Macroeconomics Qualifying Examination

August 2017

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Money in the Utility Function

Demographics: Unit mass of agents who live forever.

Preferences: $E_0\left[\sum_{t=0}^{\infty}\beta^t\left\{\ln C_t^i + \ln\frac{M_t^i}{P_t} - \frac{(N_t^i)^2}{2}\right\}\right]$, where C_t^i denotes consumption of a final good of agent i, M_t^i is the holding of money, N_t^i is hours worked, and P_t is the price level.

Technology: $C_t = AN_t$ where C_t is aggregate consumption and N_t is aggregate hours worked. $A > 0$ is a parameter.

Government: The government costlessly prints money and hands it to households as lump-sum transfers: $T_t \equiv M_t - M_{t-1}$. The money supply evolves according to $\ln M_t = \ln M_{t-1} + \epsilon_t$, where ϵ_t is an i.i.d. shock. ϵ_t is known at the beginning of period t.

Markets: All markets are competitive. A representative firm rents labor from households at wage rate W_t. It sells consumption to households at price P_t. Money is the only asset and the numeraire.

Budget constraint: A household's budget constraint in period t is given by: $P_t C_t^i + M_t^i \le W_t N_t^i + M_{t-1}^i + T_t$.

Questions:

1. [6 points] Define a competitive equilibrium.
2. [6 points] Derive the optimality conditions for the problem of household and firm.
3. [9 points] Determine the equilibrium levels of aggregate employment. Explain how monetary shocks affect real economic activities.

2 Small Open Endowment Economy

Demographics: Unit mass of agents who live forever.

Preferences: $E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$ over a single consumption good, where $u(c_t) = -(c_t - \bar{c})^2$, and β and $\bar{c}$ are a parameters. Assume throughout that $c_t < \bar{c}$, so that u is increasing in c_t.

Endowment: Each individual receives identical stochastic endowments $\{y_t\}_{t=0}^{\infty}$.

Credit market: Agents can borrow and lend in an international credit market. The world interest rate is given by $1 + r = \frac{1}{\beta}$.

Optimization problem: Taking the interest rate $1 + r$ as given, the individual solves

$$\max_{\{c_t \geq 0, d_t\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

subject to the budget constraint

$$c_t + (1 + r)d_{t-1} = y_t + d_t$$

and the no-Ponzi scheme condition

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1 + r)^j} \leq 0, \forall t \geq 0.$$

Questions:

1. [3 points] Show that individual consumption is a martingale.

2. [9 points] Show that optimal consumption is given by

$$c_t = \frac{r}{1 + r}\sum_{j=0}^{\infty}\frac{E_t y_{t+j}}{(1 + r)^j} - rd_{t-1} \qquad (1)$$

3. [6 points] Assume $y_t = \rho y_{t-1} + \epsilon_t$, where $0 < \rho < 1$ and ϵ_t is an i.i.d. shock with mean zero. Derive equilibrium consumption c_t and the current account ca_t as a function of y_t and inherited net debt position d_{t-1}.

4. [6 points] Repeat the analysis of part 3, but assume that ϵ_t, the shock that affects y_t, is already known in $t - 1$.

3 Endogenous growth with two technologies

Demographics: A unit mass of infinitely lived households.

Preferences: $\sum_{t=0}^{\infty} \beta^t \ln c_t$

Endowments: k_0 at the start of $t = 0$. $l_t = e^{gt}$ units of labor in period t. $g > 0$ is exogenous.

Technology:

$$k_{t+1} = Ak_{At} + Bk_{Bt}^{\alpha} l_t^{1-\alpha} + (1-\delta) k_t - c_t \tag{2}$$

where $k_t = k_{At} + k_{Bt}$.

Consider the planner's problem.

Questions:

1. [5 points] State the planner's Dynamic Program. Note: This should be detrended first, but you may solve the undetrended version of the problem.
2. [8 points] Derive the first-order and envelope conditions.
3. [3 points] Under what conditions is the Ak technology used in a given period?
4. [6 points] Find the balanced growth rates. There are two cases, depending on whether the Ak technology is used asymptotically.
5. [6 points] Under what conditions is the Ak technology used asymptotically?

4 Recursive Competitive Equilibrium

We study the *Recursive Competitive Equilibrium* in the following economy.

Demographics: There is a unit mass of infinitely lived households, indexed by $j \in [0, 1]$.

Preferences: Household j maximizes $\mathbb{E} \sum_{t=0}^{\infty} \beta^t u\left(c_{jt} - bc_{j,t-1}, h_{c,j,t} + h_{i,j,t}\right)$ with $0 < b < 1$. $h_{c,j,t}$ denotes hours worked in the consumption goods sector and $h_{i,j,t}$ denotes hours worked in the investment goods sector.

Endowments: At time 0, the households are endowed with capital stocks k_{j0} that sum to $K_0 = \int_0^1 k_{j0} dj$. In each period, each household has 1 unit of time, so that $h_{c,j,t} + h_{i,j,t} \leq 1$.

Technologies:

$$C_t = F\left(K_{c,t}, z_t H_{c,t}\right) \tag{3}$$

$$K_{t+1} = G\left(K_{i,t}, H_{i,t}\right) \tag{4}$$

where $K_t = K_{c,t} + K_{i,t}$ and $H_{i,t} = \int_0^1 h_{i,j,t} dj$. $H_{c,t}$ is defined analogously. C_t is aggregate consumption. F and G have constant returns to scale. z_t is a technology shock that evolves according to a finite Markov chain.

Timing:

1. The economy enters the period with capital stocks $k_{j,t}$.
2. Households decide how much labor to supply to the two sectors $(h_{c,j,t}, h_{i,j,t})$.
3. The technology shock z_t is realized.
4. Firms rent capital and labor services in competitive spot markets. Note that the capital allocation is chosen after z_t is realized. Households can move capital freely between sectors.

Questions:

1. [4 points] What is the aggregate state of the economy? This part is important for the remainder of the question.
2. [5 points] Write out the firm's problem in each sector.
3. [9 points] Write out the household problem and define a solution. Hint: prices depend on z_t, which the households do not know when they choose $h_{c,j,t}$ and $h_{i,j,t}$.
4. [9 points] Define a Recursive Competitive Equilibrium. Make sure that you use recursive language.

End of exam.

5 Answers

5.1 Money in the utility function

1. Standard definition.

2. Profit maximization of perfectly competitive firms and labor market clearing imply that equilibrium wage $W_t = P_t A$. Household optimizations: they maximize lifetime utility subject to budget constraints

$$P_t C_t + M_t \leq W_t N_t + M_{t-1}.$$

   Lagrangian:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(C_t) + \ln \frac{M_t}{P_t} - \frac{N_t^2}{2} + \lambda_t \left[W_t N_t + M_{t-1} - P_t C_t - M_t \right] \right\}$$

   FOCs:

$$\frac{1}{P_t} \ln'(C_t) = \lambda_t$$
$$\frac{1}{P_t} \ln' \left(\frac{M_t}{P_t} \right) = \lambda_t - \beta E_t \lambda_{t+1}$$
$$N_t = \lambda_t W_t.$$

   Note that the first and the third equations yield the standard intra-temporal FOC, and the first and the second yield the inter-temporal Euler equation.

3. Given nominal variables $\{M_t, P_t\}$, the real variables of employment and consumption are determined via the intra-temporal first order condition:

$$N_t C_t = A$$

   and the budget constraint:

$$C_t = A N_t + \frac{M_{t-1} - M_t}{P_t} + T_t = A N_t.$$

   These two equations imply that real variables are independent of monetary policy shocks:

$$N_t = 1, \; C_t = A.$$

   Then, given the money supply process $\{M_t\}$, the inflation $\{\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}\}$ is determined via the inter-temporal Euler equation:

$$\ln'(C_t) = \ln' \left(\frac{M_t}{P_t} \right) + \beta E_t \frac{\ln'(C_{t+1})}{\Pi_{t+1}}$$

   As seen above, monetary policy shocks have no effects on the real economy. This should be intuitive as money is neutral in this kind of model without nominal rigidity.

5.2 Small open endowment economy

1. Euler equation implies that consumption follows a Martingale:

$$c_t = E_t c_{t+1} = E_t c_{t+j}, \forall j \geq 0.$$

2. The budget constraint holds state by state. Hence, for each history we have

$$(1+r)d_{t-1} = \sum_{j=0}^{T} \frac{y_{t+j} - c_{t+j}}{(1+r)^j} + \frac{d_{t+T}}{(1+r)^T},$$

If this holds for each history, then it must hold in expectation (sum over histories):

$$(1+r)d_{t-1} = \mathbb{E}_t \sum_{j=0}^{T} \frac{y_{t+j} - c_{t+j}}{(1+r)^j} + \frac{d_{t+T}}{(1+r)^T},$$

Now take $\lim_{T\to\infty}$ and apply the No Ponzi condition to get

$$(1+r)\, d_{t-1} = \hat{y}_t^p + \mathbb{E} \sum_j \frac{c_{t+j}}{(1+r)^j} \tag{5}$$

where $\hat{y}_t^p$ is the expected present value of y. Now from the FOC, the present value of consumption is $c_t\,(1+r)\,/r$. Therefore,

$$c_t = \frac{r}{1+r}\hat{y}_t^p - rd_{t-1} \tag{6}$$

or equivalently

$$c_t = y_t^p - rd_{t-1}$$

where non-financial permanent income is defined as $y_t^p \equiv \frac{r}{1+r}\sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}$.
From these equations it follows that the current account ($ca_t \equiv -(d_t - d_{t-1})$) in equilibrium is equal to:

$$ca_t = y_t - y_t^p$$

which says that in this model, the country runs a current account surplus (or deficit) when the current income is higher (or lower) than the nonfinancial permanent income.

3. Given $y_t = \rho y_{t-1} + \epsilon_t$, it follows that $E_t y_{t+j} = \rho^j y_t$ for each $j \geq 0$. Hence the permanent income is

$$y_t^p = \frac{r}{1+r}\mathbb{E}_t \sum_{j=0}^{\infty} \frac{\rho^j}{(1+r)^j} y_t = \frac{r}{1+r} \frac{y_t}{1-\frac{\rho}{1+r}} = \frac{r}{1+r-\rho} y_t.$$

Thus consumption is

$$c_t = \frac{r}{1+r-\rho} y_t - rd_{t-1}$$

and current account is

$$ca_t = y_t - \frac{r}{1+r-\rho} y_t = \frac{1-\rho}{1+r-\rho} y_t.$$

4. Given $y_t = \rho y_{t-1} + \epsilon_{t-1}$, it follows that y_{t+1} is non-random given the information set at t, and $E_t y_{t+1} = y_{t+1} = \rho y_t + \epsilon_t$. By induction and the law of iterated expectation:

$$E_t y_{t+j} = \rho^j y_t + \rho^{j-1}\epsilon_t,\ \forall j \geq 1.$$

Hence the permanent income is

$$y_t^p = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{\rho^j y_t + \rho^{j-1}\epsilon_t}{(1+r)^j} = \frac{r}{1+r-\rho}(y_t + \frac{\epsilon_t}{\rho}).$$

Thus consumption is

$$c_t = \frac{r}{1+r-\rho}(y_t + \frac{\epsilon_t}{\rho}) - rd_{t-1}$$

and current account is

$$ca_t = y_t - \frac{r}{1+r-\rho}(y_t + \frac{\epsilon_t}{\rho}) = \frac{1-\rho}{1+r-\rho}y_t - \frac{r}{1+r-\rho}\frac{\epsilon_t}{\rho}.$$

5.3 Endogenous growth with two technologies[1]

1. Bellman

$$V(k,l) = \max_{c,\phi} \ln c + \beta V\left(A\phi k + B\left[(1-\phi)k\right]^{\alpha} l^{1-\alpha} + (1-\delta)k - c, le^{g}\right) \tag{7}$$

subject to $0 \leq \phi \leq 1$. It is important that l is a state variable. If you try to solve the detrended problem, you first have find the balanced growth rates.

2. FOC

$$1/c = \beta V_k(') \tag{8}$$

$$Ak - Bk^{\alpha}l^{1-\alpha}\alpha(1-\phi)^{\alpha-1} \leq 0 \tag{9}$$

with equality if $\phi > 0$.

Envelope: Let

$$R(k/l,\phi) = A\phi + \alpha B(1-\phi)^{\alpha}(k/l)^{\alpha-1} + 1 - \delta \tag{10}$$

denote the "rate of return." Note: If both sectors are used, then

$$R(k/l,\phi) = A + 1 - \delta = \alpha B(k/l)^{\alpha-1} + 1 - \delta \tag{11}$$

Then the envelope condition can be written as

$$V_k = R(k/l,\phi) \tag{12}$$

Euler is standard:

$$1 + g(c) = \beta R(k/l,\phi) \tag{13}$$

[1]Based on UCLA qualifying exam 2014.

3. The Ak technology is used if the marginal product of capital in the A sector is larger than in the B sector when $\phi = 0$. The cutoff value of capital satisfies $A = \alpha B \left(k^*/l\right)^{\alpha-1}$.

4. The balanced growth rates are standard. If the Ak sector is used, $g_c = g_k = \beta\left(A + 1 - \delta\right) - 1$. If only the B sector is used, we live in a Ramsey model with population growth, so that

$$g_c = g_k = g \tag{14}$$

5. The Ak technology is used asymptotically, if, as $k \to \infty$, k/l remains above k^*/l . This can only happen if capital grows faster than labor. Hence we need $\beta\left(A + 1 - \delta\right) > 1 + g$. In that case, $\phi \to 0$ as $t \to \infty$.

5.4 Recursive Competitive Equilibrium

1. Aggregate state: $\hat{z} = z_{t-1}$, joint distribution of $k_{j,t}, c_{j,t-1}$. Call that distribution S. Because of the timing, we also need z_t.

2. Firms solve standard problems.

   (a) Consumption sector:

   $$\max_{K_c, H_c} p\left(S, z\right) F\left(K_c, z H_c\right) - q\left(S, z\right) K_c - w_c\left(S, z\right) H_c \tag{15}$$

   (b) Investment goods sector

   $$\max_{K_i, H_i} G\left(K_i, H_i\right) - q\left(S, z\right) K_i - w_i\left(S, z\right) H_i \tag{16}$$

   where I have imposed that q cannot differ across sectors. Wages do differ because workers cannot move across sectors once z has been revealed.

3. Households:

   (a) One key item: prices are stochastic. The budget constraint is

   $$k_j' + c_j = q\left(S, z\right) k_j + w_c\left(S, z\right) h_{c,j} + w_i\left(S, z\right) h_{i,j} \tag{17}$$

   (b) Bellman

   $$V\left(k, \hat{c}; S, \hat{z}\right) \max_{c, h_c, h_i} u\left(c - b\hat{c}, h_i + h_c\right) + \beta \sum_n \Pr\left(z_n | \hat{z}\right) V\left(k', c; S', z_n\right) \tag{18}$$

   subject to budget constraint (with z_n for z) and law of motion for S: $S' = \Lambda\left(S, z'\right)$. Note that households do not choose a single k' but rather one k' for each realization of z.

   (c) Recursive CE:

i. household policy functions, such as $c(k, \hat{c}, S, \hat{z})$, and value function that "solve" the Bellman equation
ii. firm policy functions, such as $K_c(S, z)$.
iii. price functions, such as $q(S, z)$
iv. market clearing conditions
    A. goods: resource constraints
    B. labor: $H_c(S, z) = \int_0^1 h_c(k_j, \hat{c}_j, S, \hat{z})$ for all z (because supply is fixed); same for H_i
    C. capital rental: $K_c(S, z) + K_i(S, z)$ equals integral over individual capital stocks using marginal density as weights.
v. law of motion for aggregate state: $S' = G(S, \hat{z}, z)$
vi. law of motion for $\hat{z}$: $\hat{z}' = z$.
vii. transition matrix for z (from Markov process)
viii. consistency of expectations

End of exam.