Macroeconomics Qualifying Examination

August 2016

Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Nominal Wage Rigidity

Consider a small open economy that exists for two periods, $t = 1$ and $t = 2$. There is a unit measure of households and a unit measure of firms.

In each period, the representative household receives an exogenous endowment y_t^T of a non-storable tradable good. The household can borrow/lend the tradable good at an international interest rate $1 + r$.

In each period the household also supplies one unit of labor inelastically to firms that produce the non-tradable good (which is also non-storable). The nominal wage (in the local currency) is W_t. The nominal profit (in the local currency) of a representative firm is Φ_t.

The price in local currency of the nontradable good and the tradable good are P_t^N and P_t^T, respectively, and are taken as given by households and firms.

The optimization problem of the representative household is:

$$\max_{c_1^T, c_1^N, c_2^T, c_2^N, d_1} (\ln c_1^T + \ln c_1^N) + \beta(\ln c_2^T + \ln c_2^N)$$

subject to the following budget constraints (written in units of the local currency):

$$P_1^T c_1^T + P_1^N c_1^N = P_1^T y_1^T + W_1 h_1 + \frac{P_1^T d_1}{1+r} + \Phi_1$$
$$P_2^T c_2^T + P_2^N c_2^N = P_2^T y_2^T + W_2 h_2 - P_2^T d_1 + \Phi_2$$

The optimization problem of the representative firm that takes prices as given is:

$$\Phi_t = \max_{h_t} P_t^N y_t^N - W_t h_t$$

subject to $y_t^N = h_t^\alpha$.[1]

Given exogenous tradable prices $\{P_t^T\}_{t=1,2}$ and endowments $\{y_t^T\}_{t=1,2}$, an equilibrium consists of prices P_t^N, W_t, profit Φ_t and quantities $y_t^N, c_t^T, c_t^N, d_1, h_1, h_2$ such that

- the quantities solve the optimization problems of the representative household and firm, taking prices as given
- the good markets clear: $c_t^N = y_t^N$ and $c_1^T = y_1^T + \frac{d_1}{1+r}$, $c_2^T = y_2^T - d_1$,
- the labor market clears at $t = 1$: $h_1 = 1$,
- the labor market does not necessarily clear at $t = 2$; instead the following complementary slackness condition and downward-sticky nominal wage condition must hold:

$$(1 - h_2)(W_2 - \gamma W_1) = 0, \tag{1}$$
$$W_2 \geq \gamma W_1 \tag{2}$$

where $\gamma \in [0, 1]$ is an exogenous measure of downward nominal wage rigidity.

[1] Note that, as there is no entry, it is possible to make positive profits in equilibrium.

Throughout, assume for simplicity that $\gamma = 1$, $\beta(1 + r) = 1$, $y_t^T = 1$, and $P_1^T = P_2^T = \mathcal{E}$ (an exogenous positive constant).

Questions:

1. Find

   (a) c_1^T, c_2^T and d_1.

   (b) h_t, P_t^N and W_t. Is there involuntary unemployment?

2. Assume the economy is in the equilibrium as described in part 1 above. However, at $t = 2$, an *unanticipated* shock reduces y_2^T to 1/2. The shock is unanticipated in the sense that, In $t = 1$, agents were certain that $y_2^T = 1$. Find

   (a) c_1^T, c_2^T and d_1.

   (b) h_t, P_t^N and W_t. Is there involuntary unemployment, and why?

2 Two Capital Goods

Consider a closed economy consisting of a unit measure of identical households and a unit measure of identical firms.

Preferences of the representative household are given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where $u(c) = \frac{c^{1-\nu}}{1-\nu}$, $\nu > 0$.

At the beginning of time, households are endowed with two capital stocks: $k_{g,0}$ and $k_{p,0}$.

The resource constraint is

$$c_t + i_{p,t} + i_{g,t} \leq y_t.$$

Output is produced from two capital goods

$$y_t = k_{g,t}^{1-\alpha} k_{p,t}^{\alpha}, \ 0 < \alpha < 1.$$

The capital goods accumulate according to

$$k_{p,t+1} = (1 - \delta_p) k_{p,t} + i_{p,t}$$

and

$$k_{g,t+1} = (1 - \delta_g) k_{g,t} + i_{g,t},$$

where $k_{j,t}$ is the beginning-of-period t stock of capital type $j \in \{g, p\}$, $i_{j,t}$ is period t gross investment, and the depreciation rates are $0 < \delta_j < 1$.

Consumption and capital stock cannot be negative. Investment may be negative.

Questions:

1. State the planner's optimization problem in sequence language.
2. State the necessary and sufficient conditions for the solution to the planner's optimization problem.
3. Show that in any solution to the planner's problem, $k_{g,t}/k_{p,t}$ is a constant over time.
4. State the planner's dynamic programming problem. There should be a single state variable.
5. Derive the first-order conditions.

3 Complete Markets

Demographics: There are 2 types of agents who live forever. Each type has unit mass.

Preferences: Both agents have identical preferences: $U_i = \mathbb{E}\sum_{t=0}^{\infty} \beta^t u\left(c_{it}\right)$ where $u\left(c\right) = c^{1-\sigma}/\left(1-\sigma\right)$.

Endowments: At $t = 0$, agent i is endowed with k_{i0}.

Technology: $k_{t+1} = z_t f\left(k_t\right) - c_{1t} - c_{2t}$ where $f\left(k\right) = k^{\alpha}$. The shock z_t follows a Markov chain with transition matrix Π.

Questions:

1. State the planner's dynamic program. The planner maximizes $\sum_i \lambda_i U_i$, where $\lambda_i > 0$ is the weight the planner attaches to the utility of type i.
2. Characterize its solution.
3. Describe a market economy that decentralizes the planner's solution. What kinds of assets / markets do you need to introduce?

4 Ben-Porath Model

Consider a worker in continuous time who maximizes the present value of lifetime earnings.

The worker lives for a period of length T. At birth, he is endowed with h_0 units of human capital. At each moment, he can produce additional human capital using time n and goods x as inputs:

$$\dot{h} = G(h, n, x) - \delta h = (nh)^{\alpha} x^{\beta} - \delta h \tag{3}$$

The price of goods is p. Earnings are then given by $wh(1-n) - px$. The present value of earnings is $\int_0^T e^{-rt} [wh_t(1-n_t) - px_t]\, dt$. Assume an interior solution.

Questions:

1. State the Hamiltonian. Call the co-state q.
2. Derive the first-order conditions and boundary conditions.
3. Verify that the solution to q is given by

   $$q_t = \frac{w}{r+\delta} \left[1 - e^{-(r+\delta)(T-t)}\right] \tag{4}$$

   That is, show that (4) solves the differential equation that governs q and its boundary conditions.

   Explain what (4) means in words.
4. Derive a closed form solution for human capital investment hn. Hint: According to (4), q_t does not depend on current value of h_t.

End of exam.

5 Answers

5.1 Two-period SOE New Keynesian model with nominal wage rigidity

1. (a) The FOC of d_1 with $\beta(1+r) = 1$ combined with the market clearing conditions for the tradable good implies that

$$c_1^T = c_2^T = c^T \equiv \frac{y_1^T + \frac{y_2^T}{1+r}}{1 + \frac{1}{1+r}} = 1. \tag{5}$$

It follows that $\frac{d_1}{1+r} = c^T - y_1^T = 0$.

(b) The FOCs of consumption imply that $\frac{\log'(c_t^N)}{\log'(c_t^T)} = \frac{P_t^N}{P_t^T}$, or $P_t^N c_t^N = \mathcal{E} c_t^T = \mathcal{E}$.

To find the equilibrium wage, we start by trying a full employment equilibrium, so that $h_2 = 1$. Nontradable goods market clearing implies that $c_t^N = h_t^\alpha = 1$, and hence the price of the nontradable good is $P_t^N = \mathcal{E}$. Substitute $h_t = 1$ into the FOC of firms and get the nominal wage:

$$W_t = P_t^N \alpha = \mathcal{E}\alpha.$$

This satisfies the wage rigidity condition. Hence, we have found an equilibrium.

2. (a) c_1^T and d_1 are as in part 1, because the shock is unanticipated. Tradable consumption at $t = 2$ is smaller: $c_2^T = y_2^T - \bar{d} = \frac{1}{2} - 0 = \frac{1}{2}$.

(b) $h_1 = 1$, $P_1^N = \mathcal{E}$, $W_1 = \mathcal{E}\alpha$ as the shock is unanticipated.

The FOCs of consumption imply that $\frac{\log'(c_2^N)}{\log'(c_2^T)} = \frac{P_2^N}{P_2^T}$, or $\frac{c_2^T}{c_2^N} = \frac{P_2^N}{\mathcal{E}}$, or

$$P_2^N c_2^N = \mathcal{E} c_2^T = \frac{1}{2}\mathcal{E}. \tag{6}$$

To find W_2, let us first find the wage associated with full employment. Assuming $h_2 = 1$, then $c_2^N = h_2^\alpha = 1$, and hence from equation (6) above we get $P_2^{N,full} = \frac{1}{2}\mathcal{E}$. On the other hand, the FOC of firms with $h_2 = 1$ implies $W_2^{full} = P_2^{N,full}\alpha = \frac{1}{2}\mathcal{E}\alpha$. However, this violates the nominal rigidity constraint that the wage in $t = 2$ cannot be below $W_1 = \mathcal{E}\alpha$. Hence, in equilibrium

$$W_2 = W_1 = \mathcal{E}\alpha.$$

P_2^N and h_2 solve the FOC of firm: $P_2^N \alpha h_2^{\alpha-1} = W_2$, and (6) above with $c_2^N = h_2^\alpha$, i.e., $P_2^N h_2^\alpha = \frac{\mathcal{E}}{2}$. Dividing the two equations yields $\frac{h_2}{\alpha} = \frac{\mathcal{E}/2}{W_2} = \frac{\mathcal{E}/2}{\mathcal{E}\alpha} = \frac{1}{2\alpha}$, or

$$h_2 = \frac{1}{2}.$$

It then follows that $c_2^N = (1/2)^\alpha$.

There is involuntary unemployment in equilibrium because the unanticipated shock lowers the aggregate demand for the nontradable good. Without rigidity, nominal wage would fall to clear the labor market. But since nominal wage cannot fall, there is rationing in the labor market.

5.2 Answer: Two Capital Goods

1. The planner's problem is: choose $\{c_t, k_{g,t+1}, k_{p,t+1}\}_{t\geq 0}$ to maximize the discounted utility of the household. After substituting out consumption using the resource constraint, the problem becomes:

$$\max_{k_{g,t+1},k_{p,t+1}} \sum_{t=0}^{\infty} \beta^t u[k_{g,t}^{1-\alpha} n^{1-\alpha} k_{p,t}^{\alpha} + (1-\delta_g)k_{g,t} + (1-\delta_p)k_{p,t} - k_{g,t+1} - k_{p,t+1}],$$

subject to the object in square brackets (consumption) as well as capital $k_{g,t}, k_{p,t}$ being non-negative at all periods.

2. The FOCs are:

$$\begin{aligned} u_{c,t} &= \beta u_{c,t+1} \cdot [f_{k_{p,t+1}} + 1 - \delta_p] \\ u_{c,t} &= \beta u_{c,t+1} \cdot [f_{k_{g,t+1}} + 1 - \delta_g], \end{aligned}$$

at all $t \geq 0$. Given the functional forms, the FOCs can be written as:

$$\begin{aligned} (\frac{c_{t+1}}{c_t})^{\nu} &= \beta \left[\alpha k_{g,t+1}^{1-\alpha} (\frac{n}{k_{p,t+1}})^{1-\alpha} + 1 - \delta_p\right] \\ &= \beta \left[(1-\alpha) k_{g,t+1}^{-\alpha} n^{1-\alpha} k_{p,t+1}^{\alpha} + 1 - \delta_g\right]. \end{aligned}$$

The conditions, along with the laws of motion for the capital stocks, the resource constraint, and a transversality condition $\lim_{T\to\infty} \beta^T u'(c_T)[k_{g,T} + k_{p,T}] = 0$ are necessary and sufficient conditions to pin down a solution to the planner's optimization problem.

3. Combining the FOCs of the planner yields:

$$\begin{aligned} &\beta[\alpha n^{1-\alpha} \left(\frac{k_{g,t+1}}{k_{p,t+1}}\right)^{1-\alpha} + 1 - \delta_p] \\ =&\beta[(1-\alpha) n^{1-\alpha} \left(\frac{k_{p,t+1}}{k_{g,t+1}}\right)^{\alpha} + 1 - \delta_g] \end{aligned}$$

which implies that $\frac{k_{p,t+1}}{k_{g,t+1}}$ is time-invariant.

4. Bellman equation:

$$V(k) = \max u\left([vk]^{1-\alpha} [(1-v)k]^{\alpha} + (1-\delta_g) vk + (1-\delta_p)(1-v)k - k'\right) + \beta V(k') \quad (7)$$

where v is the fraction of the capital stock employed as type g in the period.

5. FOCs

$$(1-\alpha)\left(\frac{v}{1-v}\right)^{-\alpha} + \alpha\left(\frac{v}{1-v}\right)^{1-\alpha} = \delta_p - \delta_g \quad (8)$$

which implies the same ratio of capital stocks as derived for the sequence problem, and

$$u'(c) = \beta u'(c')\left[v^{1-\alpha}(1-v)^{\alpha} + (1-\delta_g) v + (1-\delta_p)(1-v)\right] \quad (9)$$

5.3 Answer: Complete Markets

1. $V(k,j) = \max \sum_i \lambda_i u(c_i) + \beta \mathbb{E} V(k',j')$ subject to the resource constraint. j indexes the z state.

2. Characterization:

   (a) ratio of marginal utilities is constant across agents (static condition)

   (b) Euler equation holds for each agent: $u'(c_i) = \beta \mathbb{E} u'(c_i') z' f'(k')$

3. Market economy:

   Perhaps the easiest answer would be to construct an Arrow-Debreu economy. But we'll set up a sequence of markets equilibrium with Arrow securities instead.

   Markets: labor (wage w), capital rental (price q), Arrow security that pays out 1 unit of consumption in state j tomorrow (price p_j).

   Household

   $$W(k,x,j) = \max u(c) + \beta \mathbb{E} W(k',x',j') \tag{10}$$

   subject to

   $$k' + c + \sum_{j'} x'_{j'} p_{j'} = w + qk + x_j \tag{11}$$

   Note: If we omit the Arrow securities, we cannot ensure that households choose the same consumption growth rate state-by-state.

   Equilibrium: stochastic sequences $\{c_{i,t}, k_{i,t}, x_{i,j,t}; w_t, q_t, p_{j,t}\}$ that satisfy

   (a) household

   (b) firm (trivial)

   (c) market clearing: resource constraint; $k_t = \sum_i k_{i,t}$; $\sum_i x_{i,j,t} = 0$

5.4 Answer: Ben-Porath Model[2]

1. Hamiltonian:

   $$\Gamma = wh(1-n) - px + q[G(h,n,x) - \delta h] \tag{12}$$

2. FOCs:

   $$whn = q\alpha G \tag{13}$$
   $$p_w x = q\beta G \tag{14}$$
   $$\dot{q} = rq - q\{\alpha G/h - \delta\} - w(1-n) \tag{15}$$
   $$q_T = 0 \tag{16}$$

[2]This problem is based on Manuelli & Seshadri (AER 2014).

3. Verify: Simplify the first-order conditions to $\dot{q} = (r + \delta)\, q - w$. Differentiate the proposed solution:

$$\dot{q} = -\frac{w}{r+\delta} e^{-(r+\delta)(T-t)} (r + \delta) = (r + \delta) [q - w/ (r + \delta)] \tag{17}$$

and the solution also satisfies $q_T = 0$.

Interpretation: The marginal value of human capital is the present value of the earnings it generates.

4. Start from the definition of G and sub in the static condition

$$px = whn\beta/\alpha \tag{18}$$

That yields

$$G = (nh)^{\alpha+\beta} \left[\frac{w}{p}\frac{\beta}{\alpha}\right]^{\beta} \tag{19}$$

Sub that into the first order condition for n and rearrange to obtain

$$(nh)^{1-\alpha-\beta} = \frac{q\alpha}{w} \left[\frac{w}{p}\frac{\beta}{\alpha}\right]^{\beta} \tag{20}$$

This is, really, the key result of the paper. The sensitivity of human capital investment with respect to technology parameters and prices depends on the returns to scale of the human capital technology $(1 - \alpha - \beta)$.

End of exam.