Macroeconomics Qualifying Examination

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Department of Economics

UNC Chapel Hill

Instructions:

- This examination consists of three questions. Answer all questions.
- Answering only two questions (or providing a cursory answer to the third question) will greatly reduce your chances of passing the exam.
- If you believe a question is ambiguously stated, ask for clarification. Unnecessary simplifications will be penalized.
- Do not consult any books, notes, calculators, or cell phones.
- **Write legibly**.
- Good luck!

1 Overlapping Generations

Consider an overlapping generations model with a government.

Demographics: Time is discrete: $t = 0, 1, 2, ...$ In each period $N_t = (1+n)^t$ persons are born. Each person lives for 2 periods.

Preferences:

$$U(c_{1t}, c_{2t+1}) = log(c_{1t}) + log(c_{2t+1})$$

Endowments: Young persons are endowed with one unit of labor. The *initial* old are each endowed with x_0 units of capital.

Technology: The good is produced from capital and labor according to

$$F(K_t, L_t) = K_t^{\alpha}(A_t L_t)^{(1-\alpha)}.$$

The good can be consumed or saved as capital: $Y_t + (1-\delta)K_t = C_t + K_{t+1}$. A_t is an exogenous productivity sequence.

Government: The government levies lump sum taxes T_t on each young and issues $N_t b_{t+1}$ one period bonds.

Markets: All markets are competitive. Firms rent capital and labor from households are rental prices q_t and w_t.

Questions:

1. Solve for the household's optimal consumption and saving rules in closed form. If you cannot solve this part, assume that the household saves a constant fraction θ of her after tax income.

2. Prove that the evolution of the capital stock per capita (NOT per young person) is given by:

$$k_{t+1} = \frac{1}{2(1+n)}(1-\alpha)A_t^{(1-\alpha)}k_t^{\alpha} - \frac{1}{2(1+n)}T_t - \frac{1}{(1+n)}b_{t+1}.$$

3. For the case in which labor productivity, taxes and bond emissions are constant ($A_t = A, T_t = T, b_{t+1} = b$) determine the expression that defines the steady state level of capital per capita.

4. Draw the law of motion for capital in (k_{t+1}, k_t) space. Be precise about the details of the graph, in particular the points at which the curves cut the different axes. How many steady states does this economy exhibit? How many are stable?

5. Assume that initially the economy is in the highest stable steady state. Then the economy experiences a productivity based recession, that is A falls permanently from A_0 to A_1 ($A_1 < A_0$).

(a) What happens with the steady state level of capital and output?

(b) Show the effect of the fall in productivity in a (k_{t+1}, k_t) graph.

6. Suppose that the government wants to stimulate the economy, that is to increase the steady state level of output.

   (a) Can the government achieve its goal by reducing taxes and increasing bond emissions in the same amount($\Delta b = -\Delta T$)? Show the effect of this policy in a (k_{t+1}, k_t) graph.

   (b) What is the economic reason?

   (c) Do you think this is a realistic prediction?

   (d) What feature of the model is behind this result?

2 Heterogeneous Preferences

Consider a standard growth model with two modifications:

1. Households have different utility functions.
2. Shares of the firm are traded.

The details are as follows.

Households: There are N_i infinitely lived households of type i who maximize $\sum_{t=0}^{\infty} \beta^t \ u_i(c_{it})$. Each household is endowed with k_{i0} units of capital and x_{i0} shares of the firm. The household works one unit of time in each period at wage rate w_t. Savings in the form of capital earn a gross interest rate of R_t. Households choose consumption c_{it}, saving in the form of capital k_{it+1}, and saving in the form of shares x_{it+1}. The share price is p_t. The dividend per share is d_t.

Firms: There is a single representative firm that maximizes the present value of profits. The firm produces the good from capital and labor according to the production function $F(K_t, L_t)$ which has *diminishing* returns to scale. Profits are given by $\pi_t = F(K_t, L_t) - q_t K_t - w_t L_t$ and paid out as dividends in each period ($d_t = \pi_t$).

Questions:

1. Write down the household's period budget constraint and the Bellman equation.
2. Derive the household's first-order conditions and define a solution to the household problem.
3. Define a solution to the firm's problem.
4. Define a competitive equilibrium.
5. Define a steady state. Can you determine the steady state interest rate without knowing the initial distribution of assets (k_{i0}, x_{i0}).
6. Suppose the initial capital endowments of all households add up to the steady state capital stock. What will happen to consumption inequality over time? Will it vanish or will it remain constant? [*Hint:* Which aggregate quantities can you determine in steady state without knowing the distribution of capital (and thus consumption)?]

3 R&D With Capital

Demographics: Households are infinitely lived. The population grows at rate n: $L_t = e^{nt}$. Time is continuous.

Preferences:

$$\int_0^\infty e^{-\rho t} u(c_t) dt$$

with $u(c) = c^{1-\sigma}/(1-\sigma)$.

Endowments: At the beginning of time ($t = 0$) households are endowed with capital K_0 and knowledge stock A_0.

Technologies:

- Final goods are produced from labor and intermediates. They are used for consumption and capital accumulation:

$$Y_t = \left(\int_0^{A_t} x_{it}^\theta di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} = L_t c_t + \dot{K}_t - \delta K_t \tag{1}$$

  Intermediate inputs perish in the production of the final good (they are not durable).

- New ideas are produced from ideas and labor:

$$\dot{A}_t = \nu A_t^\phi L_{At}^\lambda \tag{2}$$

  with $\phi < 1$, $\lambda > 0$.

- Intermediate goods are produced from capital. Each unit of capital yields a flow of intermediate goods which is constrained by

$$\int_0^{A_t} x_{it} di = K_t \tag{3}$$

The government taxes capital income at rate τ and rebates the revenues to households in lump-sum fashion.

The household solves a standard problem with a complicated budget constraint. He holds capital and shares of firms. But we know that the result is a standard Euler equation of the form

$$g(c_t) = \dot{c}_t / c_t = \frac{(1-\tau) r_t - \rho}{\sigma} \tag{4}$$

and we don't worry about the details. r is the rate of return on capital and shares.

Questions:

1. State the problem of the final goods producing firm. It behaves competitively in all markets. The input prices are w_t and p_{it}. The price of the final good is normalized to 1. Derive the first-order conditions.

2. State the problem of the intermediate goods producer. It rents capital at interest rate r_t from the household, so that r_t is also it's marginal cost. Derive the profit maximizing price p_{it}.

3. Innovators see their production function as having constant returns in labor: $\dot{A}_t = \bar{\nu}_t L_{At}$ where $\bar{\nu}_t = A_t^{\phi} L_{At}^{\lambda-1}$. The fact that, in equilibrium, higher L_A leads to diminishing returns (congestion) is not taken into account by the innovators. Innovators become intermediate goods producers after the innovation. State the problem of the innovating firm and show that the value of a patent is given by $P_{At} = w_t/\bar{\nu}_t$.

4. Explain the free entry condition for innovators:

$$r_t P_{At} = \pi_{it} + \dot{P}_{At} \tag{5}$$

   where π_{it} is the flow profit earned by selling x_{it}.

5. For the remainder of the question, assume a symmetric equilibrium. Derive the balanced growth rate for A. Hint: This depends on feasibility conditions only.

   (a) Why does the balanced growth rate not depend on first-order conditions?

6. How does a change in the tax rate affect the long-run after-tax interest rate and the growth rate of consumption? What is the intuition for this?

4 Answers

4.1 Answer: Heterogeneous Preferences

1. Household budget constraint:

$$k_{it+1} + p_t x_{it+1} + c_{it} = w_t + R_t k_{it} + (d_t + p_t)\, x_{it}$$

2. Household solution: Dynamic program:

$$\begin{aligned} V_i(k, x) &= \max\; u_i(c) + \beta\; V_i(k', x') \\ &\quad + \lambda\,[w + Rk + (d + p)\, x - c - px' - k'] \end{aligned}$$

First-order conditions:

$$\begin{aligned} u_i'(c) &= \beta\; R'\; u_i'(c') \\ R' &= \frac{p' + d'}{p} \end{aligned}$$

Solution: Sequences $\{c_{it}, k_{it+1}, x_{it+1}\}_{t=0}^{\infty}$ that satisfy 2 FOC, budget constraint, TVC.

3. Firm: Since the firm does not own the capital stock, it simply maximizes period profits. FOCs:

$$\begin{aligned} F_K &= q \\ F_L &= w \end{aligned}$$

4. Competitive Equilibrium Sequences $\{c_{it}, k_{it+1}, x_{it+1}, C_t, K_t, L_t, d_t, w_t, R_t, q_t, p_t\}$ that satisfy:

- Household conditions (3).
- Firm conditions (3).
- $R = q + 1 - \delta$.
- Market clearing:

$$\begin{aligned} F(K_t, L_t) &= C_t + K_{t+1} - (1 - \delta)\, K_t \\ C_t &= \sum N_i c_{it} \\ K_t &= \sum N_i k_{it} \\ L_t &= \sum N_i \\ X &= \sum N_i x_{it} \end{aligned}$$

The last equation equates the fixed supply of shares to demand for shares.

5. Steady state: A vector (K, L, R, w, q, p) and a distribution over household values (c_i, k_i, x_i) that satisfy:

- Household: $\beta R = 1, R = \frac{d+p}{p}$ and present value budget constraint for each household.
- Firm: unchanged with $\pi = d = F(K, L) - wL - qK$.
- $R = \frac{d+p}{p}$.
- Market clearing: unchanged.

For each household type, constant consumption implies $\beta R = 1$.

6. Steady state inequality: Consumption inequality will remain constant over time. The economy is immediately in steady state. To see this note that from the definition of steady state any distribution of assets such that $K_t = K_{ss}$ is a steady state. This is because the steady state has a recursive structure:

- $\beta R = 1 \rightarrow R$.
- $R \rightarrow F_K \rightarrow K, w, q$.
- $d = F(K, L) - wL - qK$.
- $R = \frac{d+p}{p} \rightarrow p$.
- Market clearing $\rightarrow C$.
- Distribution c_i is determined from household's present value budget constraints. These have the key feature that c_i is linear in household wealth (with constant prices):

  $$c_{i0}/(R-1) = w/(R-1) + k_{i0} + p_0 x_{i0}$$

  All households have the same marginal propensity to consume (in steady state). Redistributing wealth leaves aggregate consumption unchanged. Any distribution of assets that adds up to K_{ss} is a steady state.

4.2 Answer: R&D

This question is based on "Dynamic Scoring in a Romer-style Economy" by Dean Scrimgeour.

1. Final goods producer:

$$max_{x_{it},L_{Yt}} \left(\int_0^{A_t} x_{it}^{\theta} di \right)^{\alpha/\theta} L_{Yt}^{1-\alpha} - w_t L_{Yt} - \int_0^{A_t} p_{it} x_{it} di \tag{6}$$

FOCs:

$$p_{it} = \alpha x_{it}^{\theta-1} \left(\int_0^{A_t} x_{it}^{\theta} di \right)^{\alpha/\theta-1} L_{Yt}^{1-\alpha} \tag{7}$$

$$w_t = (1-\alpha) Y_t / L_{Yt} \tag{8}$$

2. Intermediate goods producer:

$$max_{p_{it}} x(p_{it}) (p_{it} - r_t) \tag{9}$$

Given the constant price elasticity of demand, the optimal price is given by the standard monopoly pricing rule

$$p_{it} = \frac{r_t}{\theta} \tag{10}$$

3. Innovators:

$$\max P_{At} \bar{\nu}_t L_{At} - w_t L_{At} \tag{11}$$

FOC:

$$P_{At} = w_t / \bar{\nu}_t \tag{12}$$

4. Free entry: This is standard asset pricing. The return on owning a patent is given by the current profit flow plus the capital gain. Under free entry, the return must equal the interest rate.

5. Balanced growth rate: We can derive the balanced growth rate from feasibility alone. The reason is that, due to diminishing returns in innovation, the growth rate is exogenous:

$$g(A) = \frac{\lambda n}{1-\phi} \tag{13}$$

6. Symmetric equilibrium: The resource constraint on capital implies $x_t = K_t/A_t$ for every variety i. Substitute into the production function to obtain

$$Y_t = A_t^{\alpha/\theta} (K_t/A_t)^{\alpha} L_{Yt}^{1-\alpha} = A_t^{\alpha \frac{1-\theta}{\theta}} K_t^{\alpha} L_{Yt}^{1-\alpha} \tag{14}$$

so that $g(Y) = \alpha \frac{1-\theta}{\theta} g(A) + \alpha g(Y) + (1-\alpha) n$. Now we have $g(c) = g(Y) - n = \frac{(1-\tau)r-\rho}{\sigma}$. Changing the tax has no effect on growth or the after tax interest rate. This is the standard result with exogenous growth and infinitely lived agents.

End of exam.