Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of **three** questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**. Number your answers.
- Good luck!

1 Trees and Bonds

Consider a Lucas fruit tree economy with multi period bonds.

Demographics: There is a representative consumer who lives forever.

Preferences: $\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_t)$ with $u(c) = c^{(1-\sigma)}/(1-\sigma)$.

Endowments: The household is endowed with one tree $t = 0$. The tree yields a dividend that evolves according to

$$\ln d_{t+1} = \ln d_t + g + \varepsilon_{t+1} \tag{1}$$

where $\varepsilon_{t+1} \sim N(0, v)$. v is the variance of the shock. g is the trend growth rate of the dividend. A statistical result: the distribution of $\ln d_{t+n}$ conditional on $\ln d_t$ is $N(\ln d_t + ng, nv)$.

Market arrangements: Households trade in competitive markets: (a) goods at price 1, (b) trees at prices q_t. q_t is "ex dividend" - it values the type i tree that has already paid its period t dividend.

The new feature: In addition to the tree, agents trade n period discount bonds at price p_{nt} for some fixed $n > 1$. The bond issued at t pays one unit of consumption at $t + n$.

Questions:

1. State the household's dynamic program.
2. Derive the Lucas asset pricing equations for the tree and the bond.
3. Show that the equilibrium bond price is given by

$$p_{nt} = \beta^n \exp\left(-\sigma n[g + v/2]\right) \tag{2}$$

   Note that $\mathbb{E}x^{-\alpha} = \mathbb{E}\left\{\exp(-\alpha \ln x)\right\} = \exp\left(-\alpha\mathbb{E}\ln x - \frac{\alpha}{2}Var(\ln x)\right)$ when $\ln x$ is a Normal random variable (as is the dividend in the model).

4. Derive the per period return for the n period bond. Is the yield curve upward sloping or downward sloping?
5. What is the intuition why the bond price does not depend on the current state d_t?

2 An Investment Problem

Consider the problem of an infinitely lived firm that invests in capital K_t subject to an adjustment cost.

Time is continuous. The profit stream is given by

$$\pi_t = f(k_t) - I_t - \phi(I_t) \tag{3}$$

where f obeys Inada conditions and the adjustment cost is convex: $\phi' > 0$ and $\phi'' > 0$. $\phi(0) = 0$.

The firm maximizes the discounted present value of profits

$$\max_{I_t, K_t; t \geq 0} \int_0^\infty e^{-rt} \pi_t dt \tag{4}$$

subject to the law of motion

$$\dot{k}_t = I_t - \delta k_t \tag{5}$$

Questions:

1. Derive the necessary conditions for the firm's optimal investment plan, including the transversality condition.
2. From the necessary conditions, derive
$$\dot{I}_t = \frac{(r+\delta)(1+\phi'(I_t)) - f'(k_t)}{\phi''(I_t)} \tag{6}$$
3. Draw a phase diagram in (k_t, I_t) space. For simplicity, assume that the $\dot{I} = 0$ locus is downward sloping.
4. Discuss the stability properties of the steady state.

3 New Keynesian Model

This question will start from the point where the firm's and household's problems are already solved and the economy is reduced to two equations representing the dynamic IS and New Keynesian Phillips curve. Later we will add the monetary authority's reaction function to close the economy.

Model I: Consider the following forward-looking model:

$$y_t = E_t(y_{t+1}) - \beta r_t + v_t \tag{7}$$

$$\pi_t = E_t(\pi_{t+1}) + \alpha y_t + u_t \tag{8}$$

Equation (1) is the dynamic IS schedule while equation (2) is the New Keynesian Phillips curve, where:

- y - output gap
- r - the real interest rate (the policy instrument)
- π - the inflation rate
- v, u - zero mean serially uncorrelated innovations
- $\alpha, \beta > 0$

Assume that the policymaker minimizes the following loss function

$$L = Var(y_t) + \varphi Var(\pi_t) \tag{9}$$

and that optimal policy implies a systematic relationship between the output gap and inflation of the form:

$$\theta y_t + \pi_t = 0 \tag{10}$$

1. Derive the optimal value for θ. Interpret.
2. What policy response does the optimal θ imply for each shock?

Model II: The IS and Phillips curves are given by

$$y_t = E_t(y_{t+1}) - \beta(i_t - E_t(\pi_{t+1})) + v_t \tag{11}$$

$$\pi_t = \lambda E_t(\pi_{t+1}) + \alpha y_t + u_t \tag{12}$$

where the nominal interest rate (i_t) is now the policy instrument. Let us close the model with the following policy rules

- rule A: $i_t = \varepsilon_t$
- rule B: $i_t = \phi\pi_t + \varepsilon_t$

where ε_t is an exogenous stochastic process.

Compare the above rules for determinacy of equilibrium. Give intution for both rules. Is inflation self-fulfilling under one or both rules? *For this question it is more important to get the intuition right.*

4 Answers

4.1 Answer: Trees and Bonds

1. This is standard.
2. Asset pricing equation for the bond:

$$p_{nt} = \beta^n \mathbb{E}\left\{\frac{u'(d_{t+n})}{u'(d_t)}\right\} = \beta^n d_t^{\sigma} \mathbb{E}\left\{d_{t+n}^{-\sigma}\right\} \tag{13}$$

3. Note that $\ln d_{t+n} = \ln d_t + ng + \sum_{i=1}^{n} \varepsilon_{t+i} \sim N(\ln d_t + ng, nv)$. From the hint,

$$\mathbb{E} d_{t+n}^{-\sigma} = \exp\left(-\sigma(\ln d_t + ng) - \frac{\sigma}{2} nv\right) \tag{14}$$

   In the pricing equation, the d_t terms cancel and we have 2.

4. The yield per period is $1 + r_{nt} = (1/p_{nt})^{1/n} = \beta^{-1} \exp\left(\sigma(g + \frac{v}{2})\right)$. The model implies that the yield curve is flat.
5. Think of a detrended model - divide everything through by d_t. Then going forward the household problem looks exactly the same each period. With homothetic preferences, this is valid. Doubling all quantities does not change the relative marginal utilities (and hence prices) of the quantities.

Extension: Show that the price of the tree is given by

$$q_t = d_t \sum_{j=1}^{\infty} \beta^j \exp\left((1-\sigma) j (g + v/2)\right) \tag{15}$$

Here you want to start from

$$\mathbb{E} d_{t+j}^{1-\sigma} = \exp\left((1-\sigma)\,\mathbb{E} \ln d_{t+j} + \frac{1-\sigma}{2} Var \ln d_{t+j}\right) \tag{16}$$

and then sub in for the moments of $\ln d_{t+j}$ using the hint.

4.2 An Investment Problem[1]

1. Hamiltonian:

$$H = f(k_t) - I_t - \phi(I_t) + q_t[I_t - \delta K_t] \tag{17}$$

[1]Based on chapter 7 of Acemoglu, D. (2011). Introduction to modern economic growth. Princeton University Press.

First-order conditions:

$$I_t : q_t = 1 + \phi'(I_t) \tag{18}$$

$$K_t : f'(k_t) - \delta q_t = r q_t - \dot{q}_t \tag{19}$$

Transversality:

$$\lim_{t \to \infty} e^{-rt} q_t k_t = 0 \tag{20}$$

2. Differentiate (18) and substitute into (19):

$$\dot{I}_t = \frac{(r + \delta)(1 + \phi'(I_t)) - f'(k_t)}{\phi''(I_t)} \tag{21}$$

3. Phase diagram: $\dot{k} = 0$ is evidently an upward sloping straight line. For k above the locus $\dot{k} > 0$. By assumption $\dot{I} = 0$ is downward sloping (this assumption does not matter for the results because the locus is always downward sloping near the steady state). For k above the locus $\dot{I} > 0$.

4. The initial condition pins down k_0 but not I_0. We need to argue that a unique I_0 is admissible for any k_0 and that the dynamics leads to the steady state. The argument is analogous to the growth model.

   Aside from paths that lead to the steady state, paths either feature k and $I \to \infty$ or k and $I \to 0$.

   To see that the first type of paths violates transversality, note that (19) implies that $g(q) \to r + \delta$. The second type of path leads to $I = 0$ in finite time. At the same time $k \to 0$ which violates (19). It follows that the steady state is saddle path stable.

4.3 New Keynesian Model

Model I: 1. Plug (1) and (2) into (4) and solve for r_t

$$r_t = \frac{E_t(\pi_{t+1}) + \alpha y_t + u_t + \theta \left[E_t(y_{t+1}) + v_t\right]}{\theta \beta} \tag{22}$$

now plug into dynamic IS

$$y_t = \left(\frac{-1}{\alpha + \theta}\right) \left[E_t(\pi_{t+1}) + u_t\right] \tag{23}$$

and plug above equation into NKPC

$$\pi_t = \left(\frac{\theta}{\alpha + \theta}\right) \left[E_t(\pi_{t+1}) + u_t\right] \tag{24}$$

Use method of undertermined coeeficients. Propose

$$y_t = \phi_{11} u_t \tag{25}$$
$$\pi_t = \phi_{21} u_t \tag{26}$$

this implies that

$$y_{t+1} = \phi_{11} u_{t+1} \tag{27}$$
$$\pi_{t+1} = \phi_{21} u_{t+1} \tag{28}$$

this implies $E_t(\pi_{t+1}) = E_t(y_{t+1}) = 0$

now this implies that

$$y_t = \left(\frac{-1}{\alpha + \theta}\right) u_t \tag{29}$$

and

$$\pi_t = \left(\frac{\theta}{\alpha + \theta}\right) u_t \tag{30}$$

and that

$$Var(y_t) = \left(\frac{1}{\alpha + \theta}\right)^2 \sigma_u^2 \tag{31}$$

and

$$Var(\pi_t) = \left(\frac{\theta}{\alpha + \theta}\right)^2 \sigma_u^2 \tag{32}$$

the loss function now becomes

$$L = \left(\frac{1}{\alpha + \theta}\right)^2 \sigma_u^2 + \varphi \left(\frac{\theta}{\alpha + \theta}\right)^2 \sigma_u^2 \tag{33}$$

minimize with respect to θ to get

$$\theta^* = \frac{1}{\varphi \alpha} \tag{34}$$

Interpret: notice the more the CB care about inflation φ the lower the weight on output θ in the target rule. The larger the weight on the output gap α in the NKPC the lower weight the CB needs to put on output in the target rule.

2. To see what policy response is implied for both shocks use

$$r_t = \frac{E_t(\pi_{t+1}) + \alpha y_t + u_t + \theta \left[E_t(y_{t+1}) + v_t\right]}{\theta \beta} \tag{35}$$
$$= \frac{\alpha \left(\frac{-1}{\alpha+\theta}\right) u_t + u_t + \theta v_t}{\theta \beta} \tag{36}$$
$$= \frac{1}{\beta} v_t + \frac{u_t}{\beta(\alpha + \theta)} u_t \tag{37}$$

at optimum $\theta^* > 0$.

For positive v shocks, y and π increase. The CB increases r from above which then reduces y and then lowers π. The intial increases in ouput and inflation are subsequently reversed.

For positive u shocks, π increases, the CB increases r from above which then reduces y and then lowers π. Notice we now have a fall in output that is not offset by an initial increase. The CB controls inflation by sacrificing output growth.

Model II: Solving for determinacy:

First substitute $i_t = \varepsilon_t$ into DIS and set up matrix

$$\begin{bmatrix} E_t(y_{t+1}) \\ E_t(\pi_{t+1}) \end{bmatrix} = \begin{bmatrix} 1+\frac{\alpha\beta}{\lambda} & -\frac{\beta}{\lambda} \\ -\frac{\alpha}{\lambda} & \frac{1}{\lambda} \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \beta v_t - u_t \\ 0 \end{bmatrix} \tag{38}$$

for determinacy check that the roots of $\begin{bmatrix} 1+\frac{\alpha\beta}{\lambda} & -\frac{\beta}{\lambda} \\ -\frac{\alpha}{\lambda} & \frac{1}{\lambda} \end{bmatrix}$ lie inside the unit circle. It does not. Lets says there is an increased in expected inflation, with no feedback to i the real rate will fall, leading to an increase in the output gap, leading to an increase in inflation. Here inflation is self-fulfilling.

Now lets use $i_t = \phi\pi_t + \varepsilon_t$ we get

$$\begin{bmatrix} E_t(y_{t+1}) \\ E_t(\pi_{t+1}) \end{bmatrix} = \begin{bmatrix} 1+\frac{\alpha\beta}{\lambda} & \frac{\beta(\lambda\phi-1)}{\lambda} \\ -\frac{\alpha}{\lambda} & \frac{1}{\lambda} \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \beta v_t - u_t \\ 0 \end{bmatrix} \tag{39}$$

here we have the Taylor principle. Stationary (unique) equlibrium exists when there is a sizeable weight on inflation $\phi > 1$. Now an increase in expected inflation will lead to a fall in real rate, an increase in output gap, an increase in inflation, but then an increase in the nominal rate which is contractionary leading to a fall in inflation. Not self-fulfilling.

End of exam.