Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **4** questions. Answer all questions.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Number your answers.
- Explain your answers.
- Budget your time wisely. Don't get hung up on one question.
- Good luck!

1 Small Open Economy Real Business Cycles

Notation:

- d_t: debt position chosen at t and due in $t+1$ (negative d_t means saving), r_t: interest rate on debt d_t, c_t: consumption, h_t: hours worked, k_t: capital stock chosen in $t-1$, w_t: wage rate, r_t^k: rental rate of capital, x_t: net exports.
- A_t: productivity

Parameters: $\alpha, \beta, \rho \in (0,1)$; $\zeta, \sigma > 0$; $\bar{D}$. Assumption: $\beta(1+r^*) = 1$

Demographics: There is a representative agent of unit mass who lives forever.

Preferences: $E_0 \sum_{t=0}^{\infty} \beta u(c_t)$ where $u(c) = \ln c$.

Endowments:

- At the beginning of time: k_0 units of capital, d_0 debt.
- In each period: 1 unit of work time.

Technologies: $y_t = A_t k_t^{\alpha} h_t^{1-\alpha}$. $y_t + (1-\delta) k_t = c_t + k_{t+1} + x_t$.

Shocks: $\ln A_{t+1} = \rho \ln A_t + \sigma \epsilon_{t+1}$ where $\epsilon_t \sim N(0,1)$ (iid).

Markets: There are competitive international markets for debt and goods. There are competitive domestic markets for capital and labor.

The debt market features a **risk premium**. If **aggregate** debt is D_t, the interest rate is $r_t = r^* + p(D_t)$ where $p(D) = \zeta \times \left(e^{D-\bar{D}} - 1\right)$. Hence, if $D = \bar{D}$, then $r = r^*$. If $D > \bar{D}$, then $r > r^*$. The household, of course, takes aggregate bond holdings as given.

Budget constraint of representative agent: $c_t + (k_{t+1} - (1-\delta)k_t) + (1+r_{t-1})d_{t-1} = w_t + r_t^k k_t + d_t$.

Questions:

1. Define a competitive equilibrium. Define the current account. You may define the equilibrium in terms of stochastic sequences.
2. Solve for the steady state values of the endogenous variables. Here the assumption that $\beta(1+r^*) = 1$ is important.
3. Log-linearize the model around the steady state.

2 "Liquidity trap" (Eggertsson and Krugman, QJE 2012)

Consider a closed economy.

Notation: C: consumption, D: debt, Y: endowment, r: real interest rate

Parameters: $\beta(i) \in (0,1)$ for $i \in \{b, s\}$

Demographics: There are two representative agents, indexed by $i \in \{b, s\}$. Each has unit mass and lives forever. The two agents are identical, except for their discount factors: $\beta(s) = \beta > \beta(b)$.

Preferences:

$$E_t \sum_{t=0}^{\infty} \beta(i)^t \ln C_t(i)$$

Endowments: Each agent receives $Y/2$ units of goods in every period.

Technologies: Goods are not storable. They can only be eaten.

Markets: There are a competitive market for goods and for one period risk free loans D. $D > 0$ indicates that a household is in debt. Loans are in zero net supply.

Borrowing limits: We assume a financial friction of the simplest possible form. Agents can borrow up to an exogenous amount given by $(1 + r_t)D_t(i) \leq D^{high}$. Assume that the debt limit is smaller than the present discounted value of output of each agent: $D^{high} < \frac{1}{2}\frac{\beta}{1-\beta}Y$.

The budget constraint of each agent is:

$$C_t(i) + (1 + r_{t-1})D_{t-1}(i) = \frac{1}{2}Y + D_t(i).$$

Questions:

1. Find the steady state.

2. *Deleveraging*: Suppose the economy reaches the steady state in period $t = 0$. Then, unexpectedly, in $t = 1$, there is a financial shock that reduces the borrowing limit from D^{high} to $D^{low} \in (0, D^{high})$, that is:

   $$(1 + r_1)D_1(i) \leq D^{low}.$$

   We assume that the borrower must "deleverage," that is, to reduce borrowing, within a single period to the new debt limit; hence, $D_1(b) = D^{low}/(1 + r_1)$.

   (a) Derive $C_1(i)$.

   (b) Derive $C_2(i)$.

   (c) Find r_1. Is r_1 increasing or decreasing in D^{low}? Explain your intuition.

3 Growth Model With Public Capital

Notation: c: consumption, N: hours worked, K: private capital, G: public capital, Y: output, r: rental price of capital, w: wage rate, τ: tax rate.

Parameters: $0 < \beta < 1$, $0 < \alpha < 1$, $A > 0$.

Demographics: There is a single, infinitely lived, representative household.

Preferences: $\sum_{t=0}^{\infty} \beta^t \ln(c_t)$.

Technology: There is a single good that can be made (one-for-one) into consumption, private or public capital.

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha} G_t^{1-\alpha} = c_t + K_{t+1} + G_{t+1} \tag{1}$$

Key assumption: K and G can be costlessly transformed into each other. They are the same good.

Endowments: At the start of time, the household is endowed with X units of the good. In each period, he has 1 unit of work time.

Government: The government taxes household income at rate τ to pay for public capital. The government budget constraint is

$$G_{t+1} = \tau (r_t K_t + w_t) \tag{2}$$

Markets: There are competitive markets for goods, capital and labor rental.

Questions:

1. State the planner's Bellman equation.
2. Show that the planner's optimal capital input ratio K/G equals $\alpha/(1-\alpha)$.
3. Show that the planner's optimal output growth rate is $\beta A\phi$ where $\phi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$.
4. Define a Competitive Equilibrium.
5. Can the government find a tax rate τ that replicates the planner's allocation? Prove your answer.

4 Detrending a Growth Model

Notation:

- c: consumption (per capita), h: hours worked, z: productivity shock, y: Y/N, $k = K/N$.
- K: capital, L: land, N: population size, Y: output

Parameters: $\alpha, \beta, \phi \in (0,1)$; $\rho > 0$; $\eta, \gamma > 1$.

Demographics: There is a representative agent with mass $N_t = \eta^t$ who lives forever.

Preferences:

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t N_t \left\{\ln c_t + \rho \ln\left(1 - h_t\right)\right\} \tag{3}$$

Endowments: The household has 1 unit of time (per person) in each period. At the beginning of time, he also has 1 unit of land and K_0 units of capital. Land is in fixed supply.

Technology: $Y_t = \gamma^t e^{z_t} K_t^{\alpha} \left(N_t h_t\right)^{\phi} L_t^{1-\alpha-\phi} = N_t c_t + K_{t+1}$.

Shocks: z_t follows a finite Markov chain with transition matrix $\Pi_{z',z}$.

Questions: The goal is to write down the planner's problem as a *stationary* dynamic program. To do so, we need to make the growing economy stationary. We do so in steps.

1. First, express all model equations in per capita terms. Hint: Keep L as an aggregate, as it is in fixed supply (no need to detrend it).
2. Find the balanced growth rates for y, k, h, c. You should find that $g\left(y\right)\left(1-\alpha\right) = \gamma - 1 + \left(\alpha + \phi - 1\right)\left(\eta - 1\right)$. What is the intuition for the term involving $\eta - 1$?
3. Next, define appropriately detrended objects $\tilde{y}, \tilde{k}, \tilde{c}, \tilde{h}$ that are constant on the balanced growth path. Write all model equations in terms of the detrended variables and parameters.
4. State the planner's Bellman equation in terms of detrended objects.
5. Show that the detrended planner's problem implies a "standard" Euler equation.

End of exam.

5 Answers

5.1 Answer: Small Open Economy Real Business Cycles

1. Given TFP process A_t and initial conditions d_{-1} and k_0, a competitive equilibrium is a set of processes $d_t, c_t, h_t, k_{t+1}, w_t, r_t^k, D_t$ satisfying:

   (a) Household: first order conditions:

   $$\begin{aligned} u'(c_t) &= \beta(1 + r^* + p(D_t))E_t u'(c_t + 1) \\ &= \beta E_t u'(c_{t+1})\left[r_{t+1}^k + 1 - \delta\right] \end{aligned}$$

   budget constraint (given in the question), and transversality condition:

   $$\lim_{j\to\infty} E_t \frac{d_{t+j}}{\Pi_{s=0}^{j}(1 + r^* + p(d_s))} = 0.$$

   (b) Firm: $w_t = A_t F_h(k_t, h_t)$ and $r_t^k = A_t F_k(k_t, h_t)$

   (c) Market clearing: $h_t = 1$, aggregate resource constraint (given in the question).

   (d) Identity: $D_t = d_t$.

   The current account is given by $ca_t = d_{t-1} - d_t$.

2. Steady state:

   $$\begin{aligned} d &= \bar{D} \\ h &= 1 \\ k &= \left(\frac{\beta^{-1} - 1 + \delta}{\alpha}\right)^{1/(\alpha-1)} \\ c &= -r^*\bar{d} + Ak^\alpha - \delta k \end{aligned}$$

3. Let $\hat{x}_t \equiv \ln(x_t/x)$. Then:

   $$\begin{aligned} E_t[\hat{a}_{t+1}] &= \rho\hat{a}_t \\ \hat{a}_t + \alpha\hat{k}_t + \frac{d}{F(k,1)}\hat{d}_t &= \frac{d}{F(k,1)}[\zeta d + 1 + r^*]\hat{d}_{t-1} + \frac{c}{F(k,1}\hat{c}_t + \frac{\delta k}{F(k,1)}[\hat{k}_{t+1} - (1-\delta)\hat{k}_t] \\ -\hat{c}_t &= \zeta d/(1 + r^*)\hat{d}_t - E_t\hat{c}_{t+1} \\ &= \frac{r^* + \delta}{1 + r^*}\left[E_t\hat{a}_{t+1} + (\alpha - 1)E_t\hat{k}_{t+1}\right] - E_t\hat{c}_{t+1}. \end{aligned}$$

5.2 Answer: Debt Deleveraging

1. The saver is never borrowing-constrained, hence their Euler equation in steady state is $1/C(s) = \beta(1+r)1/C(s)$, implying that the steady state interest rate is $r = \frac{1-\beta}{\beta}$. Since $\beta(b) < \beta$, it must be that the borrower is borrowing-constrained (otherwise their Euler equation will also hold with equality, leading to a contradiction). So $D = D^{high}$ an

$$\begin{aligned} C(b) &= \frac{1}{2}Y - \frac{r}{1+r}D^{high} \\ C(s) &= Y - C(b). \end{aligned}$$

Note that the condition $D^{high} < \frac{1}{2}\frac{\beta}{1-\beta}Y$ guarantees that $C(b)$ is positive.

2. Deleveraging:

(a) $C_1(b) = \frac{1}{2}Y + \frac{D^{low}}{1+r_1} - D^{high}$ and $C_1(s) = \frac{1}{2}Y - \frac{D^{low}}{1+r_1} + D^{high}$.

(b) $C_2(s) = \frac{1}{2}Y + \frac{r}{1+r}D^{low} = \frac{1}{2}Y + (1-\beta)D^{low}$ and $C_2(b) = \frac{1}{2}Y - (1-\beta)D^{low}$.

(c) $\frac{1}{C_1(s)} = \beta(1+r_1)\frac{1}{C_2(s)}$ so $1 + r_1 = \frac{\frac{1}{2}Y + D^{low}}{\beta\frac{1}{2}Y + \beta D^{high}}$. As a corollary, r_1 is increasing in the new debt limit D^{low}. The intuition is straightforward: a smaller new debt limit means smaller consumption from the borrower; as the aggregate goods market must clear, this means the saver must be induced to save less and consume more to make up for the decreased consumption from the borrower. For this to happen, the real interest rate must fall, inducing the saver to save less. So a smaller D^{low} is associated with a smaller r_1.

5.3 Answer: Growth Model with Public Capital[1]

1. The state variable is $X = K + G$. The technology may be written as

$$\begin{aligned} Y &= A\left[\kappa X\right]^{\alpha}\left[(1-\kappa)X\right]^{1-\alpha} && (4) \\ &= AX\kappa^{\alpha}(1-\kappa)^{1-\alpha} && (5) \end{aligned}$$

Hence,

$$V(X) = \max_{\kappa, X'} \ln\left(AX\kappa^{\alpha}\left[1-\kappa\right]^{1-\alpha} - X'\right) + \beta V(X') \quad (6)$$

2. Optimality: The choice of κ maximizes current period output:

$$\kappa = \arg\max_{z} z^{\alpha}(1-z)^{1-\alpha} \quad (7)$$

so that $\kappa = \alpha$. Hence, the optimal K/G ratio is $\alpha/(1-\alpha)$.

[1] Based on a qualifying exam question at UCLA.

3. Conditional on κ, the planner solves an "AK" model with "interest rate" $A\phi$ where $\phi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. The Euler equation is $c'/c = \beta A\phi$, which determines the optimal growth rate.

4. CE objects: c, K, G, N, w, r.

   Equations:

   (a) household: Euler $c'/c = \beta r'(1-\tau)$ and budget constraint with boundary conditions K_0 given and TVC.

   (b) firm: $w = (1-\alpha) AG^{1-\alpha}(K/N)^{\alpha}$ and $r = \alpha AG^{1-\alpha}(K/N)^{\alpha-1}$.

   (c) government: budget constraint with boundary condition G_0 given.

   (d) Market clearing: $N = 1$ and resource constraint for the goods market.

5. No. One way of showing this is to consider an equilibrium where the government sets τ such that $K/G = \alpha/(1-\alpha)$. Then $r = \alpha A\phi$, which is below the planner's "rate of return." This is because $r = \alpha AG^{1-\alpha}K^{\alpha-1}$. Now plug in $G = K\alpha/(1-\alpha)$. But then the household chooses consumption growth rate $c'/c = \alpha\beta A\phi(1-\tau)$, which is lower than the planner's.

5.4 Answer: Detrending a Growth Model[2]

1. Preferences: just replace N_t with η^t. Technology:

$$y_t = \gamma^t e^{z_t} k_t^{\alpha} h_t^{\phi} L_t^{1-\alpha-\phi} \eta^{(\alpha+\phi-1)t} = c_t + k_{t+1}\eta \tag{8}$$

2. The resource constraint requires $g(y) = g(k) = g(c)$ and $g(y)(1-\alpha) = \gamma - 1 + (\alpha+\phi-1)(\eta-1)$.

3. Define a detrending factor $\chi^t = \gamma^{1/(1-\alpha)t}\eta^{(\alpha+\phi-1)/(1-\alpha)t}$. Then $\tilde{y}_t = y_t/\chi_t$ is constant on the balanced growth path.

   Detrended utility (so to speak) is given by

$$\mathbb{E}\sum_{t=0}^{\infty}(\beta\eta)^t\{\ln\tilde{c}_t + \ln\chi_t + \rho\ln(1-h_t)\} \tag{9}$$

   The detrended resource constraint is

$$\tilde{y}_t = e^{z_t}\tilde{k}_t^{\alpha} h_t^{\phi} L_t^{1-\alpha-\phi} = \tilde{c}_t + \eta\chi\tilde{k}_{t+1} \tag{10}$$

4. Bellman

$$V\left(\tilde{k}, z\right) = \max_{\tilde{k}', h} \ln\left\{e^z\tilde{k}^{\alpha}h^{\phi}L^{1-\alpha-\phi} - \eta\chi\tilde{k}'\right\} + \rho\ln(1-h) + \beta\eta\mathbb{E}V\left(\tilde{k}', z'\right) \tag{11}$$

   where I omitted the discounted value of χ_t, which is just a constant that does not affect decisions.

[2] Based on a 2013 UCLA exam.

5. The FOCs are a static condition, $\partial\tilde{y}/\partial h \times 1/\tilde{c} = \rho/(1-h)$, and

$$\eta\chi/c = \beta\eta\mathbb{E}V_k(.') \tag{12}$$

with envelope condition

$$V_k = \frac{\partial\tilde{y}}{\partial\tilde{k}}\frac{1}{\tilde{c}} \tag{13}$$

Combining those yields an Euler equation in detrended consumption

$$\chi/\tilde{c} = \beta\mathbb{E}\left\{\frac{1}{\tilde{c}'}\frac{\partial\tilde{y}}{\partial\tilde{k}'}\right\} \tag{14}$$

Apply $c_t = \tilde{c}\chi^t$ and cancel χ terms to obtain

$$1/c = \beta\mathbb{E}\left\{\frac{1}{c'}\frac{\partial\tilde{y}}{\partial\tilde{k}'}\right\} \tag{15}$$

Now it only remains to be shown that

$$\frac{\partial\tilde{y}}{\partial\tilde{k}'} = \frac{\partial y}{\partial k} \tag{16}$$

End of exam.