# Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

**Instructions:**

- This examination consists of **three** questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**. Number your answers.
- Good luck!

# 1 Technology Shocks in the New Keynesian Model

Consider a New Keynesian economy with key equations describing equilibrium:

$$y_t = E_t\left\{y_{t+1}\right\} - \frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}\right\} - \rho\right) \tag{1}$$

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \kappa\left(y_t - y_t^n\right) \tag{2}$$

where $y$ is output, $y^n$ is the natural, full employment output level, $i$ is the nominal interest rate, and $\pi$ is the level of inflation. Monetary policy is described by the following rule

$$i_t = \rho + \phi_\pi \pi_t \tag{3}$$

with $\phi_\pi > 1$ (Taylor principle holds). Labor productivity is given by

$$a_t = y_t - n_t \tag{4}$$

where $a_t$ is an exogenous technological process with law of motion

$$a_t = \rho_a a_{t-1} + \varepsilon_t \tag{5}$$

with $\rho_a \in (0,1)$ and $\{\varepsilon_t\}$ is an iid stochastic process.

In this model it is assumed that the natural level of output is completely determined by technology

$$y_t^n = \psi_y a_t \tag{6}$$

where $\psi_y > 1$.

1. Describe in words where equations (1) and (2) come from
2. Determine the equilibrium response of output, employment, and inflation to the technology shock (Hint: technology is the only impetus in the model)
3. How do these responses depend on the value of $\phi_\pi$ and $\kappa$. Provide intuition. What happens when $\phi_\pi$ approaches infinity? What happens when the degree of price rigidity changes?
4. Analyze the joint response of employment and output to a technology shock and discuss briefly the implications for assessment of the role of technology as a source of business cycle fluctuations.

## 2 Static Model of Differentiated Goods

Demographics: The world lasts for 1 period. There is a representative worker of mass 1.

Endowments: Each worker has $L$ units of labor time.

Commodities: There are $N$ (endogenous) consumption goods.

Preferences: $U = \int_0^N \frac{1-e^{-bc_i}}{b} di$.

Technologies:

- Introducing a good costs $\bar{l}/q$ units of labor. This is for now exogenous.
- Units of the good are produced at 0 marginal cost.

Markets:

- Labor: competitive (wage $w$).
- Goods: monopolistic competition (prices $p_i$). Normalized to 1.

**Questions:**

1. Solve the household problem for the demand functions

$$c_i = \bar{c} - \frac{\ln(p_i)}{b} \tag{7}$$

   where

$$\bar{c} = \frac{R + b^{-1} \int_0^N p_i \ln(p_i)\, di}{\int_0^N p_i di} \tag{8}$$

   and $R$ is the household's income.

2. Solve the pricing problem of a firm. Show that the equilibrium price is $p_i = p = e^{b\bar{c}-1}$.
3. Show that free entry implies $w = \bar{c}q/\bar{l}$.

Now introduce heterogeneous managerial abilities, $q$, drawn from a uniform distribution with cdf $F(q) = (q - q_{min}) / (q_{max} - q_{min})$ and pdf $f(q) = 1/(q_{max} - q_{min})$. The timing is now: Firms enter and hire managers at competitive wages $\omega(q)$. Free entry drives profits to 0. Firms hire $\bar{l}/q$ workers to start a new good. Then they produce at marginal cost 0 as before. Each person can be either a worker or a manager.

1. Show that $\omega(q) = 1/b - w\bar{l}/q$. $w$ is now the wage paid to workers.

2. Let $q^*$ be the ability cutoff for managers, so that all persons with $q > q^*$ becomes managers. Explain the condition $LF(q^*) = \int_{q^*}^{q_{max}} (\bar{l}/q) f(q) \, dq$.

3. Show that the equilibrium wage is given by $w = \left(b \left[L + \bar{l}/q^*\right]\right)^{-1}$.

4. Show that an increase in overhead costs $\bar{l}$ increases both $q^*$ and $\bar{l}/q^*$.

## 3 Bonds in a Lucas Tree Model

Demographics: A representative agent of mass 1 lives forever.

Preferences: $\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t)$ with $u(c) = c^{1-\theta}/(1-\theta)$ and $0 < \theta < 1$.

Endowments: At $t = 0$, each household owns 1 tree. The tree's dividend follows $d_{t+1} = d^* d_t^{\phi}\varepsilon_{t+1}$ with $0 < \phi < 1$ and $\ln(\varepsilon) \sim N(0, \sigma^2)$. Note that this implies

$$\ln(d_{t+j}) - \ln(d_t) = \left(\phi^j - 1\right)\ln(d_t) + \frac{1-\phi^j}{1-\phi}\ln(d^*) + \sum_{k=0}^{j-1}\phi^k \ln(\varepsilon_{t+j-k}) \tag{9}$$

Markets: All markets are competitive. There are markets for goods (numeraire) and discount bonds of maturities $j = 1, ..., J$, traded at prices $q_{j,t}$.

**Questions:**

1. State the household's dynamic program.
2. Derive the Lucas asset pricing equations for the bonds.
3. Define a recursive competitive equilibrium.
4. Show that $q_{j,t} = \mathbb{E}_t\left\{\beta^j u'(c_{t+j})/u'(c_t)\right\}$.
5. Show that $\ln(q_{j,t})/j = a(j) - b(j)\ln(c_t)$ where $b(j) = \theta(1-\phi^j)/j$. Note that $|b(j)|$ decreases in maturity.
6. Note that the yield on a $j$ period bond is given by $R_{j,t} = q_{j,t}^{-1/j}$. What happens to the yield curve and to the sensitivity of yields to consumption as consumption becomes highly persistent ($\phi \to 1$)? What is the intuition?

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End of exam.

# 4 Answers

## 4.1 Answer: Static Model of Differentiated Goods

[Based on a question due to Gilles Saint-Paul]

1. Household: Budget constraint: $R = wL = \int_0^N p_i c_i di$. Set up Lagrangian and take first-order conditions. $\partial U/\partial c_i = \lambda p_i = e^{-bc_i}$. $c_i = -(\ln \lambda + \ln p_i)/b$. Integrate so that expenditure matches $R$.

2. Firm: $\max p_i [\bar{c} - \ln (p_i)/b]$. FOC: $\bar{c} - \ln (p_i)/b - 1/b = 0$.

3. Free entry: $w\bar{l}/q = \pi$. Equilibrium profit is the same as revenue (zero marginal cost): $\pi = pc = \bar{c}$ because $c_i = \bar{c}$ when $p = 1$. Free entry: $w = \bar{c}q/\bar{l}$.

4. Equilibrium quantities and prices: $\bar{c} = b^{-1}$ from firm FOC. Labor market clearing: $N = Lq/\bar{l}$. Income: $wL = Lq/\left(\bar{l}b\right)$.

Heterogenous abilities:

1. This is the 0 profit condition. Starting a firm yields revenue $1/b$ and costs $w\bar{l}/q$ for workers plus $\omega(q)$ for the manager.

2. This is labor market clearing. The mass of managers is $L(1 - F(q^*))$. The mass of workers is $LF(q^*)$. The right hand side is labor demand.

3. This follows from the fact that the marginal manager with $q = q^*$ must be indifferent between working and managing: $\omega(q^*) = w$.

4. Write labor market clearing as $G\left(q^*, \bar{l}\right) = LF(q^*) - \int_{q^*}^{q_{max}} \left(\bar{l}/q\right) f(q)\, dq = 0$. Obviously, $\partial G/\partial \bar{l} < 0$ and $\partial G/\partial q^* > 0$. It follows that $\partial q^*/\partial \bar{l} > 0$. To see that $\bar{l}/q^*$ rises, compute $\partial q^*/\partial \bar{l} \times \bar{l}/q^*$ and show that it is less than 1.

## 4.2 Answers: Bonds in a Lucas Tree Model

[Based on a question due to Rody Manuelli]

1. Household: $V(s, B_0, ..., B_{J-1}) = \max u(c) + \beta \mathbb{E} V\left(s', B_0', ..., B_{J-1}'\right)$ subject to the budget constraint

$$c + ps' + \sum_{j=0}^{J-1} q_{j+1} B_j' = s(p + d) + \sum_{j=0}^{J-1} q_j B_j \tag{10}$$

2. For all bonds:
$$q_{j+1} = \mathbb{E}\left\{\beta \frac{u'(c')}{u'(c)} q'_j\right\} \tag{11}$$
with $q_0 = 1$.

3. Equilibrium:

   Market clearing: $s = 1$, $B_j = 0$, $c = d$.

   Objects: $V(s, B_0, ..., B_{J-1}, d)$ and policy functions and price functions $q_j(d)$ and $p(d)$.

   Equations: Value and policy functions solve the household problem. Market clearing. Exogenous law of motion of the aggregate state.

4. Iterate over the bond pricing equation and use the law of iterated expectations.

5. $\ln(q_{j,t}) = \ln(\beta^j) - \theta \mathbb{E}_t\{\ln(d_{t+j})\} + \theta \ln(d_t)$. Substitute in from (9). Done.

6. $\ln(R_{j,t}) = -\ln(q_{j,t})/j = -a(j) + b(j)\ln(c_t)$. As $\phi \to 1$, the yield curve gets flat and does not respond to consumption. Intuition: The yield curve responds to consumption because consumption contains information about future consumption growth (the MRS). This disappears as consumption approaches a random walk.

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End of exam.