Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **three** questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**. Number your answers.
- Good luck!

1 Barro-Gordon Rules versus Commitment

Consider an economy in which a central bank tries to minimize a loss function of the form

$$V = \frac{1}{2}\delta\left(y - (y_n + \kappa)\right)^2 + \frac{1}{2}\pi^2 \tag{1}$$

where y denotes output, y_n is the exogenous full employment output level associated with zero inflation, π inflation, $\delta, \kappa > 0$. Assume the central bank can control inflation up to a random mean-zero error ν with variance σ_ν^2,

$$\pi = \widetilde{\pi} + v \tag{2}$$

where $\widetilde{\pi}$ is set by the central bank *(here we ignore difference between target and instrument)*. Suppose aggregate output is is given by

$$y = y_n + a(\pi - \pi^e) + \varepsilon \tag{3}$$

where π^e denotes inflation expectations. ε is a random mean-zero supply shock with variance σ_e^2. $a > 0$

The timing of the model is such that households first make decisions and form expectations before the ε shock hits the economy. The central bank, upon observing ε and π^e, and sets $\widetilde{\pi}$. Finally, v is realized and inflation and output determined.

Questions:

1. Does policy ineffectiveness hold in this version of the Barro-Gordon model? That is, demonstrate the extent of any inflation bias under discretionary policy.

2. Derive the equilibrium inflation and output under discretion.

3. Suppose the monetary authority can commit to a rule

   $$\widetilde{\pi} = \phi_0 + \phi_1\varepsilon \tag{4}$$

   such that inflation deviates from a constant only due to shocks. Derive the optimal weights ϕ_0 and ϕ_1.

4. Compare the welfare costs under discretion versus commitment.

2 OLG Model With Money

Demographics: In period t $(1+n)^t$ young are born. Each lives for 2 periods.

Endowments: The young receive $e_1$units of consumption; the old receive e_2. The initial old at $t=1$ hold M_1 units of paper.

Technology: This is an endowment economy. Goods cannot be produced or stored.

Preferences: $u(c_t^y) + \beta u(c_{t+1}^o)$.

Government: hands out θ units of money for each unit of money held at the beginning of each period.

Questions:

1. Write down the Euler equation. You need not derive it as it is well known.
2. State the market clearing conditions.
3. Derive an implicit equation for the offer curve that relates m_{t+1} to m_t, where m_t denotes real money holdings per young household.
4. Assume $u(c) = c^{1-\sigma}/(1-\sigma)$ and simplify the offer curve.
5. Solve for the steady state m.
6. Show that offer curve is upward sloping for $\sigma = 1$ (log utility).

3 Guvenen, Kuruscu and Ozkan

Consider a household who lives for T periods. At birth, he is endowed with human capital h_1 and assets a_1. He solves the following problem

$$\max_{\{c_t, n_t, a_{t+1}, i_t\}} \sum_{t=1}^{T} \beta^t u(c_t, 1-n_t) \tag{5}$$

subject to

$$c_t + a_{t+1} = (1-\tau(y_t))\, y_t + R a_t \tag{6}$$
$$h_{t+1} = h_t + A(h_t i_t n_t)^\alpha \tag{7}$$
$$y_t = w h_t (1-i_t) n_t \tag{8}$$

Here a is assets, c is consumption, n is hours worked, y is labor income, $\tau(y)$ is a labor income tax, R is the gross interest rate, i is the fraction of time spent on human capital investment, w is the wage rate. $A > 0$ and $0 < \alpha < 1$ are parameters.

It is convenient to define human capital investment as $Q_t = A(h_t i_t n_t)^\alpha$. Then $h_{t+1} = h_t + Q_t$ and $y_t = w h_t n_t - wC(Q_t)$ where $wC(Q_t) = wQ_t^{1/\alpha}/A$ is the cost of human capital investment.

Questions:

1. State the household's dynamic program. It is helpful to keep the budget constraint and law of motion for h as separate constraints with their own multipliers.

2. Derive the first-order conditions and envelope equations.

3. Derive the consumption Euler equation and the static labor-leisure condition. Explain the latter in words. Note that $\hat{\tau}(y) = 1 - \tau(y) - \tau'(y)y$ is 1 minus the marginal tax rate.

4. Starting from $C'(Q) = \gamma/(w\lambda)$, where λ is the Lagrange multiplier for the budget constraint and γ is the one for the law of motion for h, *sketch* how one could derive a closed form solution for human capital investment

$$C'(Q_t) = \sum_{j=1}^{T-t} (\beta R)^j \frac{\hat{\tau}(y_{t+j})}{\hat{\tau}(y_t)} n_{t+j} \tag{9}$$

   I only ask that you start the derivation and then explain in words how to proceed. You need not work out all the details. Interpret this condition in words. Trick: What is λ_{t+j}/λ_t according to the Euler equation?

5. Based on this solution, how would you expect the following to affect human capital investment:

(a) a flat income tax $\tau(y) = \bar{\tau}$;

(b) a progressive income tax where $\tau'(y) > 0$.

End of exam.

4 Answers

4.1 Barro-Gordon

1. Central Bank chooses $\widetilde{\pi}$ taking π^e as given and minimnize loss function equation (1) subject to equations (2) and (3). The FOC is

$$\widetilde{\pi} = \frac{\delta\alpha^2\pi^e + \delta\alpha(\kappa - \varepsilon)}{1 + \delta\alpha^2}$$

we know from monetary policy rule (equation 4) that $\pi^e = \widetilde{\pi}$. Taking expectations and substituting we get

$$\pi^e = \frac{\delta\alpha^2\pi^e + \delta\alpha\kappa}{1 + \delta\alpha^2}$$

Solve to get

$$\pi^e = \delta\alpha\kappa > 0$$

2. Given π^e the central bank sets

$$\widetilde{\pi} = \delta\alpha\kappa - \frac{\delta\alpha\varepsilon}{1 + \delta\alpha^2}$$

actual inflation is

$$\pi = \delta\alpha\kappa - \frac{\delta\alpha\varepsilon}{1 + \delta\alpha^2} + \nu$$

and output

$$y = y_n + \frac{\varepsilon}{1 + \delta\alpha^2} + \alpha\nu$$

3. Now carry out the minimization of the loss function by choosing ϕ_0 and ϕ_1. The new monetary policy rule implies that $\pi^e = \phi_0$. The FOC for ϕ_0 results in $\phi_0 = 0$. The FOC for ϕ_1 implies that

$$\phi_1 = -\frac{\delta\alpha}{1 + \delta\alpha^2}$$

4. $E(V^{Discretion}) - E(V^{Commitment}) = \frac{1}{2}\delta\left(\frac{\delta\alpha^2}{1+\delta\alpha^2}\right)\sigma_\varepsilon^2 + \frac{1}{2}(\delta\alpha\kappa)^2 > 0$ Expected loss greater under discretionary policy.

4.2 OLG model with money

1. Euler: $u'(c_t^y) = \beta R_{t+1} u'(c_{t+1}^o)$ where $R_{t+1} = (1 + \theta)P_t/P_{t+1}$.

2. Market clearing:

   (a) goods: $N_t e_1 + N_{t-1} e_2 = N_t c_t^y + N_{t-1} c_t^o$.

   (b) money: $m_t = e_1 - c_t^y$.

3. Sub in the budget constraints:

$$u'(e_1 - x_t) = \beta R_{t+1} u'(e_2 + R_{t+1} x_t) \tag{10}$$

Market clearing implies $m_t = x_t$. The law of motion for nominal money is $M_{t+1} = (1+\theta)M_t$. For real, per young money balances: $(1+n)m_{t+1}P_{t+1} = (1+\theta) P_t m_t$. So $(1+n) m_{t+1} = R_{t+1} m_t$. Substituting both into the Euler equation yields

$$u'(e_1 - m_t) = \beta R_{t+1} u'(e_2 + (1+n)m_{t+1}) \tag{11}$$

Next: $R_{t+1} = (1+\theta)P_t/P_{t+1} = (1+n)m_{t+1}/m_t$. Substitute that into the Euler equation as well to obtain the offer curve.

4. Let $\rho = -1/\sigma$. Then

$$(e_1 - m_t)^{-\sigma} = \beta(1+n)\frac{m_{t+1}}{m_t}(e_2 + (1+n)m_{t+1})^{-\sigma} \tag{12}$$

$$m_t^{\rho} e_1 - m_t^{1+\rho} = [\beta(1+n)]^{\rho}\left[m_{t+1}^{\rho} e_2 + (1+n)m_{t+1}^{1+\rho}\right] \tag{13}$$

5. Divide by m:

$$e_1 - m^{\rho} = [\beta(1+n)]^{\rho}[e_2 + (1+n)m^{\rho}] \tag{14}$$

Rearrange:

$$m^{\rho} = \frac{e_1 - e_2[\beta(1+n)]^{\rho}}{1 + (1+n)[\beta(1+n)]^{\rho}} \tag{15}$$

which has the plausible properties that higher e_1 raises m while higher e_2 lowers it.

6. $\sigma = 1 \implies \rho = -1$ and $1 + \rho = 0$. Offer curve becomes

$$m_t^{-1} e_1 = [\beta(1+n)]^{-1} e_2 m_{t+1}^{-1} \tag{16}$$

which is upward sloping.

4.3 Guvenen, Kuruscu and Ozkan

1. Dynamic program

$$V(h, a, t) = \max_{h', a', Q, y, n} u([1 - \tau(y)]\, y + Ra - a', 1 - n) + \beta V(h', a', t+1) \tag{17}$$

$$+\lambda (whn - wC(Q) - y) \tag{18}$$

$$+\gamma (h + Q - h') \tag{19}$$

2. FOC:

$$\beta V_h(.') = \gamma \tag{20}$$
$$u_c = \beta V_a(.') \tag{21}$$
$$\lambda w C'(Q) = \gamma \tag{22}$$
$$u_c \left[1 - \tau(y) - \tau'(y)\, y\right] = \lambda \tag{23}$$
$$u_l = \lambda w h \tag{24}$$

Envelope

$$V_a = u_c R \tag{25}$$
$$V_h = \lambda w n + \gamma \tag{26}$$

3. We get a standard Euler equation $u_c = \beta R' u_c(.')$ and a standard static condition for labor-leisure choice

$$u_c \left[1 - \tau(y) - \tau'(y)\, y\right] = u_l / (wh) \tag{27}$$

In words: Give up a unit of leisure at cost u_l. That adds h units of effective labor supply at marginal wage $\hat{\tau}(y)w$, which can be eater at marginal utility u_c.

4. Optimal human capital investment: Start from $\gamma = \beta\left(\gamma' + \lambda' w' n'\right)$. Next,

$$C'(Q_t) = \frac{\gamma_t}{w\lambda_t} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{w_{t+1}}{w_t} n_{t+1} + \beta \frac{\gamma_{t+1}}{w\lambda_t} \tag{28}$$

Assume that wages are constant and iterate forward to obtain the discounted sum of λ_{t+j}/λ_t, which from the Euler equation equals $(\beta R)^j$. In words: the marginal cost of another unit of human capital equals the present discounted value of the (marginal) after tax incomes it generates over an agent's remaining lifetime.

5. Taxes

   (a) If leisure were exogenous, the flat tax would not affect human capital investment as the $\hat{\tau}$ terms cancel. However, the flat income tax would reduce labor supply and therefore human capital investment.

   (b) A progressive that has an additional effect. For young agents, who do most of the investment, future earnings are higher than current earnings, so that the tax wedge $\hat{\tau}\left(y_{t+j}\right)/\hat{\tau}\left(y_t\right) < 1$, which further reduces investment.

End of exam.