Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **four** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Price Level Interest Rate Rule

Consider an economy described by the following equilibrium conditions

$$\widetilde{y}_t = E_t \{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n)$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

Questions:

1. What are these equations? Where do they come from?
2. Show that the interest rate rule
$$i_t = r_t^n + \phi_p \widehat{p}_t$$
where $\widehat{p}_t \equiv p_t - p^*$, where p^* is a price level target, generates a unique stationary equilibrium, if and only if $\phi_p > 0$. *In case you have issues with the math state clearly how you would solve the problem.*

2 Inflation Persistence and Monetary Policy

In the presence of partial price indexation by firms, the second-order approximation to the household's welfare loss function is of the form:

$$\frac{1}{2} E_0 \sum \beta^t \left[\alpha_x x_t^2 + (\pi_t - \gamma \pi_{t-1})^2\right]$$

where x is the output gap and γ denotes the degree of price indexation to past inflation. The equation describing the evolution of inflation is given by

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \beta E_t\{(\pi_{t+1} - \gamma \pi_t)\} + u_t$$

where u_t represents an exogenous *i.i.d.* cost-push shock.

Questions:

1. Determine the optimal policy under discretion.
2. Determine the optimal policy under commitment.
3. Discuss how the degree of indexation γ affects the optimal responses to a transitory cost-push shock under discretion and commitment.

3 Stochastic CIA Model

Demographics: There is a single representative household who lives forever. Time is discrete.

Preferences: $\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[u\left(c_t\right)+v\left(1-l_t\right)\right]$ where c is consumption.

Endowments: At the beginning of time, the household is endowed with $m_0=\bar{m}_0$ units of fiat money (pieces of green paper), and $b_0=\bar{b}_0=0$ one period real discount bonds.

Technology: $c_t=l_t$.

Government: The government hands out money as lump-sum transfers: $\bar{m}_{t+1}=\bar{m}_t+\tau_t$. τ is stochastic with transition matrix $G\left(\tau'|\tau\right)$.

Markets: goods (price p), money (numeraire), bonds (pq), labor (w).

Cash-in-advance constraint: $p_t c_t \leq m_t+\tau_t$.

Questions:

1. State the household's dynamic program. Hint: the budget constraint is
$$pc+pqb'+m'=wl+m+\tau+pb \tag{1}$$
2. State the first-order and envelope conditions.
3. State a solution in sequence language.
4. Interpret the first-order conditions.
5. Define a recursive competitive equilibrium. For simplicity, assume that the CIA constraint never binds.

4 Quality Ladders

Demographics: Time is continuous. There is a representative household of mass L who lives forever.

Preferences: $U=\int_0^{\infty}e^{-\rho t}u\left(c_t\right)$ where $u\left(c\right)=c^{1-\epsilon}/\left(1-\epsilon\right)$ with $\epsilon,\rho>0$. c is consumption.

Endowments: Households have $\bar{L}$ units of work time at each instant. At the beginning of time, households own all intermediate goods producing firms (values V_{it}) and capital K_0.

Technologies:

1. Final goods (numeraire): $Y_t=\int_0^1 A_{it}x_{it}^{\alpha}L_t^{1-\alpha}di=C_t+\dot{K}_t+Z_t$, where A is an (endogenous) quality parameter, x denotes intermediate inputs (rental prices p_{it}), and L is work time (rental price w_t).

2. Intermediate goods: $x_{it} = K_{it}/A_{it}$ where K is capital used in production of intermediates (rental price r_t).

3. Innovation: Spending n_t units of final goods (a flow) yields an innovation with probability $\lambda n_t / A_t^{max}$. The innovation yields quality A_t^{max}.

4. A_t^{max} evolves according to $g\left(A_t^{max}\right) = \sigma \lambda n_t$. This is an externality. Agents take the path of A_t^{max} as given.

Markets: Final goods and labor markets are competitive. Intermediate goods producers have monopolies. There is free entry into innovation.

As usual, the household problem is complicated. From it, we just take the usual Euler equation: $g(c) = (r - \rho)/\epsilon$.

Questions:

1. State and solve the problem of the final goods producer.

2. State and solve the problem of an intermediate goods producer. Note that intermediates are not durable (even though K is). Show that $p_{it} = A_{it} r_t / \alpha$.

3. State and explain the free entry condition. Show that $V_{it} = A_t^{max}/\lambda$ for any good where innovation occurs.

4. State the market clearing conditions.

5. From now on assume balanced growth. Show that profits at the time of innovation are given by $\pi_{it} = \frac{1-\alpha}{\alpha} r \left(K_t/A_t\right) A_t^{max}$ where $A_t = \int_0^1 A_{it}$. Hint: $x_{it} = x_t$.

   Note that this implies that profits are constant until the monopoly is destroyed by another innovation (because K/A is constant over time).

6. Show that the value of the firm is given by $V_{it} = V_t = \frac{1-\alpha}{\alpha} r \left(K/A\right) A_t^{max} / \left(r + \phi\right)$, where ϕ is the flow probability that an innovation destroys the monopoly.

7. Balanced growth requires that $\phi = \lambda n$. Derive $1/\lambda = \frac{1-\alpha}{\alpha} r \left(K/A\right) / \left(r + \lambda n\right)$ and $r = \sigma \lambda \epsilon n - \rho$.

8. Together with a 3rd equation, these two could be solved for $r, n, K/A$. Briefly, where would you get the 3rd equation from (you don't have to derive it)?

End of exam.

5 Answers

5.1 Price Level Interest Rate Rule

1. Equation 1 is the Dynamic IS curve which is the solution to the households problem. We also assume market clearing $Y = C$. Equation 2 is the NKPC derived from the fiorm's problem and Calvo price setting.

2.

$$\widetilde{y}_t = E_t\left\{\widetilde{y}_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}\right\} - r_t^n\right)$$

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa \widetilde{y}_t$$

substitute

$$\begin{aligned} i_t &= r_t^n + \phi_p \widehat{p}_t \\ &= r_t^n + \phi_p(p_t - p_{t-1} + p_{t-1} - p^*) \\ &= r_t^n + \phi_p \pi_t + \phi_p \widehat{p}_{t-1} \end{aligned}$$

get

$$\widetilde{y}_t = E_t\left\{\widetilde{y}_{t+1}\right\} - \frac{1}{\sigma}\left(\phi_p \pi_t - E_t\left\{\pi_{t+1}\right\} - \phi_p \widehat{p}_{t-1}\right)$$

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa \widetilde{y}_t$$

In Matrix

$$\begin{bmatrix} 1 & \frac{\phi_p}{\sigma} \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} E_t\left\{\widetilde{y}_{t+1}\right\} \\ E_t\left\{\pi_{t+1}\right\} \end{bmatrix} + \begin{bmatrix} \frac{\phi_p}{\sigma}\widehat{p}_{t-1} \\ 0 \end{bmatrix}$$

....

$$\begin{aligned} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} &= \frac{1}{1 + \frac{\kappa\phi_p}{\sigma}} \begin{bmatrix} 1 & \frac{1}{\sigma} - \frac{\beta\phi_p}{\sigma} \\ \kappa & \frac{\kappa}{\sigma} + \beta \end{bmatrix} \begin{bmatrix} E_t\left\{\widetilde{y}_{t+1}\right\} \\ E_t\left\{\pi_{t+1}\right\} \end{bmatrix} + V_t \\ &= \Psi_t \begin{bmatrix} E_t\left\{\widetilde{y}_{t+1}\right\} \\ E_t\left\{\pi_{t+1}\right\} \end{bmatrix} + V_t \end{aligned}$$

show that roots/eigenvalues of Ψ_t inside unit circle. The restriction on the coefficients are enough.

5.2 Inflation Persistence and Monetary Policy

1. Under discretion minimize the period by period problem

$$\frac{1}{2}\left[\alpha_x x_t^2 + \left(\pi_t - \gamma\pi_{t-1}\right)^2\right]$$

subject to

$$\pi_t - \gamma\pi_{t-1} = \kappa x_t + -\beta\gamma\pi_t + v_t$$

FOC

$$\begin{aligned} \alpha_x x_t + \lambda_t \kappa &= 0 \\ \pi_t - \gamma\pi_{t-1} + \lambda_t(-\beta\gamma - 1) &= 0 \end{aligned}$$

substitute into constraints

$$\pi_t - \gamma\pi_{t-1} = \frac{\alpha_x(\beta\gamma+1)}{\alpha_x(\beta\gamma+1) - \kappa^2}\beta E_t\left(\pi_{t+1} - \gamma\pi_t\right) + \frac{\alpha_x(\beta\gamma+1)}{\alpha_x(\beta\gamma+1) - \kappa^2}u_t$$

Steps: solve for π_t (in terms of $E_t\left(\pi_{t+1}\right)$ and u_t); iterate that equation forward; the coefficient on $E_t\left(\pi_{t+1}\right)$ goes to zero.

2. Under discretion minimize

$$\frac{1}{2}E_0\sum\beta^t\left\langle\left[\alpha_x x_t^2 + \left(\pi_t - \gamma\pi_{t-1}\right)^2\right] + \lambda_t\left[\kappa x_t + \beta\left(\pi_{t+1} - \gamma\pi_t\right) - \pi_t + \gamma\pi_{t-1}\right]\right\rangle$$

Find FOCs with respect to x_t and π_t (realize there is a π_t present from the previous period; see the π_{t+1} term). Then foolow the above steps

3. Once you solve the the model in terms of u_t what you have are the impulse responses. State the effects of γ on said coefficient.

5.3 Answer: Stochastic CIA model

1. Household

$$V\left(m,b,S\right) = u\left(\frac{w}{p}l + \frac{m+\tau}{p} + b - \frac{m'}{p} - qb'\right) + v\left(1-l\right) + \beta\mathbb{E}V\left(m',b',S'\right) + \lambda\left(\frac{m+\tau}{p} - l\right) \quad (2)$$

with aggregate state $S = \left(\bar{m}, \bar{b}, \tau\right)$ obeying some law of motion $S' = F\left(S|\tau'\right)$.

2. FOC:

$$u'w/p = v' + \lambda \quad (3)$$

$$u'/p = \beta\mathbb{E}V_m\left(.'\right) \quad (4)$$

$$u'q = \beta\mathbb{E}V_b\left(.'\right) \quad (5)$$

Envelope:

$$V_m = u'/p + \lambda/p \quad (6)$$

$$V_b = u' \quad (7)$$

3. Solution in sequence language: Stochastic sequences $\{c, l, m, b, \lambda\}$ that solve:

   (a) first-order conditions:

$$u'w/p = v' + \lambda \tag{8}$$
$$u' = \beta\mathbb{E}\left\{\left(u'\left(.'\right) + \lambda'\right)p/p'\right\} \tag{9}$$
$$u' = \beta\mathbb{E}\left\{u'\left(.'\right)/q\right\} \tag{10}$$

   (b) $\lambda\left(\frac{m+\tau}{p} - l\right) = 0$ with $\lambda \geq 0$ and CIA

   (c) boundary conditions: m_0, b_0 given. TVC: $\lim_{t\to\infty}\mathbb{E}\beta^t u'\left(c_t\right)\left(m_t/p_t + b_t\right) = 0$.

4. Interpretation: with $\lambda = 0$, the first-order conditions are a standard static consumption-leisure condition and 2 Lucas asset pricing equations. λ captures the fact that holding money has an additional benefit and that consuming has an additional cost (in the CIA constraint).

5. Market clearing:

   (a) goods: $c = l$

   (b) bonds: $b = 0$

   (c) money: $m = \bar{m}$

6. RCE:

   Objects:

   (a) $l\left(m, b, S\right), m'\left(m, b, S\right), b'\left(m, b, S\right), V\left(m, b, S\right)$

   (b) $q\left(S\right), p\left(S\right)$

   (c) $S' = F\left(S|\tau'\right)$

   Conditions:

   (a) household optimality

   (b) market clearing

   (c) government law of motion for $\bar{m}$

   (d) consistency: $\bar{m}' = F_{\bar{m}}\left(S;\tau'\right) + \tau'$; $\bar{b}' = F_{\bar{b}}\left(S;.\right)$; $\tau' = F_{\tau}\left(.;\tau'\right)$. I am inventing notation here, but it's hopefull obvious.

5.4 Answer: Quality Ladders[1]

1. Final goods producer: $\max Y_t - w_t L_t - \int_0^1 p_{it} x_{it} di$.

   FOC: $p_{it} = \alpha Y_t / x_{it} = \alpha A_{it} x_{it}^{\alpha-1} L_t^{1-\alpha}$ and $w_t = (1-\alpha) Y_t / L_t$

2. Intermediates:

   Static problem. Profits: $\pi_{it} = p_{it} x_{it} - r_t A_{it} x_{it}$. FOC: $x_{it} = L\left(\alpha^2 / r_t\right)^{1/(1-\alpha)}$ or $p_{it} = A_{it} r_t / \alpha$. And max profits are $\pi_{it} = A_{it}\left[(1-\alpha)/\alpha\right] r_t x_{it}$.

3. Innovation:

   Free entry equalizes expected profits $\lambda n_t / A_t^{max} V_t = n_t$ or $V_t = A_t^{max} / \lambda$.

4. Market clearing:

   $K_t = \int A_{it} x_{it}$. Labor: $L_t = \bar{L}_t$. Intermediates: implicit. Goods: resource constraint with $Z_t = \int n_{it}$.

5. Profits: Start from the profit equation derived above. Impose $x_{it} = K_t / A_t$ and note that $A_{it} = A_t^{max}$ at the time of innovation.

6. Value of the firm: $V_t = \int_t^\infty e^{-(r+\phi)z} \pi_z dz = \pi_t / (r+\phi)$. Sub in profits.

7. The first equation is just free entry with V_t plugged in. The second equation is the Euler equation:

$$g(C) = \frac{r-\rho}{\epsilon} = g(A) = \sigma \lambda n \tag{11}$$

8. The final equation would have to pin down K/A (intuitively). We would get it from the household's Euler equation plus lifetime budget constraint (which determines C/K.

End of exam.

[1]Based on Zeng, J and H Du, "Allocation of Tax Revenue and Growth Effects of Taxation."