Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **three** questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**. Number your answers.
- Good luck!

1 Monetary Policy and Wage Setting

Assume that the representative household's utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

with

$$U(C_t, N_t) = C_t - \frac{1}{2} N_t^2$$

where C_t is consumption and N_t denotes hours worked. Firms' production technology is given by

$$Y_t = A_t N_t$$

where Y_t denotes output and A_t denotes exogenous technology. There is no storage facility in this setup so all output is consumed.

Firms set prices via a Calvo staggered mechanism resulting in the following equation for inflation,

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \widehat{mc}_t$$

where

$$\widehat{mc}_t \equiv mc_t - mc$$

is the log deviation of the real marginal cost from its level in the zero inflation steady state.

Questions:

1. Derive the marginal rate of substitution between consumption and leisure.
2. Given the model setup derive an expression for the (log) of the efficient level of employment (denoted n_t^*) that a benevolent social planner would choose.
3. Assume the (log) of nominal wage w_t is set each period is set according to

   $$w_t = p_t + \frac{1}{1+\alpha} n_t$$

   where $\alpha > 0$. Compare this schedule for the real wage to the schedule that would be observed under competitive labor markets. Is there a sense in which $\alpha > 0$ can be viewed as "real rigidity"?
4. Derive the implied (log) natural level of employment (denote by $\overline{n}_t$) defined as the equilibrium level of employment under flexible prices (when all firms keep a constant (log) markup μ).
5. Derive an expression for the real marginal cost $\widehat{mc}_t$ as a function of the real employment gap ($\widetilde{n}_t \equiv n_t - \overline{n}_t$).

6. Derive the inflation equation in terms of the welfare-relevant employment gap $(n_t - n_t^*)$. How does the presence of $\alpha > 0$ generate a trade-off between stabilization of inflation and stabilization of the welfare-relevant employment gap.

7. Suppose the monetary authority has a loss function given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\theta(n_t - n_t^*)^2 + \pi_t^2\right] \tag{1}$$

Using the equation for inflation in the previous question, solve for the equilibrium process for inflation and output under the optimal monetary policy under discretion, under the assumption of an **i.i.d. technology process**. For this question assume the frictionless markup μ is small enough to ignore.

8. Derive *(just)* the first order condition under commitment.

2 Asset Pricing with Habits

Consider the problem of an infinitely lived household with utility $\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t u\left(c_t, x_t\right)\right\}$ where x_t is a habit stock.

The household receives a stochastic endowment e_t in each period. He can trade an asset with stochastic gross return R_{t+1}.

Questions part A: Start by assuming that the household takes x_t as given

1. State the household problem as a dynamic program.

2. Derive the Euler equation that determines asset prices.

Questions part B: Now assume that the habit depends on the household's own past consumption: $x_t = \sum_{j=1}^{\infty} \phi^j c_{t-j}$ with $0 < \phi < 1$.

1. State the household problem as a dynamic program. Key: what is the law of motion for x?

2. Derive the first-order and envelope conditions.

3. Derive the Euler equation that determines asset prices. Substituting out the marginal values of x is messy, so just leave them in the equation. Just sketch how you could substitute out those terms.

   Provide an interpretation of the Euler equation.

3 Money as a Bubble

Demographics: There is a unit mass of 2 types of agents, indexed by i. Agents live forever.

Endowments: Agent i receives e_t^i units of "the good" in period t.

Preferences: $\sum_{t=1}^{\infty} \beta^t u\left(c_t^i\right)$

Technology: The good can only be eaten: $\sum_i c_t^i = \sum_i e_t^i$.

Questions part A:

1. Characterize the set of Pareto efficient allocations.
2. Define an Arrow-Debreu equilibrium.
3. Characterize the Arrow-Debreu equilibrium when the endowments alternate. That is, agent 1's endowments are $\{e, 0, e, 0, ...\}$ and agent 2's endowments are $\{0, e, 0, e, ...\}$.

Questions part B: For the remaining questions, we introduce money as follows. At the start of period 1, each agent is endowed with $M_1/2$ units of fiat money. In each period, each agent receives a lump-sum transfer of $\tau M_t/2$ units of money, so that $M_{t+1} = (1+\tau) M_t$.

1. Write down the problem of agent i and characterize its solution.
2. Define a competitive equilibrium where money is valued.
3. Assuming alternating endowments (as in Q.A3), find the value of τ that maximizes the average utility of both types of agents.
   (a) Characterize the resulting allocation.
   (b) Is the resulting allocation Pareto efficient?
   (c) Is it Pareto superior to the Arrow-Debreu allocation of Q.A3?

End of exam.

4 Answers

4.1 Monetary Policy and Wage Setting

Households max

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{2}$$

$$U(C_t, N_t) = C_t - \frac{1}{2} N_t^2 \tag{3}$$

$$Y_t = A_t N_t \tag{4}$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \widehat{mc}_t \tag{5}$$

$$\widehat{mc}_t \equiv mc_t - mc \tag{6}$$

1. mrs = mpn (in logs):

$$n_t^* = a_t \tag{7}$$

2. Planner:

$$w_t - p_t = \frac{1}{1+\delta} n_t \tag{8}$$

under part 1.

$$w_t - p_t = n_t \tag{9}$$

with $\delta > 0$ real wages are less sensitive to changes in n_t implying real wages more "rigid"

3. Real rigidity:

$$\mu = p_t - (w_t - a_t) \tag{10}$$

mark-up equals price above real marginal cost

$$\mu = p_t - \left(\frac{\overline{n}_t}{1+\delta} + p_t \right) + a_t \tag{11}$$

$$\overline{n}_t = (1+\delta)(a_t - \mu) \tag{12}$$

4. Independent of stickiness

$$\begin{aligned} mc_t &= (w_t - p_t) - a_t && (13) \\ &= \frac{1}{1+\delta} n_t - a_t && (14) \end{aligned}$$

under flexible prices

$$\begin{aligned} mc &= -\mu && (15) \\ &= \frac{\overline{n}_t}{1+\delta} - a_t && (16) \end{aligned}$$

therefore,

$$\widehat{mc}_t = \frac{1}{1+\delta}\widetilde{n}_t \tag{17}$$

5. Marginal cost:

$$\begin{aligned}
\pi_t &= \beta E_t\left\{\pi_{t+1}\right\} + \lambda\left(\frac{1}{1+\delta}\widetilde{n}_t\right) &(18)\\
&= \beta E_t\left\{\pi_{t+1}\right\} + \frac{\lambda}{1+\delta}\left(n_t - n_t^* + n_t^* - \overline{n}_t\right) &(19)\\
&= \beta E_t\left\{\pi_{t+1}\right\} + \frac{\lambda}{1+\delta}(n_t - n_t^*) + \lambda\mu + \frac{\lambda\delta}{1+\delta}a_t &(20)
\end{aligned}$$

6. Inflation:

$$MinE_0 \sum \beta^t\left[\alpha(n_t - n_t^*)^2 + \pi_t^2\right] \tag{21}$$

subject to

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \frac{\lambda}{1+\delta}(n_t - n_t^*) + \lambda\mu + \widetilde{a}_t \tag{22}$$

under discretion lump $\beta E_t\left\{\pi_{t+1}\right\}$ with $\widetilde{a}_t$, FOC (lagrange multiplier γ_t)

$$\begin{aligned}
2\alpha(n_t - n_t^*) - \gamma_t\left(\frac{\lambda}{1+\delta}\right) &= 0 &(23)\\
2\pi_t &= \gamma_t &(24)
\end{aligned}$$

$$(n_t - n_t^*) = \frac{-\lambda}{\alpha\left(1+\delta\right)}\pi_t \tag{25}$$

plug in to get

$$\begin{aligned}
\pi_t &= \beta E_t\left\{\pi_{t+1}\right\} + \frac{\lambda}{1+\delta}\left(\frac{-\lambda}{\alpha\left(1+\delta\right)}\pi_t\right) + \frac{\lambda\delta}{1+\delta}a_t &(26)\\
&= \frac{\alpha\left(1+\delta\right)^2}{\alpha\left(1+\delta\right)^2+\lambda^2}\beta E_t\left\{\pi_{t+1}\right\} + \frac{\alpha\left(1+\delta\right)\lambda\delta}{\alpha\left(1+\delta\right)^2+\lambda^2}a_t &(27)
\end{aligned}$$

assuming $a_t \sim iid$

$$a_t = \varepsilon_t^a \tag{28}$$

guess

$$\pi_t = \psi_\pi a_t \tag{29}$$

$$E_t\left\{\pi_{t+1}\right\} = 0 \tag{30}$$

this imples that

$$\psi_\pi a_t = \frac{\alpha\left(1+\delta\right)\lambda\delta}{\alpha\left(1+\delta\right)^2+\lambda^2}a_t \tag{31}$$

4.2 Answer: Asset pricing with habits

English language advisory: The word *stock* does not only mean a type a financial asset, it also means something that has been accumulated over time (in contrast to a *flow*).

Part A: When habits are external, the answer is the standard Lucas asset pricing equation.

Part B: Internal habits.

1. Household: States are k and x with laws of motion $k' = Rk + e - c$ and $x' = \phi(c + x)$. Bellman:
$$V(k, x) = \max u(c, x) + \beta \mathbb{E}\{V(Rk + e - c, \phi c + \phi x)\} \tag{32}$$

2. First order condition: $u_1 + \beta \mathbb{E}\{V_2(.')\phi\} = \beta \mathbb{E}\{V_1(.')\}$

   Envelope: $V_1 = \beta \mathbb{E}\{V_1(.')R\}$ and $V_2 = u_2 + \beta \mathbb{E}\{V_2(.')\phi\}$

   In words: eating another unit today gains u_1 immediately, but reduces k' by 1 at a cost of $V_1(.')$. In addition, it raises x' by ϕ at a cost of $V_2(.')$.

   A higher x costs u_2 today and raises future habits by ϕ at cost $V_2(.')$.

3. Euler: $\beta \mathbb{E}\{V_1(.')\} = u_1 + \beta \mathbb{E}\{V_2(.')\phi\} = \beta^2 \mathbb{E}\{V_1(.'')R'\} = \beta \mathbb{E}\{u_1(.')R' + \beta V_2(.'')\phi R'\}$

   Think of $u_1 + \beta \mathbb{E}\{V_2(.')\phi\}$ as the total marginal utility of consumption and the Euler equation is pretty standard.

   Collect terms:

   $u_1 = \beta \mathbb{E}\{u_1(.')R'\} + \phi \beta \mathbb{E}(\beta R' V_2(.'') - V_2(.'))$

   To get rid of the V_2 terms, expand the envelope condition into $V_2 = u_2 + \beta \phi u_2(.') + (\beta \phi)^2 u_2(.'') + ...$

   Interpretation: In addition to the usual trade-off between u_1 today and tomorrow, the household also has to consider how damaging today's versus tomorrow's consumption is in terms of driving up the habit stocks tomorrow versus two days from today.

4.3 Answer: Money as a Bubble

Part A:

1. The set of Pareto optimal allocations maximizes $\sum_i \lambda_i U_i$ subject to a sequence of static constraints $\sum_i e_t^i - c_t^i = 0$. The obvious outcome: the ratio of marginal utilities must be constant over time.

2. Budget constraint: $\sum_t p_t(e_t^i - c_t^i) = 0$. Euler: $u'(c_t^i) = \beta u'(c_{t+1}^i) p_t / p_{t+1}$. Again, the ratio of marginal utilties is constant over time. The same as the Pareto optimal allocation.

3. With symmetry: $p_t = \beta^t$ and $c_t^i = c^i$. The budget constraints imply that c^1/c^2 equals the ratio of lifetime incomes. A little algebra shows that this ratio is $1/\beta$. The household who gets paid first is richer. Then $c^1 = (1+\beta)^{-1}$ and $c^2 = \beta/(1+\beta)$.

Part B:

Notation: At the start of period t, agent i holds M_t^i units of money, so that $\sum_i M_t^i = M_t$. $m_t^i = M_t^i/p_t$.

1. Budget constraint: $M_t^i + \tau M_t/2 + p_t e_t^i = p_t c_t^i + M_{t+1}^i$. In real terms: $m_t^i + \tau m_t/2 + e_t^i = c_t^i + m_{t+1}^i \pi_{t+1}$. Euler: $u'(c_t^i) = \beta u'\left(c_{t+1}^i\right) p_t/p_{t+1}$.
2. Market clearing: resource constraint and $\sum_i m_t^i = (1+\tau) m_t = m_{t+1}/\pi_{t+1}$.
3. Optimal τ: The policy maker maximizes the sum of agents' lifetime utilities subject to resource constraint, Euler equations, budget constraints, market clearing for money.

   Approach: Drop all constraints except the resource constraint. Then show that the resulting allocation satisfies all the other constraints.

   Maximizing utility subject to resource constraints only yields $c_t^i = e/2$. Conjecture that prices are constant and $\tau = 0$ (Friedman rule). Then the Euler equation is satisfied. The budget constraint becomes $m_t^i + e_t^i - e/2 = m_{t+1}^i$. If the rich agent sells half his endowment to the poor one and if $p_t e/2 = m_t = m$, then the money market clears. This is also consistent with $\tau = 0$, so that m is constant over time.

   In spirit, this is the same allocation is Q.A3. If the policy maker had the proper weights, it would be the same allocation. But with equal weights, it makes agent 2 better off and agent 1 worse off.

End of exam.