4}$$

   Bellman:

$$V(a,K) = \max_c u(c) + \beta V\left(R(K)\,a + w(K) + d(K) - c, K'\right) \tag{5}$$

   where $K' = G(K)$.

   Solution: V and policy rules $c(a,K), a'(a,K)$ that solve the usual thing.

3. Firm:

$$W(k,K) = \max_{l,k'} f(k,l) + (1-\delta)\,k - wl - k' + R(K')^{-1} W(k',K') \tag{6}$$

[2] Based on Bianchi, American Economic Review 2011

4. FOCs:

$$R(K') = W_k(') \tag{7}$$

$$f_l = w \tag{8}$$

Envelope

$$W_k = f_k + (1-\delta) \tag{9}$$

Therefore

$$R(K') = f_k(k', l') + (1-\delta) \tag{10}$$

Solution: W and policy rules $l(k,K), k'(k,K)$ that solve the usual thing.

5. RCE:

Objects:

(a) household: V, policy functions for a' and c

(b) firm: W, policy functions for k', l

(c) price functions $R(K), w(K)$

Conditions:

(a) households and firm optimize

(b) market clearing: $l=1$, $a=0$, goods (resource constraint)

(c) consistency: $k'(K,K) = G(K)$

5.4 Answer: Matching With Random Productivities[3]

1. Nash bargain: Worker surplus $V_w(y) - U$. Firm surplus $J(y) - V_V$. The Nash bargain solves $\max_w (V_w(y) - U)^\theta (J(y) - V_V)^{1-\theta}$.

2. Unemployed: $rU = b + \alpha_w \int_{y_R}^B \{V_w(y) - U\} dF(y)$. Employed: $rV_w(y) = w(y) + \lambda(U - V_w(y))$. Filled vacancy: $rJ(y) = y - w(y) - k + \lambda(V_V - J(y))$. Unfilled vacancy: $rV_V = -k + \alpha_e \int_{y_R}^B \{J(y) - V_V\} dF(y)$.

3. $rU = b + \alpha_w \int \theta S_y dF(y)$. $rV_V = -k + \alpha_e \int (1-\theta) S_y dF(y)$. Set the two integrals equal to obtain the equation in the answer.

End of exam.

[3]This is based on p. 971 in Rogerson, Richard, Robert Shimer, and Randall Wright. "Search-theoretic models of the labor market: A survey." Journal of economic literature 43.4 (2005): 959-988.