Macroeconomics Qualifying Examination

January 2016

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **3** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 OLG Model With Imperfect Capital Markets

The world consists of two countries, North and South. The setup in the two countries are identical, except for the value of the capital wedges (or tax rates) τ and τ^*. Southern variables are denoted with an asterisk $*$. Each country is characterized by the following.

Demographics: In each period, a unit mass of persons are born. Each person lives for 2 periods.

Endowments: Young agents are endowed with one unit of work time. The initial old are endowed with K_0 units of capital in the North and K_0^* in the South.

Preferences: Agents only value consumption C when old.

Technologies: The resource constraint is given by

$$Y_t = K_t^\alpha L_t^{1-\alpha} = K_{t+1} + C_t + X_t + Z_t \tag{1}$$

X denotes net exports. Z is the amount of output that vanishes from the economy due to the "capital wedge" τ (think of it as tax revenue that is eaten by the government). Capital depreciates completely after one period.

Markets (all competitive): labor rental (wage W), capital rental $\left(R^k\right)$, bonds (return R), goods (numeraire). Goods and bonds are traded internationally. Capital and labor are traded only domestically.

Capital wedges: The wedges τ act as capital income taxes. If firms pay rental price R^k, households receive $(1-\tau)\,R^k$. Therefore, $Z_t = \tau R_t^k K_t$.

Notation: As usual, K_t is accumulated in $t-1$ and used in production in t, paying rental price R_t^k. Bond holdings are denoted as debts D_t, which was borrowed in $t-1$ and pays R_t in t.

Household problem: A young agent in period t chooses capital and debt to maximize old age consumption:

$$\max_{K_{t+1}\geq 0, D_{t+1}} C_{t+1}$$

subject to the budget constraint in period t:

$$K_{t+1} = W_t + D_{t+1}$$

and the budget constraint in period $t+1$:

$$C_{t+1} = (1-\tau)R_{t+1}^k K_{t+1} - R_{t+1} D_{t+1}.$$

Questions:

1. **Closed economy:** Assume that the two countries are closed, so agents can only borrow or lend domestically.

   (a) What is the credit-market clearing condition in each country?

   (b) Define a competitive equilibrium in each economy.

   (c) Find the steady state capital stock K and interest rate R in the North.

   (d) Find the steady state capital stock K^* and interest rate R^* in the South.

   (e) Assume $\tau^* > \tau$. Compare interest rates R and R^* ($>$, $<$, or $=$). Show the math and explain your economic intuition.

2. **Open economies:** Assume that the two countries are financially integrated, so that agents can borrow or lend in a perfectly integrated credit market.

   (a) What is the credit market clearing condition?

   (b) Define a competitive equilibrium in the globalized economy.

   (c) Find the steady state capital stocks K, K^*, and the interest rate R. [Hint: K and K^* solve a system of two equations: the sum of the budget constraints of the young in the two countries and the interest-rate parity condition.]

   (d) Let R^{aut} denote the steady state interest rate when the country is closed (in autarky). Let R^{open} be the world's interest rate when the two economies are open (financially integrated). [Note that you have solved for all of these interest rates already.] Compare R^{aut}, R^{aut*}, and R^{open}. Show the math and explain your economic intuition.

   (e) Find the steady state debt stocks D and D^*.

   (f) Which country is the net debtor? Explain your economic intuition.

   (g) What is the steady state current account in each country (recall that the current account is defined as the change in the net foreign asset position)? Explain your economic intuition.

2 AK Model With Different Discount Factors

Demographics: There are two types of agents, $j \in \{1, 2\}$. Each type has unit mass. Each agent lives forever.

Preferences:

$$\sum_{t=0}^{\infty} \beta_j^t \frac{c_{jt}^{1-\sigma}}{1-\sigma} \tag{2}$$

with $\beta_1 > \beta_2$.

Endowments: $k_{1,0} = k_{2,0}$.

Technology: $K_{t+1} = AK_t - \sum_{j \in \{1,2\}} c_{j,t}$.

Markets: capital rental (q_t), goods (numeraire).

Questions:

1. Define a competitive equilibrium.
2. Which of the agents consumes more in $t = 0$?
3. What happens to the ratio $c_{1,t}/c_{2,t}$ as $t \to \infty$?

Prove your answers.

3 Indivisible labor

Demographics: There is a unit measure of identical, infinitely lived households.

Preferences: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - e_t h_t)$ where c is consumption, e is work effort, and h is hours worked.

Endowments: At $t = 0$: k_0. In each period: 1 unit of work time.

Technology: $k_{t+1} = (1 - \delta) k_t + z_t f(k_t, l_t)$ where l_t is aggregate work effort: $l_t = \int e_{it} h_{it} di$.

Indivisible labor: an individual can work either $h = 0$ or $h = 1$ hours.

The technology shock z_t follows a Markov chain with transition matrix Π.

Questions:

1. State the planner's dynamic program. In each period, the planner chooses how many individuals work (n_t). He must choose this before z_t is known. He chooses everything else (consumption, investment, effort) after z_t has been revealed.
   Hint: Think of the planner as choosing n_{t+1} in period t.
2. Derive the first-order conditions and envelope conditions.
3. Simplify these conditions to arrive at 4 conditions that can be interpreted as equating the marginal benefits and costs of changing one of the choice variables. Explain these conditions in words.

End of exam.

4 Answers

4.1 OLG Model with Imperfect Capital Markets

1. Closed economy:

   (a) $D_t = D_t^* = 0$

   (b) Given initial K_0, an equilibrium in the closed North consists of quantities $\{K_{t+1}, D_{t+1}, C_t, Z_t\}_{t\geq 0}$ and prices $\{R_t, R_t^k, W_t\}_{t\geq 0}$ that satisfy

      i. agents' optimization: budget constraints and no arbitrage $R_t = (1-\tau)\, R_t^k$
      ii. firms' optimization: $W_t = (1-\alpha)K_t^\alpha$ and $R_t^k = \alpha K_t^{\alpha-1}$
      iii. credit market clearing $D_t = 0$
      iv. goods market clearing (resource constraint)
      v. definition of Z (and of course $X = 0$).

   (c) Autarky steady-state capital stock solves $K = W = (1-\alpha)K^\alpha$. Hence $K^{aut} = (1-\alpha)^{\frac{1}{1-\alpha}}$. Since $R = (1-\tau)R^k = (1-\tau)\alpha K^{\alpha-1}$, it follows that

   $$R^{aut} = (1-\tau)\frac{\alpha}{1-\alpha}.$$

   (d) Symmetrically, $K^{aut*} = (1-\alpha)^{\frac{1}{1-\alpha}}$ and $R^{aut*} = (1-\tau^*)\frac{\alpha}{1-\alpha}$.

   (e) $R^* < R$ because $\tau^* > \tau$. Intuitively, the capital wedge in the less financially developed South is larger, leading to a lower autarky interest rate.

2. Open economies:

   (a) $D_t + D_t^* = 0$

   (b) Given initial K_0, K_0^* an equilibrium consists of quantities $\{K_{t+1}, D_{t+1}, C_t, X_t, Z_t\}_{t\geq 0}$ and prices $\{R_t, R_t^k, W_t\}_{t\geq 0}$ (for both countries) that satisfy

      i. agents' optimization (as before)
      ii. firms' optimization (as before)
      iii. interest rate parity $R_t = R_t^*$
      iv. credit market clearing $D_t + D_t^* = 0$
      v. goods market clearing (resource constraints)
      vi. definitions of Z
      vii. identity $X = -X^*$.

   (c) Open steady state capital stocks: Add the young budget constraints and apply the interest-rate parity condition:

   $$\begin{aligned} K + K^* &= (1-\alpha)(K^\alpha + K^{*\alpha}) \\ (1-\tau)K^{\alpha-1} &= (1-\tau^*)K^{*\alpha-1} \end{aligned}$$

Hence

$$K = \left((1-\alpha)\frac{1+\Delta^\alpha}{1+\Delta}\right)^{\frac{1}{1-\alpha}}$$
$$K^* = \Delta \cdot K$$

where $\Delta \equiv \left(\frac{1-\tau^*}{1-\tau}\right)^{\frac{1}{1-\alpha}}$. Therefore:

$$R = (1-\tau)\alpha K^{\alpha-1} = (1-\tau)\frac{\alpha}{1-\alpha}\frac{1+\Delta}{1+\Delta^\alpha}.$$

(d) From answers of previous questions, it follows that $R^{aut*} < R^{open} < R^{aut}$. Intuitively, financial integration of the South with a more financially developed North raises the interest rate in the South, explaining $R^{aut*} < R^{open}$. The reverse holds for the North, explaining $R^{open} < R^{aut}$.

(e) $D = K - W = K\left(1 - \frac{1-\alpha}{\alpha}\frac{1}{(1-\tau)}R\right) = K\left(1 - \frac{R}{R^{aut}}\right)$. And $D^* = -D$.

(f) Since $R < R^{aut}$, it follows that $D > 0 > -D^*$. So the North is a net debtor. Intuitively, the North is more financially developed, leading to a higher autarky interest rate. Financial integration then leads to capital flow from South to North, until the interest rates equalize. This leads to the North being a net debtor.

(g) Current account is defined as the change in net asset position $CA_t = (-D_t) - (D_{t-1})$. In steady state, it is zero, because net asset position does not change in steady state.

4.2 Answer: AK Model With Different Discount Factors[1]

1. CE: $\{c_{jt}, k_{jt}, K_t, q_t\}$ that satisfy:
   (a) household: Euler, budget constraint, boundary conditions.
   (b) firm: $q_t = A$.
   (c) market clearing: goods (RC), capital: $K_t = \sum_j k_{jt}$.

2. Household solution consists of Euler

$$\frac{c_{t+1,j}}{c_{t,j}} = (\beta_j q)^{1/\sigma} \tag{3}$$

and lifetime budget constraint

$$\sum_{t=0}^{\infty} c_{jt}/A^t = k_{j0} \tag{4}$$

Since the more patient household has faster consumption growth and the same present value of consumption as the less patient agent, his initial consumption must be lower.

3. Directly from the Euler equation: the ratio goes to ∞.

[1] Based on Minnesota Preliminary Exam Spring 2007.

4.3 Indivisible Labor[2]

1. Bellman equations

   State: z, n, k

   Controls: e, n', k', c_1, c_2

$$V(k,n,z) = \max_{e,n',c_1,c_2} nu(c_1, 1-e) + (1-n)u(c_2,1) + \beta \mathbb{E} V((1-\delta)k + zf(k,ne) - nc_1 - (1-n)c_2, n', z') \tag{5}$$

2. FOC:

$$u_1(1) = u_1(2) = \beta \mathbb{E} V_1(.')$$
$$nu_2(1) = \beta \mathbb{E} V_1(.')\{zf_2 n\} \tag{6}$$
$$0 = \beta \mathbb{E} V_2(.') \tag{7}$$

   Envelope:

$$V_1 = \beta \mathbb{E} V_1(.')\{1 - \delta + f_1\} \tag{8}$$
$$V_2 = u(1) - u(2) + \beta \mathbb{E} V_1(.')\{zf_2 e\} \tag{9}$$

3. Simplify:

$$u_1(1) = u_1(2) \tag{10}$$
$$u_1(1) = \beta \mathbb{E} u_1(1') z' f_1(.') \tag{11}$$
$$nu_2(1) = zf_2 n u_1(1) \tag{12}$$
$$0 = \beta \mathbb{E}\{u(1') - u(2') + z' f_2(.') e' u_1(1')\} \tag{13}$$

   Interpretation:

   (a) Move a unit of consumption from type 1 to type 2. Both must have the same marginal utility.

   (b) A standard Euler equation still holds.

   (c) A static condition that equates the disutility and the utility of increasing effort.

   (d) Increasing n' should not affect future value. The change in value is the utility difference between types plus the utility obtained from consuming the additional output produced by having more working people.

End of exam.

[2] Based on UCLA qualifying exam spring 2005.