Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **three** questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**. Number your answers.
- Good luck!

1 New Keynesian Model with Technology Shocks

Consider a New Keynesian model with IS curve, Phillips curve, and monetary policy rule of the forms:

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1}\right) \\ \pi_t &= \beta E_t \pi_{t+1} + \kappa \widetilde{y}_t \\ i_t &= \delta_\pi \pi_t \end{aligned}$$

where $\widetilde{y}_t = y_t - a_t$ and a is the technology process. All parameters are assumed to have the usual signs, with $\delta_\pi > 1$. Suppose that the technology shock follows a stationary AR(1) process

$$\begin{aligned} a_t &= \rho a_{t-1} + \varepsilon_t \\ |\rho| &< 1, \varepsilon_t \sim iid \end{aligned}$$

Questions:

1. Solve the model. That is write y_t, π_t and i_t as functions of a_t *(Hint: you can use method of undertermined coefficient and assume the solutions are AR(1) processes with same persistence as technology shock).*

2. Assume a production function

   $$y_t = a_t + n_t$$

   Provide intuition to the signs of the impact responses to a_t of the following variables $y_t, \pi_t, i_t, \widetilde{y}_t, n_t, y_t - n_t$

3. Describe and show how each of the responses above (in question 2) vary with the value of δ_π and σ.

2 Two Sector Model

Consider the following growth model with two capital goods.

Demographics: There is a single, representative household who lives forever.

Preferences: $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where c_t is consumption. Assume log utility: $u(c) = \ln(c)$.

Endowments: In period 0 the household is endowed with capital stocks K_{10}, K_{20}.

Technologies: Production takes place in two sectors ($i = 1, 2$). The resource constraints for sector 1 is

$$A_1 F_1(K_{11t}, K_{12t}) + (1-\delta) K_{1t} = K_{1t+1} + c_t$$

where K_{ist} is the amount of capital of type s used in sector i and $K_{st} = K_{1st} + K_{2st}$ is the total amount of capital good s used in both sectors. The resource constraint for sector 2 is similar, except that good 2 is not consumed:

$$A_2 F_2(K_{21t}, K_{22t}) + (1-\delta) K_{2t} = K_{2t+1}$$

There is no labor input. F_i has constant returns to scale.

Market arrangements: All markets are competitive. Households rent capital to firms.

Questions:

1. Define a solution to the firm's problem in each sector. Be careful to define the purchase and rental prices of the various goods consistently. Good 1 is the numeraire.

2. State the household problem and define a solution.

3. Define a competitive equilibrium. Make sure that the number of objects equals the number of equations.

4. Consider the balanced growth path. Derive the balanced growth rates of c, k_s, r_s, p_s for $s = 1, 2$, where $k_s = K_{s2}/K_{s1}$ is the input ratio in sector s, r_s is the rental price of capital good s, and p_s is its purchase price.

5. Derive 7 equations that solve for 7 (constant) objects and thus define the balanced growth path.

6. Using the 7 equations from the previous answer, determine qualitatively how the balanced growth rate and prices change when A_1 rises.

3 R&D: Durable Intermediates

Based on Barro & Sala-i-Martin (JPE 1992). Consider the following version of an R&D growth model where the intermediate inputs (x) are *durable*.

Demographics: There is a representative household who lives forever.

Preferences:

$$\int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt \tag{1}$$

where c is consumption.

Endowments: The household works one unit of time at each instant.

Technologies:

- Final goods are used for consumption, for investment in intermediates, and for R&D. The production function is given by

  $$y_t = A L_t^\alpha \int_0^{N_t} x_{j,t}^{1-\alpha} dj$$

  where $A > 0$ is a parameter, N_t is the number of intermediate inputs available at t, and $x_{j,t}$ is the quantity of input j used.

- Intermediates: Upon invention, the inventor is endowed with x_0 units of x_j. Additional units are then accumulated according to

  $$\dot{x}_{j,t} = \eta I_{j,t}^{\varphi} - \delta x_{j,t} \tag{2}$$

  where for now $0 < \varphi < 1$ and $I_{j,t}$ is investment (in the form of goods) in accumulating intermediates. Intermediates depreciate at rate δ. They are *rented* to final goods firms at price $R_{j,t}$.

- New varieties are invented according to:

  $$\dot{N} = \beta^{-1} \ z \tag{3}$$

  where z denotes goods devoted to R&D.

Market arrangements:

- The market for final goods and labor are competitive.
- Each intermediate input producer has a permanent monopoly for his variety.
- There is free entry into the market for innovation, which implies $\beta = V$ where V is the value of a new patent.

The solution to the household problem is standard. The budget constraint is

$$\dot{a}_t = r_t a_t - c_t \tag{4}$$

where a denotes asset holdings. c_t and a_t solve the Euler equation

$$\dot{c}_t / c_t = \frac{r_t - \rho}{\sigma} \tag{5}$$

the budget constraint, a_0 given, and the TVC $\lim_{t\to\infty} e^{-\rho t} u'(c_t) a_t = 0$.

We are looking for a stationary equilibrium with a constant interest rate r.

Questions:

1. Write down the problem of the final goods firm. Derive the first order conditions. Define a solution.
2. Write down the problem of an intermediate goods firm who has just invented good j. The firm maximizes the present value of profits, which are given by $R(x)x - I$. Derive the first-order conditions. Define a solution. Do not yet substitute out the co-state from the first-order conditions.
3. Define an equilibrium. Do not assume symmetry (there is no symmetric equilibrium because recently invented goods are supplied in smaller quantities than old ones). Hint: Free entry implies that β equals the value of a new patent.
4. From hereon assume $\varphi = 1$ and consider the balanced growth path with r constant. Although this is not strictly speaking correct, assume that the equilibrium conditions derived for $\varphi < 1$ continue to hold (it yields the right answer). Solve for the intermediate goods firm's optimal $R(x_t)$ and x_t as functions of r. How do they change over time?
5. Solve for the *symmetric* equilibrium values of x_t and r_t. Given the Euler equation, you have found the equilibrium growth rate. Note: With $\varphi = 1$ there is a symmetric equilibrium because it does not take time to build up the stock of x_j.

End of exam.

4 Answers

4.1 New Keynesian Model

1. Guessing

$$\begin{aligned} y_t &= \rho y_{t-1} + \varepsilon_{yt} \\ \pi_t &= \rho \pi_{t-1} + \varepsilon_{\pi t} \\ i_t &= \rho i_{t-1} + \varepsilon_{it} \end{aligned}$$

From Phillips curve

$$\begin{aligned} \pi_{t+1} &= \rho \pi_t + \varepsilon_{\pi t+1} \\ E_t \pi_{t+1} &= \rho \pi_t \end{aligned}$$

therefore,

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa \left(y_t - a_t \right) \\ &= \beta \rho \pi_t + \kappa \left(y_t - a_t \right) \\ &= \frac{1}{1 - \beta \rho} \kappa \left(y_t - a_t \right) \end{aligned}$$

from IS curve

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} \right) \\ &= \rho y_t - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) \\ &= \rho y_t - \frac{1}{\sigma} (\delta_\pi \pi_t - \rho \pi_t) \\ &= - \frac{1}{1 - \rho} \frac{1}{\sigma} (\delta_\pi - \rho) \pi_t \\ &= - \frac{1}{1 - \rho} \frac{1}{\sigma} (\delta_\pi - \rho) \frac{1}{1 - \beta \rho} \kappa \left(y_t - a_t \right) \\ &= - c \left(y_t - a_t \right) \end{aligned}$$

Note $c > 0$

Write all endogenous variables as functions of technology

$$\begin{aligned} y_t &= \frac{c}{1 + c} a_t \\ \pi_t &= \frac{1}{1 - \beta \rho} \kappa \left(\frac{c}{1 + c} a_t - a_t \right) = - \frac{1}{1 - \beta \rho} \frac{\kappa}{1 + c} a_t \\ i_t &= \delta_\pi \pi_t = - \frac{\delta_\pi}{1 - \beta \rho} \frac{\kappa}{1 + c} a_t \end{aligned}$$

2. From production function

$$\begin{aligned} y_t &= n_t + a_t \\ n_t &= y_t - a_t = \widetilde{y}_t \end{aligned}$$

therefore

$$\frac{\partial y_t}{\partial a_t} = \frac{c}{1+c} > 0$$

firms expand output to technological improvements

$$\frac{\partial \pi_t}{\partial a_t} = -\frac{1}{1-\beta\rho}\frac{\kappa}{1+c} < 0$$

prices fall as aggregate supply curve shifts to the right.

$$\frac{\partial i_t}{\partial a_t} = -\frac{\delta_\pi}{1-\beta\rho}\frac{\kappa}{1+c} < 0$$

interest rates fall as the central bank, according to its rule, lower nominal rates when there is a deflation.

$$\begin{aligned} \frac{\partial \widetilde{y}_t}{\partial a_t} &= \frac{\partial y_t}{\partial a_t} - 1 = \frac{c}{1+c} - 1 = -\frac{1}{1+c} < 0 \\ \frac{\partial n_t}{\partial a_t} &= \frac{\partial \widetilde{y}_t}{\partial a_t} = -\frac{1}{1+c} < 0 \end{aligned}$$

output adjusts less than one for one with technology so output gap (and labor) falls

Finally,

$$\begin{aligned} y_t - n_t &= a_t \\ \frac{\partial (y_t - a_t)}{\partial a_t} &= 1 \end{aligned}$$

3. Signing the responses

$$c = \frac{\kappa(\delta_\pi - \rho)}{\sigma(1-\rho)(1-\beta\rho)}$$

therefore

$$\frac{\partial c}{\partial \delta_\pi} > 0 \ \text{ and } \ \frac{\partial c}{\partial \sigma} < 0$$

$$\frac{\partial}{\partial(\delta_\pi)}\left(\frac{\partial y_t}{\partial a_t}\right) = \frac{\partial}{\partial(\delta_\pi)}\frac{c}{1+c} > 0$$

$$\frac{\partial}{\partial\left(\delta_{\pi}\right)}\left(\frac{\partial \pi_t}{\partial a_t}\right)=-\frac{\partial}{\partial\left(\delta_{\pi}\right)}\frac{1}{1-\beta\rho}\frac{\kappa}{1+c}>0$$

$$\frac{\partial}{\partial\left(\delta_{\pi}\right)}\left(\frac{\partial i_t}{\partial a_t}\right) = -\frac{\partial}{\partial\left(\delta_{\pi}\right)}\frac{\delta_{\pi}}{1-\beta\rho}\frac{\kappa}{1+c}=\frac{\kappa}{\left(1+c\right)\left(1-\beta\rho\right)}\left[1-\frac{\delta_{\pi}}{1+c}\left(\frac{\partial c}{\partial\left(\delta_{\pi}\right)}\right)\right]$$

ambiguous

$$\frac{\partial}{\partial\left(\delta_{\pi}\right)}\left(\frac{\partial \widetilde{y}_t}{\partial a_t}\right) = -\frac{\partial}{\partial\left(\delta_{\pi}\right)}\frac{1}{1+c}>0$$

same for n_t

$$\frac{\partial}{\partial\left(\delta_{\pi}\right)}\frac{\partial(y_t-a_t)}{\partial a_t}=0$$

Changing σ

$$\frac{\partial}{\partial\left(\sigma\right)}\left(\frac{\partial y_t}{\partial a_t}\right)=\frac{\partial}{\partial\left(\sigma\right)}\frac{c}{1+c}<0$$

$$\frac{\partial}{\partial\left(\sigma\right)}\left(\frac{\partial \pi_t}{\partial a_t}\right)=-\frac{\partial}{\partial\left(\sigma\right)}\frac{1}{1-\beta\rho}\frac{\kappa}{1+c}<0$$

$$\frac{\partial}{\partial\left(\sigma\right)}\left(\frac{\partial i_t}{\partial a_t}\right)=-\frac{\partial}{\partial\left(\sigma\right)}\left(\frac{\delta_{\pi}}{1-\beta\rho}\frac{\kappa}{1+c}\right)<0$$

$$\frac{\partial}{\partial\left(\sigma\right)}\left(\frac{\partial \widetilde{y}_t}{\partial a_t}\right) = -\frac{\partial}{\partial\left(\sigma\right)}\frac{1}{1+c}<0$$

same for n_t

$$\frac{\partial}{\partial\left(\sigma\right)}\frac{\partial(y_t-a_t)}{\partial a_t}=0$$

4.2 Answer: Two sector model

To begin, we define prices. r_{st} is the rental price of capital good s in terms of good 1.

1. The firm in sector i solves

$$\max A_i\, F_i(K_{i1t}, K_{i2t})\, p_i - \sum_s r_{st}\, K_{ist}$$

   The first-order conditions are $r_s = A_i\, F_{is}(K_{i1}, K_{i2})\, p_i$ for $s = 1, 2$. A solution is a pair (K_{i1t}, K_{i2t}) which satisfies the 2 first order conditions.

2. We anticipate that both capital goods must pay the same rate of return in equilibrium; call it R. Denote household wealth by $a_t = p_{1t}\, K_{1t} + p_{2t}\, K_{2t}$. Then the budget constraint is $a_{t+1} = R_t\, a_t - c_t$. The household problem is entirely standard with Euler equation $u'(c_t) = \beta\, R_{t+1}\, u'(c_{t+1})$. A solution is a sequence (c_t, a_t) which satisfies Euler equation and budget constraint (and a transversality condition).

3. A competitive equilibrium is a set of sequences $(c_t, a_t, K_{it}, K_{ist}, r_{it}, p_{it}, R_t)$ (13 objects) which satisfy:

   (a) 2 household conditions (see above).

   (b) 4 firm conditions (see above).

   (c) Definition of the rate of return: $R_{t+1} = [(1-\delta)\, p_{st+1} + r_{st+1}]\, / p_{st}$; $s = 1, 2$. Giving up $1/p_{st}$ units of good s today and investing the good as capital pays $[(1-\delta)\, p_{st+1} + r_{st+1}]$ units of the same good tomorrow. This rate of return must be the same for both goods (2 equations)

   (d) Goods market clearing in both sectors (given in the question).

   (e) Capital market clearing (also given): $K_{st} = \sum_i K_{ist}$.

   (f) Definition of a_t.

   (g) The normalization $p_{1t} = 1$.

   There are $2 + 4 + 2 + 2 + 2 + 1 = 14$ equations. One is redundant by Walras' law.

4. We know that R must be constant, otherwise the consumption growth rate would not be. By the definition of R, this requires constant prices and rental prices. The quantities grow, all at rate γ.

5. The Euler equation implies

$$1 + \gamma = \beta R.$$

   The definition of R yields 2 additional equations

$$R = 1 - \delta + r_s / p_s.$$

The firms' first-order conditions are

$$r_s = A_i p_i \; F_{is}(K_{i1}, K_{i2})$$

Note that the marginal products (b/c of constant returns to scale) only depend on the inputs ratios $k_i = K_{i2}/K_{i1}$:

$$r_s = A_i p_i \; F_{is}(1, k_i)$$

(slightly abusing notation) (4 equations). With better notation: define $f_i(k_i) = F_i(1, K_{i2}/K_{i1})$. Then the firms' FOCs become

$$r_1 = A_i \, p_i \, [f_i(k_i) - f_i'(k_i) \, k_i] \tag{6}$$

$$r_2 = A_i \, p_i \, f_i'(k_i) \tag{7}$$

for $i = 1, 2$. This is entirely analogous to a model with capital and labor. Note that a higher k_i reduces $f_i'(k_i)$ but increases $f_i(k_i) - f_i'(k_i) \, k_i$.

6. Take the ratio of (7), (6) for both sectors and write this as $r_1/r_2 = g_i(k_i)$. Note that $g_i'(k_i) > 0$. From $g_1(k_1) = g_2(k_2)$ it follows that k_1 and k_2 are positively related. Define this relationship as $k_2 = h(k_1) = g_2^{-1}(g_1(k_1))$. The positive relationship is not surprising. When r_1/r_2 increases, firms in both sectors substitute towards the cheaper capital good. Now consider the condition

$$r_1 = A_1 \, [f_1(k_1) - f_1'(k_1) \, k_1] = A_2 \, f_2'(k_2) = r_2/p_2 \tag{8}$$

This can be written as

$$\frac{f_1(k_1) - f_1'(k_1) \, k_1}{f_2'(h(k_1))} = \frac{A_2}{A_1} \tag{9}$$

The LHS of (9) is increasing in k_1. It follows that a higher A_1 reduces k_1 and k_2. The intuition is that K_1 is cheaper to produce and used relative more intensively. A lower k_2 implies a higher $r_1 = r_2/p_2$ by (8). Therefore R and the balanced growth rate γ must both increase.

4.3 Answer: R&D: Durable Intermediates

1. Final goods firm: $\{y_t, L_t, x_{j,t}\}$ solve the production function and the FOCs

$$w_t = \alpha y_t / L_t \tag{10}$$

$$R_{j,t} = (1-\alpha) \, A L^\alpha x_j^{-\alpha} \tag{11}$$

2. Intermediate goods firm: This is really the same as the problem of a firm that owns the capital stock in the standard growth model. The only difference is that the firm does not take R as given - it depends on x.

$$V = \max \int e^{-rt}[R(x_t)\, x_t - I_t]dt$$

subject to

$$\dot{x} = \eta I_t^{\varphi} - \delta x \tag{12}$$

Hamiltonian:

$$H = R(x)\, x - I + \mu\left[\eta I^{\varphi} - \delta x\right] \tag{13}$$

FOCs:

$$\begin{aligned} \partial H/\partial I &= -1 + \mu\eta\varphi I^{\varphi-1} = 0 \\ \dot{\mu} &= (r+\delta)\,\mu - R'(x)\, x - R(x) \end{aligned}$$

Solution: $\{I_t, x_t, \mu_t\}$ that solve 2 FOCs and law of motion for x. Boundary conditions: $x(0) = 0$ given, $\lim_{t\to\infty} e^{-rt}\mu_t x_t = 0$.

3. Equilibrium: $\{R_{j,t}, x_{j,t}, N_t, I_{j,t}, \mu_{j,t}, y_t, L_t, r_t, c_t, w_t\}$ that solve:

- Household: 2
- Final goods: 3
- Intermediate goods: 3
- Free entry: Spend $\beta\ dt$ to obtain $dN = \beta/\beta\ dt$ new patents worth $V\ dt$. Equate cost and profits:

$$\beta = V = \int e^{-rt}\left[R(x_t)\, x_t - I_t\right] dt \tag{14}$$

- Goods market clearing: $y = c + NI + \dot{N}\beta$.
- Labor market clearing: $L = 1$.
- Asset market clearing: This depends, as usual, on the structure of asset markets. If households own the firms $a_t = \int_0^{N_t} V_{j,t}$. Alternatively, one could assume that innovators issue bonds.

4. Case $\varphi = 1$: The FOCs imply together with the constant elasticity demand function:

$$\begin{aligned} \mu &= (\eta\varphi)^{-1} \\ (r+\delta)\,\mu &= R(x)(1-\alpha) \end{aligned}$$

Therefore, x and μ must be constant over time. With a linear technology, the best approach is to build all x in one shot, then keep x constant.

5. Equilibrium values:

$$R = \frac{r+\delta}{\eta(1-\alpha)} = A(1-\alpha)L^{\alpha}x^{-\alpha} \tag{15}$$

Zero profit solves for x:

$$\begin{aligned} \beta &= \frac{Rx - I}{r} - I_0 \\ &= \frac{x\alpha}{\eta(1-\alpha)}\frac{r+\delta}{r} \end{aligned}$$

Substitute into the first-order condition for x (15) to solve for r.

End of exam.